Discussion about electrodynamics spin

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Spin transferred to a mirror reflecting light

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We consider the incidence of a plane circularly polarized electromagnetic wave on a mirror at an angle φ. We have calculated the transfer of the momentum and the spin angular momentum to the mirror and, accordingly, the pressure and density of the torque on the mirror. The given calculations show that spin is a natural property of a plane electromagnetic wave, similar to energy and momentum.

Key Words: classical spin; circular polarization; electrodynamics torque
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1. Introduction 1. Spin density is proportional to energy density

It was suggested as early as 1899 by Sadowsky [1] and as 1909 by Poynting [2], that any usual circularly polarized light (without an azimutal phase gradient) carries angular momentum volume *density,* and the angular momentum density is proportional to the energy volume density. That is the angular momentum is present in any point of the light.

> **J.H. Poynting:** If we put E for the energy in unit volume and G for the torque per unit area, we have $G = E\lambda/2\pi$ [2, p. 565].

This sentence points that any absorption of a circularly polarized light results in a mechanical torque density acting on the absorber. We have researched this effect and have found that this torque density induces specific mechanical stresses in the absorber [3].

According to the Lagrange formalism, this angular momentum volume density is *spin density*. The spin of electromagnetic waves is described by a spin tensor [4 – 7].

$$\Upsilon^{\lambda\mu\nu} = -2A^{[\lambda}\delta^{\mu]}_{\alpha}\frac{\partial\mathscr{L}}{\partial(\partial_\nu A_\alpha)}, \tag{1.1}$$

where $\mathscr{L}$ is a Lagrangian and A^λ is the magnetic vector potential of the electromagnetic field. So, any infinitesimal 3-volume dV_ν contains spin

$$dS^{\lambda\mu} = \Upsilon^{\lambda\mu\nu}dV_\nu. \tag{1.2}$$

The spin tensor (1.1) is appeared in the company of the orbital angular momentum tensor, which is simply a moment of the energy-momentum tensor $2x^{[\lambda}T^{\mu]\nu}$. So, the total angular momentum tensor $J^{\lambda\mu\nu}$ equals the sum of orbital and spin angular momentums. In the case of the canonical Lagrangian $\mathsf{L} = -F_{\mu\nu}F^{\mu\nu}/4$ the canonical energy-momentum, total angular momentum, and spin tensors are

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$$\underset{c}{T}^{\mu\nu} = -\partial^{\mu} A_{\alpha} F^{\nu\alpha} + g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} / 4, \quad (1.3)$$

$$\underset{c}{J}^{\lambda\mu\nu} = 2x^{[\lambda} \underset{c}{T}^{\mu]\nu} + \underset{c}{\Upsilon}^{\lambda\mu\nu} = 2x^{[\lambda} \underset{c}{T}^{\mu]\nu} - 2A^{[\lambda} F^{\mu]\nu}, \quad \underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda} F^{\mu]\nu}. \quad (1.4)$$

here $F_{\mu\lambda}$ is the electromagnetic field tensor. So, the classical electromagnetic field theory provides meaningful descriptions of the spin and the orbital angular momentum separately. Note, spin tensor $\Upsilon^{\lambda\mu\nu}$ is not a moment of energy-momentum tensor $2x^{[\lambda}T^{\mu]\nu}$, and spin density Υ is not (a part of) a moment of linear momentum density $\mathbf{r} \times (\mathbf{E} \times \mathbf{H}) / c^2$.

