On the Complex Function Basis of Maxwell Equations

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Abstract

In this paper, we propose a concept of vector complex function to prove that the whole world can be reduced to a very simple function f(Z) = F + iG by introducing the knowledge of complex function theories. We can also derive Maxwell equations through the differential and integral analysis of the vector complex function.

Key words: Complex function; Virtual space-time; Maxwell equations

0 Introductions

If our space-time can be divided into real and virtual space-times, then we can construct the very symmetrical Maxwell equations. [1,2] On this basis. We can obtain a new interpretation of wave function in quantum theories [3], and give another explanation of gravitation [4].

The complex function theory is used to deal with functions with imaginary numbers. In the existing complex function theories, the real and imaginary parts of the function are scalar, but the world we live in is three-dimensional. Therefore, if we introduce a vector complex function that real and imaginary parts are three-dimensional, then we can now deal with the three-dimensional space-time problems based on complex function theories.

In this paper, we prove that the Maxwell equations can be further simplified into a very simple function f (Z) = F + iG by introducing the knowledge of complex function.

1 Vector Complex and Hyper-symmetric Space-time

1.1 Mathematical Analysis of Vector Complex Numbers

The vector complex number is a kind of complex number, which contains the real part and the imaginary part. However, unlike the ordinary complex number, there is at least one of the real or imaginary part of the vector complex is a vector.
Some laws of complex functions can be extended to vector complex numbers.

Common vector complex structure

$$Z = X + iY \qquad (1)$$

Here we use the uppercase letters to represent vectors.

1. The four operations of the vector complex

Addition

$$Z = Z_1 + Z_2 = (X_1 + X_2) + i(Y_1 + Y_2) \qquad (2)$$

Subtraction

$$Z = Z_1 - Z_2 = (X_1 - X_2) + i(Y_1 - Y_2) \qquad (3)$$

Multiplication

Since there are two different multiplications existed in vector operation, there will be also two different multiplication methods existed in vector multiplication operations.

Dot product

$$Z = Z_1 \cdot Z_2 = (X_1 \cdot X_2 - Y_1 \cdot Y_2) + i(X_2 \cdot Y_1 + X_1 \cdot Y_2) \qquad (4)$$

Dot product has nothing to do with the sequence of the two vectors.

Cross product

$$Z = Z_1 \times Z_2 = (X_1 \times X_2 - Y_1 \times Y_2) + i(Y_1 \times X_2 + X_1 \times Y_2) \qquad (5)$$

Division

$$Z=\frac{Z_1}{Z_2}=\frac{(X_1\cdot X_2+Y_1\cdot Y_2)+i(X_2Y_1-X_1Y_2)}{X_2^2+Y_2^2} \qquad (6)$$

1.2 The Function Representation of Hyper-symmetric Space-time

There are two different spaces existed in hyper-symmetric space-time, which are three dimensional real space X and three dimensional virtual space Y. All of these two spaces can be measured by vectors. The physical quantities that generated in these two space-time are the functions of these two vectors.

We use complex Z in formula (1) to represent the scale of two space-times. So we have

$$f(Z)=F(Z)+iG(Z)\equiv F+iG \qquad (7)$$

Here we use the general electric and magnetic field strength defined in paper [1] and [2]. That is

$$F=\sqrt{\varepsilon}E$$

$$G=\sqrt{\mu}H$$

2 Derivation of Maxwell 's Equations

2.1 Derivation of Source-free Maxwell's equations Maxwell Equations

Here are the commonly used differential operators

$$\partial_u=\nabla-i\hat{y}\frac{\partial}{\partial y} \qquad (8)$$

Here $y=ct$, represents the one dimensional time.

We assume that the relationships below are correct by considering the special relationships between one dimensional time and three-dimensional space.

$$\hat{y}\times F=F$$

$$\hat{y} \times G = G$$

$$\hat{y} \cdot F = 0$$

$$\hat{y} \cdot G = 0$$

Why we can make this assumption is because that this relationship is due to the fact that the one-dimensional time dimension and the three-dimensional spatial dimension are two independent dimensions. Which should have a multiplication method is consistent with the known physical laws (that is, a physical quantity can be derived by time). It is assumed that this consistency is manifested as a cross product. And the result is zero for dot product.

