# Is Spacetime Really a Four-Dimensional Continuum? 

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#### Abstract

According to the Theory of Relativity the Universe is an amalgam of time and space containing matter; a four-dimensional spacetime continuum alleged as an analytic generalisation of the Theorem of Pythagoras from three dimensions. Spacetime is said to be curved by matter and undergoes rippling due to gravitational waves travelling at the speed of light. Points in spacetime are called 'events'. The distance between two events is called the spacetime interval, which is manifest as a distance formula, often called a metric or lineelement, in terms of 'coordinates'. However, Minkowski-Einstein spacetime is not actually a four-dimensional continuum because it is self-referential via the speed of light.


The real world in which we dwell is characterised by four dimensions: length, breadth, height, and time. If the value of length is denoted by $x$, breadth by $y$, height by $z$ and time by $t$, any time and place can be denoted by the coordinates $(t, x, y, z$ ), where the $x$-axis, the $y$-axis and the $z$-axis are all perpendicular to one another (Cartesian coordinates). The $t$-axis is not perpendicular to $x, y$, or $z$, and does not form any part of the space described $x, y, z$ - the distance between any two points in this system of coordinates depends on $x, y, z$ but not $t$. To specify a meeting of objects at a time and place one must know the location of the place and at what time the meeting takes place: in other words the coordinates for the meeting must be specified. For instance, if you plan to meet a friend at a cafe for a coffee, the location of the cafe and the time of meeting must be specified. The location of the cafe can be found by reference to a street directory, which is set out as a rectangular grid, with columns denoted by some capital letters of the alphabet and the rows by some positive integers. This is two dimensional (length and breadth). For example, the location of the cafe might be on page 100, in column D and row 12. Street directories do not include time as a dimension or coordinate, although it takes time to locate a place in the directory. Neither is the height of the location given in a street directory (the cafe might be, for instance, on the $3^{\text {rd }}$ floor of the building at the location in the street directory).

Time is measured in time units T, such as seconds, hours, or years. Distance between two places is measured in length units L, such as metres, yards, or miles. Consequently $t$ cannot be added to any $x, y, z$, because $t$ has time units T whereas $x, y$ and $z$ have length units L. Similarly $t^{2}$ cannot be added to or subtracted from $x^{2}, y^{2}$, or $z^{2}$, or vice-versa.

Consider a truck travelling at a constant speed along a straight road from a place A towards a place B. If the truck travels a distance $s$ in a time $t$ then the constant speed $v$ of the truck is the distance divided by the time to travel that distance:

$$
v=\frac{s}{t}=\text { const }
$$

Solving this for $s$ gives,

$$
s=v t
$$

The quantity $v t$ has the units of length L . But $v t$ is not an independent dimension or independent coordinate, because $v$ depends upon $s$ and $t$.

If places $\mathbf{A}$ and $\mathbf{B}$ are on flat ground connected by a straight road, then the distance $\sigma$ between them is given by the Theorem of Pythagoras:


$$
\sigma^{2}=x^{2}+y^{2}
$$

Clearly $\sigma$ is not perpendicular to $x$ or $y$. If the straight road from $\mathbf{A}$ to $\mathbf{B}$ slopes constantly either upward or downward then the truck also moves though a vertical distance (height) as it travels along the straight road from $\mathbf{A}$ to $\mathbf{B}$. Then the distance $s$ that the truck travels is given again by the Theorem of Pythagoras:


But $s=v t$, therefore,

$$
v^{2} t^{2}=x^{2}+y^{2}+z^{2}
$$

Clearly, once again, although $v t$ has the units of length L , $v t$ is not an independent coordinate or independent dimension, even though time $t$ is an independent dimension or independent coordinate. And $s=v t$ is not perpendicular to $x, y$, or $z$. Subtracting $x^{2}+y^{2}+z^{2}$ from both sides of the last equation gives:

$$
v^{2} t^{2}-x^{2}-y^{2}-z^{2}=0
$$

This is not a new distance formula.
Newton's Laws of Motion and his theory of gravity encapsulate the four dimensional world ( $t, x, y$, $z$ ) in which we exist. In the Theory of Relativity the time and place coordinates ( $t, x, y, z$ ) are collectively called 'an event'. Changing the name of the set of four coordinates however does not
change their character. The four coordinates are still the values of the four dimensions time, length, breadth and height.