A *perfect* plane monochromatic circularly polarized electromagnetic wave travelling in z-direction and with infinite extension in the xy-directions is represented by the equations:

$$\breve{\mathbf{E}}_1 = E_1 (\mathbf{x} + i\mathbf{y}) \exp(ikz - i\omega t) \ [\text{V/m}], \quad \breve{\mathbf{H}}_1 = -i\varepsilon_0 c \breve{\mathbf{E}}_1 \ [\text{A/m}], \quad ck = \omega. \quad (1.5)$$

According to the definition (1.4), the spin volume density in such a wave is given by the component of the spin tensor

$$\underset{c}{\Upsilon}^{ijt} = -2A^{[i} F^{j]t} = -2A^{[i} E^{j]} = \mathbf{E} \times \mathbf{A} \quad (1.6)$$

For example, **Soper** [5] writes:

> “To describe a circularly polarized plane wave traveling in the z-direction, we can choose a potential
>
> $$A^x = a\cos[\omega(z-t)], \ A^y = -a\sin[\omega(z-t)] \quad (9.3.17).$$
>
> The corresponding electric field is $E^k = -\partial_t A^k$.
>
> $$E^x = -a\omega\sin[\omega(z-t)], \ E^y = -a\omega\cos[\omega(z-t)] \quad (9.3.18).$$
>
> Thus the spin density carried by this wave is
>
> $$\mathbf{s} = a^2 \omega \hat{\mathbf{z}} \quad (9.3.18),$$
>
> where $\hat{\mathbf{z}}$ is a unit vector pointing in the z-direction.”

Note that perfect plane electromagnetic waves used **Einstein** [8, § 7]:

> "In the system K, very far from the origin of co-ordinates, let there be a source of electrodynamic waves, which in a part of space containing the origin of co-ordinates may be represented to a sufficient degree of approximation by the equations" of type (1.5).

The canonical spin tensor (1.4) was successfully used in order to confirm the fulfillment of the conservation laws with respect to spin when a perfect plane circularly polarized electromagnetic wave with infinite extension reflects from a receding mirror [9]. These calculations prove the functionality of the spin tensor and show that spin is the same natural property of a perfect plane electromagnetic wave, as energy and momentum; and spin density is proportional to energy density.

The classical experiments [10 – 13] confirm that the spin density of plane waves is proportional to energy density. In these experiments, the angular momentum of the light was transferred to a half-wave plate, which rotated. So, work was performed in any point of the plate. This (positive or negative) amount of work reappeared as an alteration in the energy of the photons, i.e., in the frequency of the light, which resulted in moving fringes in any suitable interference experiment.

Some textbooks and articles point that infinite plane circularly polarized electromagnetic wave carries angular momentum:

> **F.S. Crawford, Jr.**: "A circularly polarized travelling plane wave carries angular momentum" [14, p. 365].
>
> **R. Feynman**: "… the photons of light that are right circularly polarized carry an angular momentum of one unit along the z-axis …light which is right circularly polarized carries an energy and angular momentum" [15].
>
> **K. Bliokh and F. Nori: “…** the plane wave carries the spin AM density **S** defined as the *local* expectation value of the operator $\hat{\mathbf{S}}$” [16, p.4].

A spin tensor is used when describing plane waves in the works [3,9,17,18] and in other papers of the author.

2. Introduction 2. Is the spin density proportional to gradient of energy density?

However, since 1939, another concept of electrodynamics spin is in use. The point is, Belinfante & Rosenfeld added specific terms,

$$\partial_\alpha(A^\mu F^{\nu\alpha}) \text{ and } 2\partial_\alpha(x^{[\lambda}A^{\mu]}F^{\nu\alpha}), \tag{2.1}$$

to the canonical energy-momentum and total angular momentum tensors (1.3) and (1.4) respectively [19,20], [5,Sec.9.4]. This procedure yields an energy-momentum tensor $\underset{st}{T}^{\mu\nu}$, which differs from the Maxwell tensor $T^{\mu\nu}$, and an total angular momentum tensor $\underset{st}{J}^{\lambda\mu\nu}$. We named these tensors "standard" [3]:

$$\begin{aligned}\underset{st}{T}^{\mu\nu} = \underset{c}{T}^{\mu\nu} + \partial_\alpha(A^\mu F^{\nu\alpha}) &= -\partial^\mu A_\alpha F^{\nu\alpha} + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4 + \partial_\alpha(A^\mu F^{\nu\alpha})\\ &= -\partial^\mu A_\alpha F^{\nu\alpha} + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4 + \partial_\alpha A^\mu F^{\nu\alpha} + A^\mu\partial_\alpha F^{\nu\alpha}\\ &= g^{\mu\lambda}(-\partial_\lambda A_\alpha F^{\nu\alpha} + \partial_\alpha A_\lambda F^{\nu\alpha}) + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4 + A^\mu\partial_\alpha F^{\nu\alpha}\\ &= -g^{\mu\lambda}F_{\lambda\alpha}F^{\nu\alpha} + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4 + A^\mu\partial_\alpha F^{\nu\alpha} = T^{\mu\nu} + A^\mu\partial_\alpha F^{\nu\alpha},\end{aligned} \tag{2.2}$$

where $T^{\mu\nu} = -g^{\mu\lambda}F_{\lambda\alpha}F^{\nu\alpha} + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4$ is the Maxwell tensor,

$$\begin{aligned}\underset{st}{J}^{\lambda\mu\nu} = \underset{c}{J}^{\lambda\mu\nu} + 2\partial_\alpha(x^{[\lambda}A^{\mu]}F^{\nu\alpha}) &= 2x^{[\lambda}\underset{c}{T}^{\mu]\nu} - 2A^{[\lambda}F^{\mu]\nu} + 2\partial_\alpha(x^{[\lambda}A^{\mu]}F^{\nu\alpha})\\ &= 2x^{[\lambda}\underset{c}{T}^{\mu]\nu} - 2A^{[\lambda}F^{\mu]\nu} + 2\delta_\alpha^{[\lambda}A^{\mu]}F^{\nu\alpha} + 2x^{[\lambda}\partial_\alpha(A^{\mu]}F^{\nu\alpha}) = 2x^{[\lambda}\underset{st}{T}^{\mu]\nu}.\end{aligned} \tag{2.3}$$

But this procedure eliminates spin tensor ($\underset{st}{Y}^{\lambda\mu\nu} = 0$). Really, the standard total angular momentum tensor (2.3) is equal to moment of the standard energy-momentum tensor only: $\underset{st}{J}^{\lambda\mu\nu} = 2x^{[\lambda}\underset{st}{T}^{\mu]\nu}$. So, the corresponding spin term is absent. As a result, in the absence of electrodynamics spin tensor, it has been declared that the electrodynamics spin is a part of a moment of linear momentum [21, p. 7]

$$\mathbf{J} = \int dV\varepsilon_0 \mathbf{r}\times(\mathbf{E}\times\mathbf{B}) = \mathbf{L} + \mathbf{S},$$

and that a plane wave has no angular momentum at all.

> **Heitler W:** "A plane wave travelling in z-direction and with infinite extension in the xy-directions can have no angular momentum about the z-axis, because $(\mathbf{E}\times\mathbf{B})$ is in the z-direction and $(\mathbf{r}\times(\mathbf{E}\times\mathbf{B}))_z = 0$" [22].

According to the nowadays conception, electrodynamics spin density is proportional to *gradient of energy density*, not to energy density:

> **Allen L., Padgett M. J.:** "… the local spin angular momentum density per photon is proportional to the radial intensity gradient of a light beam:
>
> $$j_z = -\frac{r}{2u^2}\frac{\partial(u^2)}{\partial r}\hbar\sigma$$
>
> where $\sigma = \pm 1$ for right- and left-handed circularly polarized light respectively, u^2 is the beam intensity, and *r* is the distance from the axis. For a plane wave there is no gradient and the spin density is zero." [23]
>
> **Simmonds J. W., Guttmann M. J.:** "The electric and magnetic fields can have a nonzero z-component only within the skin region of this wave. Having z-components within this region implies the possibility of a nonzero z-component of angular momentum within this region. So, the skin region is the only in which the z-component of angular momentum does not vanish” [24, p. 227]

Thus, according to the widespread opinion, an electromagnetic wave has no spin density everywhere where there is no intensity gradient, and it has no orbital angular momentum in the lack of an azimutal dependence. According to the opinion, spin of a real plane wave is carried out into the remote edge of the wave in compliance with the Humblet transformation [25].

