A new quantum algorithm in case of a special function (new version)

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We present a new quantum algorithm. It determines the property of a function. It is $f(x) = f(-x)$. How fast can we succeed? The quantum algorithm does not use the Hadamard transformation. All we need is of evaluating $|\overbrace{0, 0, ..., 1}^{N}\rangle$. And we can know the global property, that is, we can realize $f(x) = f(-x)$ for numbers. Our quantum algorithm overcomes a classical counterpart by a factor of $O(2^N)$.

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I. INTRODUCTION

The quantum theory (cf. [1–6]) gives approximate and at times remarkably accurate numerical predictions. Much experimental data approximately fits to the quantum predictions for the past some 100 years. We do not doubt the correctness of the quantum theory. The quantum theory, in these days, keeps saying modern science with respect to information theory, where the science is called the quantum information theory [6]. Therefore, the quantum theory gives us another very useful theory in order to create new information science and to explain the handling of raw experimental data in our physical world.

As for foundations of the quantum theory, Leggett-type non-local variables theory [7] is experimentally investigated [8–10]. The experiments report that the quantum theory does not accept Leggett-type non-local variables interpretation. However, there are debates for the conclusions of the experiments. See Refs. [11–13].

Meanwhile, as for applications of the quantum theory, implementation of a quantum algorithm to solve Deutsch's problem [14–16] on a nuclear magnetic resonance quantum computer is reported firstly [17]. An implementation of the Deutsch-Jozsa algorithm on an ion-trap quantum computer is also reported [18]. There are several attempts to use single-photon two-qubit states for quantum computing. Oliveira *et al.* implement Deutsch's algorithm with polarization and transverse spatial modes of the electromagnetic field as qubits [19]. Single-photon Bell states are prepared and measured [20]. Also the decoherence-free implementation of Deutsch's algorithm is reported by using such single-photon and by using two logical qubits [21]. More recently, a one-way based experimental implementation of Deutsch's algorithm is reported [22]. In 1993, the Bernstein-Vazirani algorithm was reported [23, 24]. It can be considered as an extended Deutsch-Jozsa algorithm. In 1994, Simon's algorithm was reported [25]. Implementation of a quantum algorithm to solve the Bernstein-Vazirani parity problem without entanglement on an ensemble quantum computer is reported [26]. Fiber-optics implementation of the Deutsch-Jozsa and Bernstein-Vazirani quantum algorithms with three qubits is discussed [27]. Quantum learning robust against noise is studied [28]. A quantum algorithm for approximating the influences of Boolean functions and its applications is recently reported [29]. Quantum computation with coherent spin states and the close Hadamard problem is also discussed [30]. Transport implementation of the Bernstein-Vazirani algorithm with ion qubits is more recently reported [31]. Quantum Gauss-Jordan elimination and simulation of accounting principles on quantum computers are discussed [32]. Finally, we mention that dynamical analysis of Grover's search algorithm in arbitrarily high-dimensional search spaces is studied [33].

On the other hand, the earliest quantum algorithm, the Deutsch-Jozsa algorithm, is representative to show that quantum computation is faster than classical counterpart with a magnitude that grows exponentially with the number of qubits. In 2015, it was discussed that the Deutsch-Jozsa algorithm can be used for quantum key distribution [34]. In 2017, it was discussed that secure quantum key distribution based on Deutsch's algorithm using an entangled state [35].

In this paper, we present a new quantum algorithm. It determines the property of a function. It is $f(x) = f(-x)$. How fast can we succeed? The quantum algorithm does not use the Hadamard transformation. All we need is of evaluating $|\overbrace{0,0,...,1}^{N}\rangle$. And we can know the global property, that is, we can realize $f(x) = f(-x)$ for numbers. Our quantum algorithm overcomes a classical counterpart by a factor of $O(2^N)$.

II. A NEW QUANTUM ALGORITHM IN CASE OF A SPECIAL FUNCTION

Suppose

$$\begin{aligned} f &: \{-(2^N-1), -(2^N-2), .., 2^N-2, 2^N-1\} \\ &\rightarrow \{-(2^N-1), -(2^N-2), .., 2^N-2, 2^N-1\}. \end{aligned} \quad (1)$$

is a function. The goal is of determining $f(-x) = f(x)$.

We introduce a function with $g(x)$ the transformation from binary to natural representation. We define $g^{-1}(f(g(x))) = F(x)$. We assume

$$\begin{aligned} &F(x) = F(-x) \in \{0,1\}^N, \\ &x \in \{0,1\}^N, \end{aligned} \quad (2)$$

and check if the condition (2) is true in the following.

In (1) we define a function f from a set of discrete values to the same set. The argument of (1) appears to be from a number to a number. In (2) we assume that x is a binary representation of such a number. The write-up suggests that x is a binary, i.e., 0,1 vector, a Cartesian product $\overbrace{\{0,1\} \times \{0,1\} \times \ldots \times \{0,1\}}^{N}$, instance, $x = (0,1,1,0,0,1,\ldots,1)$. And we define $-x = -(0,1,1,0,0,1,\ldots,1)$.