So for the function

$$f(Z) = \mathrm{F} + iG \qquad (9)$$

We use cross product at the first

$$\partial_u \times \mathrm{f(Z)} = \left(\nabla \times \mathrm{F} + \frac{\partial G}{\partial y}\right) + \mathrm{i}\left(\nabla \times \mathrm{G} - \frac{\partial F}{\partial y}\right) \qquad (10)$$

If $\partial_u \times$ f(Z)=0, the electric and magnetic fields can reach at some stable status.

Then we have

$$\nabla \times \mathrm{F} = -\frac{\partial G}{\partial y}$$

$$\nabla \times \mathrm{G} = \frac{\partial F}{\partial y}$$

As for dot product derivation

$$\partial_u \cdot \mathrm{f(Z)} = \left(\nabla \cdot \mathrm{F} + \hat{y}\frac{\partial}{\partial y} \cdot \mathrm{G}\right) + \mathrm{i}\left(\nabla \cdot \mathrm{G} - \hat{y}\frac{\partial}{\partial y} \cdot F\right) = 0 \qquad (11)$$

The conditions for $\partial_u \cdot \mathrm{f(Z)} = 0$

$$\nabla \cdot \mathrm{F} = 0$$

And

$$\nabla \cdot \mathrm{G} = 0$$

The above results are the source-free Maxwell’s equations.

2.2 Integration of functions

First, we will extend the Cauchy's integral to the surface integral.
In the complex function, the Cauchy integral forms are as follows

$$\oint f(z)dz = 0$$

Now we extend it to the three dimension space. If there are no singular point (source free) in any closed surface in three dimensional space, then we have

$$\oiint f(Z) \cdot dS = 0 \qquad (12)$$

To show formula (12), we can use

$$\oiint f(Z) \cdot dS = \oiint F \cdot dS + i \oiint G \cdot dS$$

We can obtain formula (12) after apply Gauss's law to the real and imaginary parts respectively.

The formula that response to the Residue Theorem is

$$\oiint f(Z) \cdot dS = 2\pi i \sum Resf(Z，a_k) \qquad (13)$$

Here we can the relationships between real and virtual space [4], we have $\mathrm{X} = 1/\mathrm{Y}$

The result of formula (13) will be zero for the conditions of source free.

However, G has no singularity in limited X, while F has singularity in X=0.

So we have

$$\oiint G \cdot dS = 0 \qquad (14)$$

$$\oiint F \cdot dS = Q \qquad (15)$$

On the other hand, F has no singularity in limited Y, while G has singularity in Y=0.

After we integral formula（10）and（11）, we can get

$$\iint [\partial_u \times \mathrm{f(Z)}] \cdot dS = \iint \left[(\nabla \times \mathrm{F} + \frac{\partial G}{\partial y}) + \mathrm{i}(\nabla \times \mathrm{G} - \frac{\partial F}{\partial y})\right] \cdot dS = C$$

Here, C is a constant.

1. if C = 0, then we can use Stokes' theorem to obtain

$\iint(\nabla\times \mathrm{F})\cdot dS \equiv \oint F\cdot dl = -\frac{\partial}{\partial y}\iint G\cdot dS$ （16）

The equation from imaginary parts is

$$\oint \mathrm{G}\cdot \mathrm{dl} = \frac{\partial}{\partial y}\iint F\cdot dS$$

Assume

$$\Sigma = \Sigma_1 + \Sigma_2$$

Represents a closed surface which include a singularity X=0.

Then we have

$$\iint_{\Sigma 1} F\cdot dS = \oiint_{\Sigma} F\cdot dS - \iint_{\Sigma 2} F\cdot dS$$

Or

$$\iint_{\Sigma 2} F\cdot dS = \oiint_{\Sigma} F\cdot dS - \iint_{\Sigma 1} F\cdot dS$$

While

$$\iint_{\Sigma 1} (\nabla\times \mathrm{G})\cdot dS = -\iint_{\Sigma 2} (\nabla\times \mathrm{G})\cdot dS$$