We have criticized [3] the sense of the Humblet transformation. We have noted [26] that this concept, "Spin is only in the skin region," threatens us with a considerable nonlocality of the electrodynamics because the concept implies that energy and momentum of photons are absorbed everywhere in the absorber, but spin is absorbed in the remote boundary of the wave only.

Note, the Belinfante-Rosenfeld procedure does not yield the Maxwell tensor and even does not symmetrize the canonical tensor (1.3). The term $A^{\mu}\partial_{\alpha}F^{\nu\alpha}$ in (2.2) vanishes only when interactions are absent, but this case has no sense.

In this paper, we confirm the Poynting's and Sadowsky's concept by a new calculation, which uses a spin tensor. The existence of a spin tensor entails the following interpretation of the **Heitler's** statement: "A plane wave travelling in z-direction and with infinite extension in the xy-directions can have no *orbital* angular momentum about the z-axis".

3. Spin density

Since 1905, when Einstein explained the photoelectric effect, it has become clear that an electromagnetic wave consists of photons. Photons have energy, momentum and spin (internal angular momentum), and if the wave is circularly polarized, spins of all the photons are directed in the same direction that is parallel to that of the momentum of the wave. Therefore, one can use such notions as volume density and flux density of momentum, energy, spin, and of number of photons in an electromagnetic wave. Densities of the energy and momentum are quantitatively described by the Maxwell energy-momentum tensor $T^{\mu\nu}$ [27 (33.1); 28 (12.113)]. The density of the spin angular momentum should be described by the spin tensor (1.1). The numeric density of photons is obtained either by dividing the energy density of a wave by the energy of a single photon $\hbar\omega$, or by dividing the spin density by the spin of a single photon $\hbar$ (if polarization is circular).

In this paper, we consider the incidence of a plane circularly polarized electromagnetic wave on a non-moving mirror at an angle of incidence, φ. In this situation, the electromagnetic energy is not transferred to the mirror, and the momentum and the spin of the photons change their direction upon reflection. As a result, the mirror receives the doubled normal component of the wave momentum in the form of pressure and the doubled tangential component of the spin in the form of a distributed torque. As is known, in the process of reflection, the wave helicity is reversed, i.e., the mutual orientation of the momentum and spin changes into the opposite one (see Fig. 1).

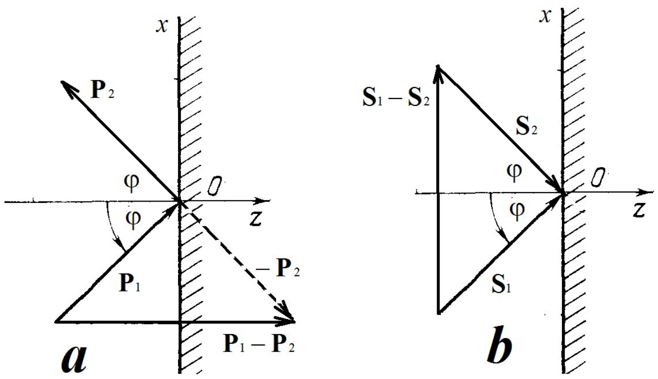

Figure 1. (***a***) Momentum of incident and reflected photons and the momentum gained by the mirror, and (***b***) spin of incident and reflected photons and the spin gained by the mirror.

We will calculate here the flux densities of the momentum and spin in the incident and reflected waves, and make sure that the change in the momentum and the spin of the reflected wave correspond to the pressure and density of the torque experienced by the mirror. The material of this paper have been published already elsewhere [29].