Now instead of a truck travelling from $\mathbf{A}$ to $\mathbf{B}$, let light travel from $\mathbf{A}$ to $\mathbf{B}$. The speed of light is the distance $s$ that light travels in some time $t$, divided by the time $t$. The speed of light $c$ is a constant, given by:

$$
c=\frac{s}{t}=\text { const. }
$$

Solving for $s$ yields:

$$
s=c t
$$

Here the quantity $c t$ has the units of length L . But it is not an independent coordinate or independent dimension, because $c$ depends upon $s$ and $t$. In the Theory of Relativity the term 'coordinate' is ambiguous because both $t$ and $c t$ are called 'the time coordinate'. Clearly $t$ and $c t$ are however not the same thing. Nevertheless, in determining distance between two 'events', the Theory of Relativity uses the 'coordinates' ( $c t, x, y, z$ ) so that all have the units of length L , to facilitate a distance formula. In the same fashion as for the truck:

$$
s^{2}=x^{2}+y^{2}+z^{2}
$$

But $s=c t$, therefore,

$$
c^{2} t^{2}=x^{2}+y^{2}+z^{2}
$$

Clearly, once again, although $c t$ has the units of length L , $c t$ is not an independent coordinate or independent dimension, even though time $t$ is an independent dimension or independent coordinate in the very same why as in Newton's theories for motion and gravity. And $s=c t$ is not perpendicular to $x, y$, or $z$. Subtracting $x^{2}+y^{2}+z^{2}$ from both sides of the last equation gives:

$$
c^{2} t^{2}-x^{2}-y^{2}-z^{2}=0
$$

This is not a new distance formula. Still $c t$ is not an independent coordinate or independent dimension and is not perpendicular to $x, y$, or $z$. Nevertheless, it is treated as an independent coordinate in the Theory of Relativity, so that the distance between two 'events' (spacetime interval) is given by:

$$
s^{2}=c^{2} t^{2}-x^{2}-y^{2}-z^{2}
$$

This is the alleged four-dimensional spacetime continuum and distance formula of Special Relativity ${ }^{1}$, where $s^{2}$ can be zero, greater than 0 , or less than zero. The term $c t$ appears in the distance formulae associated with General Relativity too, treated there again as if it is an independent coordinate, perpendicular to $x, y$, and $z$. The act of treating $c t$ as an independent coordinate or independent 'dimension' does not make it either.

Since $c t$ is not an independent coordinate, spacetime is not a four-dimensional continuum, with $c t$

[^0]perpendicular to $x, y$ and $z$. Spacetime is a fallacy [1, Appendix G]. The Theory of Relativity employs Riemannian Geometry. In Riemannian Geometry,
"Any n independent variables $x^{i}$, where $i$ takes values 1 to n, may be thought of as the coördinates of an n-dimensional space $V_{n}$ in the sense that each set of values of the variables defines a point of $V_{n}$." Eisenhart [2]

The term $c t$ is no more an independent coordinate than is $x y / z$, or $x^{2} / y$, or $\left(x^{2}+y^{2}+z^{2}\right) / x$, all of which have the units of length $L$.

## REFERENCES

[1] Crothers, S. J., General Relativity: In Acknowledgement Of Professor Gerardus 't Hooft, Nobel Laureate, 4 August, 2014, http://vixra.org/pdf/1409.0072v9.pdf
[2] Eisenhart, L.P., Riemannian Geometry, Princeton University Press, NJ, 1997.


[^0]:    1 Strictly speaking, the spacetime interval of Special Relativity is rendered in differential elements of the terms, thus: $d s^{2}=c^{2} d t^{2}-d x^{2}-d y^{2}-d z^{2}$ where $d t, d x, d y, d z$ are small changes in $t, x, y, z$ respectively, and $d t^{2} \equiv(d t)^{2}$ etc.