So

$$\oint \mathrm{G}\cdot \mathrm{dl} = \frac{\partial}{\partial y}\iint_{\Sigma 1} F\cdot dS = -\frac{\partial}{\partial y}\iint_{\Sigma 2} F\cdot dS = -\frac{\partial}{\partial y}\oiint_{\Sigma} F\cdot dS + \frac{\partial}{\partial y}\iint_{\Sigma 1} F\cdot dS$$

By combining with formula（15）, we have

$\oint \mathrm{G}\cdot \mathrm{dl} = -\frac{\partial}{\partial y}Q + \frac{\partial}{\partial y}\iint_{\Sigma 1} F\cdot dS = I + \frac{\partial}{\partial y}\iint_{\Sigma 1} F\cdot dS$ （17）

Here I is the general current intensity that is correspondence to the general current density that was defined in paper [1][2].

And

$$\mathrm{I} + \frac{\partial}{\partial y}Q = 0$$

So we can see that formulas （14）~（17） are the Maxwell's Equations in integral form.

2. if $C \neq 0$, it will be

$$\oint F \cdot dl = Re(C) - \frac{\partial}{\partial y} \iint G \cdot dS$$

It seems that even there is no change of G, the curl of F will always equal to a constant. It has no clearly physical meaning. So we don't consider this conditions.

3 Conclusions

From the above analysis, we can see that the use of the vector complex form of three dimensional spaces, combined with the theoretical knowledge of complex function, we can use a very simple form of function to represent the physical laws of the nature. It means that the representation of the natural principles can be obtained in a more concise form.

Because the form of expression becomes briefer, it helps us to better see the hidden laws behind the phenomena. Perhaps it can also help us to further solve the problems including where is the origin of the charge according to this in-depth exploration,.

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Chinese Version

麦克斯韦方程组的复变函数基础

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摘要：

本文通过引入复变函数的知识，提出一种矢量复数的概念，以此证明整个世界可以简化成一个非常简单的函数 f(Z)=F+iG，通过对该函数的微分和积分分析，可以推导出麦克斯韦方程组。

关键词：复变函数；虚时空；麦克斯韦方程组

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Abstract

In this paper, we propose a concept of vector complex function to prove that the whole world can be reduced to a very simple function f(Z) = F + iG by introducing the knowledge of complex function theories. We can also derive Maxwell equations through the differential and integral analysis of the vector complex function.

Key words: Complex function; Virtual space-time; Maxwell equations

0 引言

如果我们现在所处的时空可以被划分成虚实两个时空，则在此基础上可以构造出对称性非常好的麦克斯韦方程组[1, 2]，并能够用来实现对波函数的一种全新的诠释[3]，也可以给出万有引力的另一种解释[4]。

复变函数理论用来处理带有虚数的函数。已有的复变函数理论中，函数的实部和虚部都是标

量，然而我们所处的世界则是三维的。因此如果我们引入一个实部和虚部都是三维矢量的复变函数，则相信可以用来处理我们现在所处的三维时空问题。

本文通过引入复变函数的知识，证明麦克斯韦方程组可以进一步简化成一个非常简单的函数 f(Z)=F+iG.

1 矢量复数与超对称时空

1.1 矢量复数的数学分析

矢量复数本身也是复数，包含了实部和虚部两个部分。但是与普通的复数不同，矢量复数的实部或者虚部中，至少有一个是矢量。

复变函数的一些规律可以推广到矢量复数上来。

常见的矢量复数结构：

$$\mathrm{Z} = \mathrm{X} + \mathrm{iY} \qquad （1）$$

这里使用大写字母表示矢量。

1．矢量复数的四则运算

加法：

$$\mathrm{Z} = Z_1 + Z_2 = (\mathrm{X}_1 + X_2) + \mathrm{i}(\mathrm{Y}_1 + Y_2) \qquad （2）$$

减法：

$$\mathrm{Z} = Z_1 - Z_2 = (\mathrm{X}_1 - X_2) + \mathrm{i}(\mathrm{Y}_1 - Y_2) \qquad （3）$$

乘法：

由于矢量存在点乘和叉乘的区别，因此，矢量复数也存在点乘和叉乘结果的差异。

点乘：

$$\mathrm{Z} = Z_1 \cdot Z_2 = (\mathrm{X}_1 \cdot X_2 - \mathrm{Y}_1 \cdot Y_2) + \mathrm{i}(X_2 \cdot \mathrm{Y}_1 + X_1 \cdot Y_2) \qquad （4）$$