Our discussion combines quantum parallelism with a property of quantum mechanics known as interference.

Let us follow the quantum states through the main algorithm. Throughout the paper, we omit the normalization factor. We define $|-x\rangle = -|x\rangle$. The input state is

$$\begin{aligned} |\psi_1\rangle &= |\overbrace{0,0,...,1}^{N}\rangle|\overbrace{1,1,...,1}^{N}\rangle \\ &= \sum_{y=-(2^N-1)}^{-2} |x\rangle|\overbrace{1,1,...,1}^{N}\rangle + \sum_{y=+1}^{2^N-1} |x\rangle|\overbrace{1,1,...,1}^{N}\rangle. \end{aligned} \quad (3)$$

Now $y = g^{-1}(x)$ is a natural representation of binary x. For example, $y = 3$ implies $x = 11$.

Next, the function F is evaluated using

$$U_F : |x,z\rangle \rightarrow |x, z + F(x)\rangle, \quad (4)$$

and

$$\begin{aligned} U_F : \quad & |x,z\rangle \rightarrow |x, z+F(x)\rangle \\ &\Leftrightarrow -|x,z\rangle \rightarrow -|x, z+F(x)\rangle \\ &\Leftrightarrow |-x,z\rangle \rightarrow |-x, z+F(x)\rangle \\ &\Leftrightarrow |-x,z\rangle \rightarrow |-x, z+F(-x)\rangle, \end{aligned} \quad (5)$$

by using $F(x) = F(-x)$. We employ z. It is a binary. Here, $z+F(x) = (z_1 \oplus F_1(x), z_2 \oplus F_2(x), \ldots, z_N \oplus F_N(x))$. The symbol $\oplus$ indicates addition modulo 2.

Let us start from the following:

$$\begin{aligned} &U_F|\psi_1\rangle = |\psi_2\rangle \\ &= \sum_{y=-(2^N-1)}^{-2} |x\rangle|\overline{F(x)}\rangle + \sum_{y=+1}^{2^N-1} |x\rangle|\overline{F(x)}\rangle. \end{aligned} \quad (6)$$

Here, for example, if $F(0,0,...,1) = (0,1,1,0,0,1,\ldots,1)$, then $\overline{F(0,0,...,1)} = (1,0,0,1,1,0,\ldots,0)$. Hence we have

$$\begin{aligned} |\psi_2\rangle &= \sum_{y=-(2^N-1)}^{-2} |x\rangle|\overline{F(x)}\rangle + \sum_{y=+1}^{2^N-1} |x\rangle|\overline{F(x)}\rangle \\ &= \sum_{y=+2}^{2^N-1} |-x\rangle|\overline{F(-x)}\rangle + \sum_{y=+1}^{2^N-1} |x\rangle|\overline{F(x)}\rangle \\ &= \sum_{y=+2}^{2^N-1} |-x\rangle|\overline{F(x)}\rangle + \sum_{y=+1}^{2^N-1} |x\rangle|\overline{F(x)}\rangle \\ &= |0,0,...,1\rangle|\overline{F(0,0,...,1)}\rangle, \end{aligned} \quad (7)$$

by using $\overline{F(x)} = \overline{F(-x)}$.

Finally, we measure $|\psi_2\rangle$. If the result of a measurement on the first qubit is 1, and other results of measurements are zero (that is, we measure $|0,0,...,1\rangle$), we can determine the property of the function

$$\overline{f(x)} = \overline{f(-x)}. \quad (8)$$

That is,

$$f(x) = f(-x). \tag{9}$$

In this case, we are in success of the main algorithm. In fact, successful quantum algorithms are determining the property of a certain function [36].

Otherwise, (for example, the result of a measurement on the second qubit is +1) the function is $f(x) \neq f(-x)$. In this case, we are not in success of the main algorithm. Namely, we cannot have the following fact;

$$U_F|\overbrace{0,0,...,1}^{N}\rangle|\overbrace{1,1,...,1}^{N}\rangle = |\overbrace{0,0,...,1}^{N}\rangle|\overline{F(0,0,...,1)}\rangle. \tag{10}$$

The condition $F(x) = F(-x)$ is necessary for the condition (10) when $F(x) \neq 0$ [36].

The quantum algorithm does not use the Hadamard transformation. All we need is of evaluating $|\overbrace{0,0,...,1}^{N}\rangle$. And we can determine the global property, that is, we can realize $f(x) = f(-x)$ for numbers. Our quantum algorithm overcomes a classical counterpart by a factor of $O(2^N)$.

III. CONCLUSIONS

In conclusion, we have presented a new quantum algorithm. It has determined the property of a function. It has been $f(x) = f(-x)$. How fast can we succeed? The quantum algorithm does not have used the Hadamard transformation. All we need has been of evaluating $|\overbrace{0,0,...,1}^{N}\rangle$. And we can have known the global property, that is, we can realize $f(x) = f(-x)$ for numbers. Our quantum algorithm has overcome a classical counterpart by a factor of $O(2^N)$.

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