4. The electromagnetic waves in question

To write the expression for a wave incident at an angle φ, we will make use of the expression for a right-hand circularly polarized electromagnetic wave incident normally on the xy-surface in the coordinates x', y', z' (for convenience, we write the coordinate indexes as superscript indexes):

$$E_1^{x'} = \cos(z'-t), \quad E_1^{y'} = -\sin(z'-t), \quad B_1^{x'} = \sin(z'-t), \quad B_1^{y'} = \cos(z'-t) \tag{4.1}$$

(for simplicity we put $\omega = k = c = \varepsilon_0 = \mu_0 = 1$). The coordinate transformations

$$x' = x\cos\varphi - z\sin\varphi, \quad z' = x\sin\varphi + z\cos\varphi, \quad y' = y \tag{4.2}$$

give expressions

$$E_1^x = \cos\varphi\cos(x\sin\varphi + z\cos\varphi - t), \quad B_1^x = \cos\varphi\sin(x\sin\varphi + z\cos\varphi - t), \tag{4.3}$$

$$E_1^y = -\sin(x\sin\varphi + z\cos\varphi - t), \quad B_1^y = \cos(x\sin\varphi + z\cos\varphi - t), \tag{4.4}$$

$$E_1^z = -\sin\varphi\cos(x\sin\varphi + z\cos\varphi - t), \quad B_1^z = -\sin\varphi\sin(x\sin\varphi + z\cos\varphi - t). \tag{4.5}$$

for the right-hand circularly polarized wave incident at an angle φ.

To write the expression for a wave reflected at an angle φ, we will make use of the expression for a left-hand circularly polarized electromagnetic wave originating along the normal from the xy-surface in the coordinates x', y', z':

$$E_2^{x'} = -\cos(z'+t), \quad E_2^{y'} = -\sin(z'+t), \quad B_2^{x'} = -\sin(z'+t), \quad B_2^{y'} = \cos(z'+t). \tag{4.6}$$

The coordinate transformations

$$x' = x\cos\varphi + z\sin\varphi, \quad z' = -x\sin\varphi + z\cos\varphi, \quad y' = y \tag{4.7}$$

give expressions

$$E_2^x = -\cos\varphi\cos(-x\sin\varphi + z\cos\varphi + t), \quad B_2^x = -\cos\varphi\sin(-x\sin\varphi + z\cos\varphi + t), \tag{4.8}$$

$$E_2^y = -\sin(-x\sin\varphi + z\cos\varphi + t), \quad B_2^y = \cos(-x\sin\varphi + z\cos\varphi + t), \tag{4.9}$$

$$E_2^z = -\sin\varphi\cos(-x\sin\varphi + z\cos\varphi + t), \quad B_2^z = -\sin\varphi\sin(-x\sin\varphi + z\cos\varphi + t). \tag{4.10}$$

for the wave reflected at an angle φ.

One can easily see that the boundary conditions

$$\left[E_1^x + E_2^x\right]_{z=0} = \left[E_1^y + E_2^y\right]_{z=0} = \left[B_1^z + B_2^z\right]_{z=0} = 0. \tag{4.11}$$

are fulfilled on the surface of the mirror (an ideal conductor).

5. Momentum flux density transferred to the mirror

To calculate the momentum flux density, i.e. the pressure $\mathcal{P}$, on the mirror, it is natural to use the component of the Maxwell stress tensor [27 (33.3)]

$$T^{zz} = [-(E^z)^2 + (E^x)^2 + (E^y)^2 - (B^z)^2 + (B^x)^2 + (B^y)^2]/2. \tag{5.1}$$

The incident and reflected waves superpose, but they do not interfere with each other. This can be verified by determining that the energy–momentum tensor of the total field is equal to the sum of the energy–momentum tensors of the incident and reflected waves. Therefore, we can calculate the pressure by substituting expressions for the incident wave (4.3) – (4.5) into formula (5.1) and double the result. Thus, we obtain

$$\mathcal{P} = 2T^{zz} = 2\cos^2\varphi. \tag{5.2}$$

Einstein obtained this result by another method [8].