点乘的结果与两个矢量复数的先后顺序无关。

叉乘：

$$\mathrm{Z} = Z_1 \times Z_2 = (\mathrm{X}_1 \times X_2 - \mathrm{Y}_1 \times Y_2) + \mathrm{i}(\mathrm{Y}_1 \times X_2 + X_1 \times Y_2) \qquad （5）$$

除法：

$$\mathrm{Z} = \frac{Z_1}{Z_2} = \frac{(\mathrm{X}_1 \cdot X_2 + \mathrm{Y}_1 \cdot Y_2) + \mathrm{i}(X_2 \mathrm{Y}_1 - X_1 Y_2)}{X_2^2 + Y_2^2} \qquad （6）$$

1.2 超对称时空的函数表示

超对称时空具备两个不同的时空。三维实空间 X 以及三维虚空间 Y. 这两个三维空间中的尺度都可以用矢量来表示。而在这两个时空上面产生的物理量，则是这两个矢量的函数。

为了处理方便，使用矢量复数 Z 来表示两个空间的尺度，比直接使用六维空间进行计算要简化的多。这样我们就可以将所有物理量表示为 Z 的函数：

$$\mathrm{f(Z)} = \mathrm{F(Z)} + iG(Z) \equiv \mathrm{F} + iG \qquad (7)$$

这里使用文献【1】【2】所采用的广义电场和磁场强度的表示方法。F 表示广义电场强度，G 表示广义磁场强度。即：

$$\mathrm{F} = \sqrt{\varepsilon}E$$

$$\mathrm{G} = \sqrt{\mu}H$$

2 麦克斯韦方程组的推导

2.1 无源麦克斯韦方程组的推导

这里使用常用的微分算符：

$$\partial_u = \nabla - i\hat{y}\frac{\partial}{\partial y} \qquad (8)$$

其中 y=ct，表示一维时间。

考虑到一维时空与三维时空的特殊性，假设下面的关系成立：

$$\hat{y} \times F = F$$

$$\hat{y} \times G = G$$

$$\hat{y} \cdot F = 0$$

$$\hat{y} \cdot G = 0$$

之所以假设存在这样的关系，是因为考虑到一维时间维度与三维空间维度是两个独立的维度。其中应该有一种乘法是与已知的物理规律一致的(即一个物理量可以对时间进行求导运算)。

这里假设这种一致性体现为叉乘。而点乘结果则为零。

则对于函数：

$f(Z) = \mathrm{F} + iG$ （9）

首先进行叉乘求导，可得：

$\partial_u \times \mathrm{f(Z)} = \left(\nabla \times \mathrm{F} + \frac{\partial G}{\partial y}\right) + \mathrm{i}\left(\nabla \times \mathrm{G} - \frac{\partial F}{\partial y}\right)$ （10）

如果$\partial_u \times \mathrm{f(Z)}$=0，这样电磁场可以达到某种稳定的状态。

则：

$$\nabla \times \mathrm{F} = -\frac{\partial G}{\partial y}$$

$$\nabla \times \mathrm{G} = \frac{\partial F}{\partial y}$$

对于点乘求导：

$\partial_u \cdot \mathrm{f(Z)} = \left(\nabla \cdot \mathrm{F} + \hat{y}\frac{\partial}{\partial y} \cdot \mathrm{G}\right) + \mathrm{i}\left(\nabla \cdot \mathrm{G} - \hat{y}\frac{\partial}{\partial y} \cdot F\right) = 0$ （11）

