

# Theoretical-heuristic derivation Sommerfeld's fine structure constant by Feigenbaum's constant ( $\delta$ ): periodic logistic maps of double bifurcation

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## Abstract

In an article recently published in Vixra: <http://vixra.org/abs/1704.0365>. Its author (Mario Hieb) conjectured the possible relationship of Feigenbaum's constant  $\delta$  with the fine-structure constant of electromagnetism (Sommerfeld's Fine-Structure Constant). In this article it demonstrated, that indeed, there is an unequivocal physical-mathematical relationship. The logistic map of double bifurcation is a physical image of the random process of the creation-annihilation of virtual pairs lepton-antilepton with electric charge; Using virtual photons. The probability of emission or absorption of a photon by an electron is precisely the fine structure constant for zero momentum, that is to say: Sommerfeld's Fine-Structure Constant. This probability is coded as the surface of a sphere, or equivalently: four times the surface of a circle. The original, conjectured calculation of Mario Hieb is corrected or improved by the contribution of the entropies of the virtual pairs of leptons with electric charge: muon, tau and electron. Including a correction factor due to the contributions of virtual bosons W and Z; And its decay in electrically charged leptons and quarks.

## Introduction

The main geometric-mathematical characteristics of the logistic map of double universal bifurcation, which determines the two Feigenbaum's constants; they are: 1) The first Feigenbaum constant is the limiting ratio of each bifurcation interval to the next between every period doubling, of a one-parameter map. 2) Fractals: In the case of the Mandelbrot set for complex quadratic polynomial the Feigenbaum constant *is the ratio between the diameters of successive circles* on the real axis in the complex plane.

Mathematically these two characteristics are translated as:

1)  $x_{i+1} = f(x_i)$

Where  $f(x)$  is a function parameterized by the bifurcation parameter  $a$ .  
It is given by the limit:

$$\delta = \lim_{n \rightarrow \infty} \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}}$$

Where  $a_n$  are discrete values of  $a$  at the  $n$ -th period doubling. According to (sequence A006890 in the OEIS), this number to 30 decimal places is  $\delta = 4.669201609102990671853203821578\dots$

2)  $f(z) = z^2 + c$

The Feigenbaum  $\delta$  constant is the ratio between the diameters of successive circles on the real axis in the complex plane.

n	Period = $2^n$	Bifurcation parameter ( $c_n$ )	Ratio = $\frac{c_{n-1} - c_{n-2}}{c_n - c_{n-1}}$
1	$2^1$	-0.75	
2	$2^2$	-1.25	
3	$2^3$	-1.3680989	
4	$2^4$	-1.3940462	4.2337
5	$2^5$	-1.3996312	4.5515
6	$2^6$	-1.4008287	4.6458
7	$2^7$	-1.4010853	4.6639
8	$2^8$	-1.4011402	4.6682
9	$2^9$	-1.401151982029	4.6689
10	$2^{10}$	-1.401154502237	
$\infty$		-1.4011551890...	

Bifurcation parameter is a root point of period-  $2^n$  component. This series converges to the Feigenbaum point  $c = -1.401155\dots$ . The ratio in the last column converges to the first Feigenbaum constant.

Other maps also reproduce this ratio, in this sense the Feigenbaum constant in bifurcation theory is analogous to  $\pi$  in geometry and  $e$  in calculus.

The second Feigenbaum constant (sequence A006891 in the OEIS),  $\alpha = 2.502907875095892822283902873218\dots$ , is the ratio between the width of a tine and the width of one of its two subtines (except the tine closest to the fold). A negative sign is applied to  $\alpha$  when the ratio between the lower subtine and the width of the tine is measured.[]

These numbers apply to a large class of dynamical systems.