$\partial_u \cdot \mathrm{f(Z)}$为零的条件：

$$\nabla \cdot \mathrm{F} = 0$$

以及：

$$\nabla \cdot \mathrm{G} = 0$$

从上述结果可以看出这就是无源麦克斯韦方程组。

2.2 对函数进行积分

首先我们将柯西积分推广到面积分
在复变函数中，柯西积分的形式如下：

$$\oint f(z)dz = 0$$

现将其推广到三维情况，如果三维空间中的任意封闭曲面 S 内部没有奇点（无源），则：

$\oiint f(Z) \cdot dS = 0$ （12）

这可以通过

$$\oiint f(Z)\cdot dS=\oiint F\cdot dS+i\oiint G\cdot dS$$

对实部和虚部分别使用高斯定理就可以得到公式（12）

相对应的留数定理：

$$\oiint f(Z)\cdot dS=2\pi i\sum Resf(Z，a_k) \qquad （13）$$

这里考虑到虚实时空的特性[4]，$\mathrm{X}=1/\mathrm{Y}$

对于无源的情况，上述积分结果为零。

而在有限的 X 范围之内，G 没有奇点，在 X=0 时，F 有奇点。

则有如下关系：

$$\oiint G\cdot dS=0 \qquad （14）$$

$$\oiint F\cdot dS=Q \qquad （15）$$

在有限的 Y 范围之内，F 没有奇点，在 Y=0 时，G 有奇点。

分别对公式（10）和（11）求积分，可以得到：

$$\iint[\partial_u\times \mathrm{f(Z)}]\cdot dS=\iint\left[(\nabla\times \mathrm{F}+\frac{\partial G}{\partial y})+\mathrm{i}(\nabla\times \mathrm{G}-\frac{\partial F}{\partial y})\right]\cdot dS=C$$

其中 C 为常数。

1. 如果设：C = 0, 则使用斯托克斯公式，有：

$$\iint(\nabla\times \mathrm{F})\cdot dS\equiv\oint F\cdot dl=-\frac{\partial}{\partial y}\iint G\cdot dS \qquad （16）$$

另有：

$$\oint \mathrm{G}\cdot \mathrm{dl}=\frac{\partial}{\partial y}\iint F\cdot dS$$

设

$$\Sigma=\Sigma_1+\Sigma_2$$

表示一个封闭的曲面，其中包含了一个奇点 X=0

则：

$$\iint_{\Sigma 1} F \cdot dS = \oiint_{\Sigma} F \cdot dS - \iint_{\Sigma 2} F \cdot dS$$

或者：

$$\iint_{\Sigma 2} F \cdot dS = \oiint_{\Sigma} F \cdot dS - \iint_{\Sigma 1} F \cdot dS$$

而：

$$\iint_{\Sigma 1} (\nabla \times \mathrm{G}) \cdot dS = -\iint_{\Sigma 2} (\nabla \times \mathrm{G}) \cdot dS$$

故：

$$\oint \mathrm{G} \cdot \mathrm{dl} = \frac{\partial}{\partial y} \iint_{\Sigma 1} F \cdot dS = -\frac{\partial}{\partial y} \iint_{\Sigma 2} F \cdot dS = -\frac{\partial}{\partial y} \oiint_{\Sigma} F \cdot dS + \frac{\partial}{\partial y} \iint_{\Sigma 1} F \cdot dS$$

结合公式（15），有：

$$\oint \mathrm{G} \cdot \mathrm{dl} = -\frac{\partial}{\partial y} Q + \frac{\partial}{\partial y} \iint_{\Sigma 1} F \cdot dS = I + \frac{\partial}{\partial y} \iint_{\Sigma 1} F \cdot dS \qquad (17)$$

这里 I 为与广义电流密度[1]J 相关的广义电流强度，并使用了电流连续性方程：

$$\mathrm{I} + \frac{\partial}{\partial y} Q = 0$$

可以看出，公式（14）~（17）就是三维空间中的积分形式的麦克斯韦方程组。

2. 如果$C \neq 0$，则会出现：

$$\oint F \cdot dl = Re(C) - \frac{\partial}{\partial y} \iint G \cdot dS$$

等情况。这表明，即便没有磁场从 S 面流出或流入，F 都可能会有一个恒定的旋度。这其中具体的物理意义不太明确，也可能是没有物理意义的情况。故在这里舍弃该条件。

3 结论

由上述分析过程可以看出，利用时空的复数形式，结合复变函数的理论知识，可以用一个很简单的函数形式来表示整个世界的物理规律。这样物质世界规律的表示方法可以获得更加简洁的形式。

因为表达形式变得更加简介，这有助于我们更好地看清楚隐藏在一些现象背后的规律，或许据此深入探索下去，有助于我们进一步解决电荷的起源等比较基本的物理规律。

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