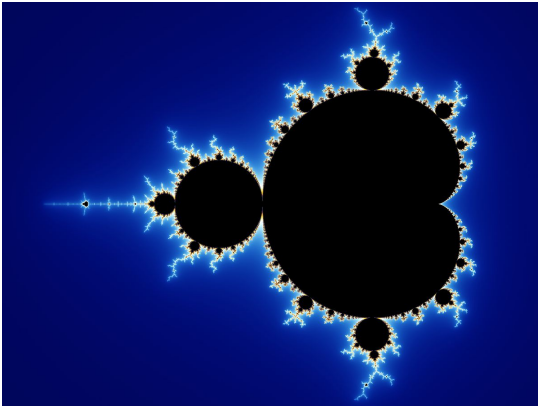


Figure 0.1:

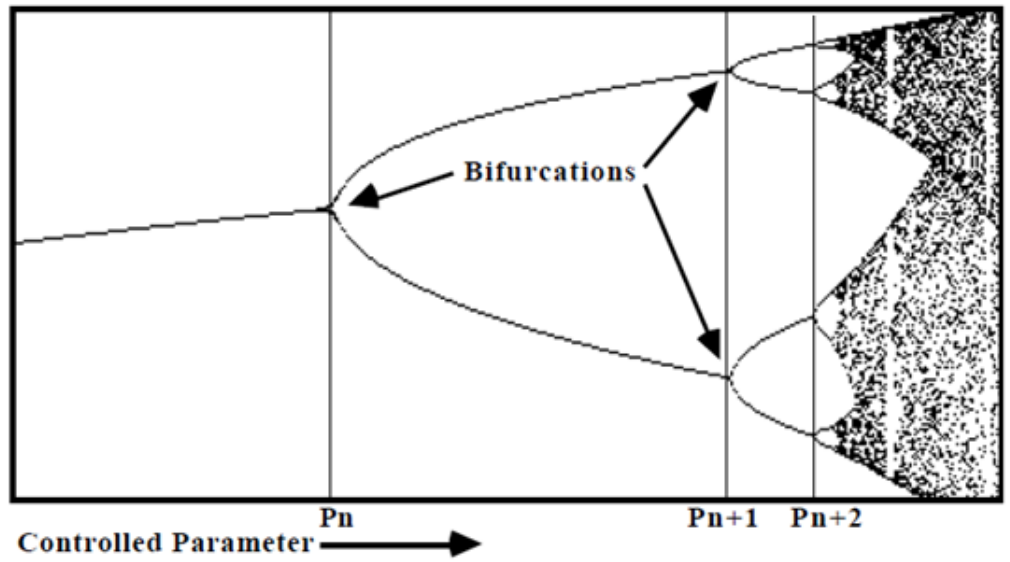


Figure 0.2:

# 1 Physical equivalence of the double bifurcation logistic map and the creation-annihilation of virtual pairs of lepton-antilepton produced by virtual photons.

A virtual photon can produce a pair of lepton-antilepton. In turn, this virtual pair can be annihilated by producing again a virtual photon. According to quantum electrodynamics, the number of virtual photons is infinite; As well as the range of wavelengths, frequency; And therefore the spectrum of energies. Graphically this scheme corresponds exactly to a branching of double arborescent bifurcation. This infinite and fractal ramification is characterized by the successive ratios of the diameters of the circles defined by the function of the Mandelbrot set. Quadratic polynomial function in the complex plane.

*This ratio between the diameters of successive circles converges to the Feigenbaum's constant.*

That is, the Feigenbaum's constant can be considered as the constant dimensionless diameter of a wavelength. Since electromagnetic waves or photons propagate with spherical symmetry in a three-dimensional space; Then immediately the surface of this sphere encodes the number of photons. This implies that the probability of emission or absorption of a photon by a pair of electric charges for virtual particles; Is precisely the fine structure constant for zero momentum or Sommerfeld's constant (virtual quantum vacuum). Or strictly: the inverse of the dimensionless spherical surface (number of photons).

Graphically we have the following diagram, with the pairs of lepton-antilepton (electron, muon and tau), the photon; Plus the internal contributions of the decay of the electroweak bosons W and Z:

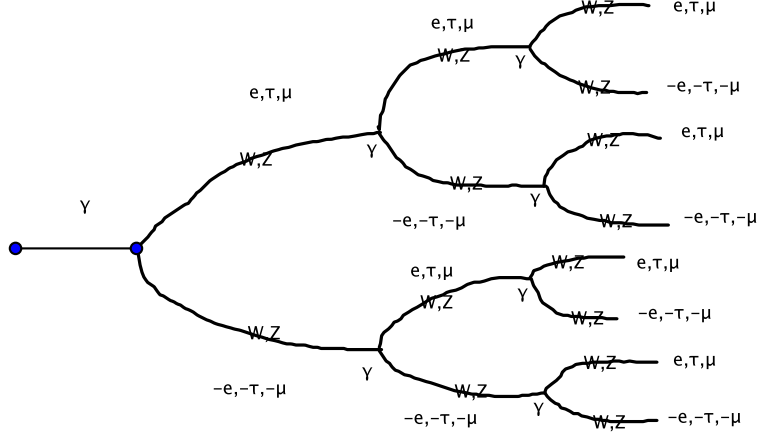


Figure 1.1:

## 2 The role of photon spin as a vector parameter of the Feigenbaum's constant.

As is well known the spin cone is defined by the angle defined by the square root of the spin module and the spin. For the photon the cosine of this angle is expressed by:

$$\cos \theta(s = 1) = \frac{s}{\sqrt{(s+1) \cdot s}} = \frac{1}{\sqrt{(1+1) \cdot 1}} = \frac{1}{\sqrt{2}}$$

It is logical to theorize that the Feigenbaum constant; Which represents the dimensionless radius (diameter for particle-antiparticle pairs) photonic is modulated by the cosine of the spin of the photon. This is:

$$R_\gamma = \delta \cdot \cos \theta(s = 1) = (4.66920160910299 \dots) / \sqrt{2} = 3.30162412052386$$

Thus; And adopting that the number of photons is the surface of the sphere defined by the previous photonic radius, by the modulation of the Feigenbaum's constant; We have a first approximation to the inverse of the fine structure constant for zero momentum (virtual quantum vacuum), expressed by:

$$\alpha_1^{-1}(0) = [\delta \cdot \cos \theta(s = 1)]^2 \cdot 4\pi = 136.982510520341 \quad (1)$$

The real value of this constant, given by the known equation:

$$\alpha^{-1}(0) = \frac{\hbar c \cdot 4\pi \cdot \epsilon_0}{(\pm e)^2}$$

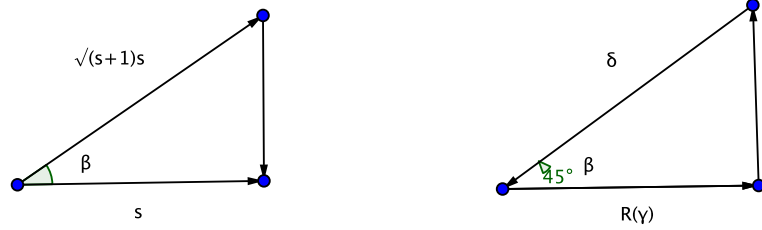


Figure 2.1:

In this work the best known experimental value will be adopted:  $\alpha^{-1}(0) = 137.035999173$

### 3 The contributions of lepton entropies with electric charge.

Two classes of entropy will be defined: 1) the entropy of equiprobable states; Which will depend on the natural logarithm between the mass of the lepton and the mass of the electron. This entropy establishes the contribution for lepton-antilepton pairs globally; Or of the whole arborescent fractal structure that determines the Feigenbaum's constant. Thus, for lepton-antilepton pairs the following entropies will have; As defined above:

$$2 \cdot \ln(m_e/m_e) ; 2 \cdot \ln(m_\tau/m_e) ; 2 \cdot \ln(m_\mu/m_e)$$

$$\frac{m_\mu}{m_e} = 206.7682826 ; \frac{m_\tau}{m_e} = 3477.1507634$$

The previous entropy will be sumatory, with a multiplicative contribution of the entropy of the double bifurcation (particle-antiparticle pairs); That is:  $\ln(2)$ .

In this way; The total entropy of equiprobable states of lepton-antilepton pairs are expressed by the following equation:

$$H_1 = \ln 2 \cdot [2 \cdot \ln(m_e/m_e) + 2 \cdot \ln(m_\mu/m_e) + 2 \cdot \ln(m_\tau/m_e)] = 18.6949658348932 \quad (2)$$

The second entropy to consider; Equally dependent on the ratios of the lepton / electron masses, will be that of non-equiprobable states for independent events and acting locally in each branch of the double bifurcation:

$$\frac{\ln(m_e/m_e)}{\left(\frac{m_e}{m_e}\right) \cdot \left(\frac{m_\mu}{m_e}\right) \cdot \left(\frac{m_\tau}{m_e}\right)} = 0; \quad \frac{\ln(m_\mu/m_e)}{\left(\frac{m_e}{m_e}\right) \cdot \left(\frac{m_\mu}{m_e}\right) \cdot \left(\frac{m_\tau}{m_e}\right)} = \frac{5.33159875853111}{718964.491689497}$$

$$\frac{\ln(m_\tau/m_e)}{\left(\frac{m_e}{m_e}\right) \cdot \left(\frac{m_\mu}{m_e}\right) \cdot \left(\frac{m_\tau}{m_e}\right)} = \frac{8.15396849126341}{718964.491689497}$$

The previous entropies will give a minimum joint entropy; Which will be the subtraction expressed by:

$$\frac{\ln(m_\tau/m_e)}{\left(\frac{m_e}{m_e}\right) \cdot \left(\frac{m_\mu}{m_e}\right) \cdot \left(\frac{m_\tau}{m_e}\right)} - \frac{\ln(m_\mu/m_e)}{\left(\frac{m_e}{m_e}\right) \cdot \left(\frac{m_\mu}{m_e}\right) \cdot \left(\frac{m_\tau}{m_e}\right)} - \frac{\ln(m_e/m_e)}{\left(\frac{m_e}{m_e}\right) \cdot \left(\frac{m_\mu}{m_e}\right) \cdot \left(\frac{m_\tau}{m_e}\right)} =$$

$$\frac{\ln(m_\tau/m_\mu)}{\left(\frac{m_e}{m_e}\right) \cdot \left(\frac{m_\mu}{m_e}\right) \cdot \left(\frac{m_\tau}{m_e}\right)}$$

Finally, the previous entropy will contain a correcting factor due to the contributions of the decays of the W and Z bosons; Namely, as the sum of the branching ratios for leptons and quarks. This sum can be expressed as three times the cosine square of the electroweak Weinberg angle:  $\cos = M_w / M_z$ .

Hence, this second entropy is expressed by the following equation:

$$H_2 = \frac{\ln(m_\tau/m_\mu)}{3 \cdot \cos^2 \theta_W \cdot \left(\frac{m_e}{m_e}\right) \cdot \left(\frac{m_\mu}{m_e}\right) \cdot \left(\frac{m_\tau}{m_e}\right)} = \frac{2.8223697327323}{2.33125032922288 \cdot 718964.491689497} \quad (3)$$

$$\frac{m_W^2}{m_Z^2} = \frac{(80.384 \text{ GeV})^2}{(91.1876 \text{ GeV})^2} = \cos^2 \theta_W; \quad \frac{\delta}{2} \simeq 3 \cdot \cos^2 \theta_W$$

Once the entropic contributions have been calculated; The final result can be obtained for the inverse of the Sommerfeld fine structure constant:

$$\alpha^{-1}(0) = [\delta \cdot \cos \theta(s=1)]^2 \cdot 4\pi + \frac{1}{H_1} - H_2 = 137.035999172221$$

*I thank Almighty God, creator of all things (visible and not visible) for allowing me to know these wonders. Thanks also, to our only Lord and Savior: Jesus the Christ, the Son of the living God.*