On the other hand, this result can be regarded as the action of the Lorentz force on charges and currents induced in the mirror. Indeed, we express the force $d\mathcal{F}$ acting on an infinitesimal area of the mirror surface da_z through the divergence of the component of the energy–momentum tensor (see Fig. 2)

$$d\mathcal{F} = \mathcal{P}da = T^{zz}da_z = -\oint_{\partial dV} T^{zi}da_i = -\partial_i T^{zi}dV. \tag{5.3}$$

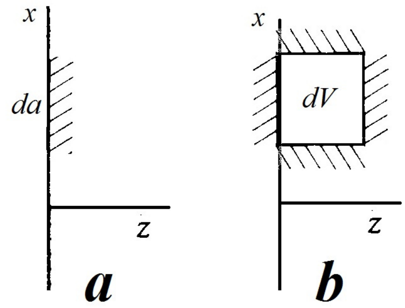

Figure 2. (***a***) Area da on the mirror and (***b***) area da forming a closed surface which is the boundary of the mirror material volume dV.

Here we assume integration over the boundary of volume dV, which is obtained by closing the area *da* inside the mirror material with changing the external orientation to the opposite one. Since [27 (33.7)] $-\partial_\mu T^{\lambda\mu} = j^\mu F^{\lambda\nu} g_{\mu\nu}$, the divergence of the tensor component is expressed in terms of the Lorentz force density. According to [27], or [28], $F^{zt} = E^z$, $F^{zx} = -B^y$, $F^{zy} = B^x$, but here we must take into account the metric signature $(+---)$. So, $g_{tt} = 1$, $g_{xx} = g_{yy} = -1$, and

$$-\partial_i T^{zi} = \partial_t T^{zt} + j^t F^{zt} - j^x F^{zx} - j^y F^{zy} = \rho E^z + j^x B^y - j^y B^x = \rho E^z + [\mathbf{jB}]_z . \tag{5.4}$$

The momentum density T^{zt} in the direction of the mirror, as well as the Poynting vector $\Pi^z = T^{tz}$, are zero. Therefore, we obtain

$$d\mathcal{F} = \mathcal{P} da = (\rho E^z + [\mathbf{jB}]^z) dV . \tag{5.5}$$

6. Spin tensor

To describe the spin volume density and spin flux density in a perfect plane wave, the canonical spin tensor (1.4) was successfully used in [9]. However, for this paper, it is very important that the canonical spin tensor *incorrectly* describes the spin flux in the directions that do not coincide with the wave propagation direction. This was pointed out in [3]. Really, consider the Soper's wave [5]

$$A^x = \cos(z-t),\ A^y = -\sin(z-t) .$$

$$E^x = -\sin(z-t),\ E^y = -\cos(z-t),\ B^x = \cos(z-t),\ B^y = -\sin(z-t),$$

A calculation of components of the canonical spin tensor yields

$$\underset{c}{\Upsilon}^{zxy} = A^x B^x = \cos^2(z-t), \quad \underset{c}{\Upsilon}^{yzx} = A^y B^y = \sin^2(z-t) .$$

This result is not adequate because it means that there are spin fluxes in the directions, which are perpendicular to the direction of the wave propagation."

A spin tensor derived from the Lagrangian of the massless vector field $\mathcal{L} = -\partial_\mu A^\nu \partial^\mu A_\nu / 2$,

$$\Upsilon^{\lambda\mu\nu} = -2A^{[\lambda}\delta^{\mu]}_\alpha \frac{\partial \mathcal{L}}{\partial(\partial_\nu A_\alpha)} = A^\lambda \partial^\nu A^\mu - A^\mu \partial^\nu A^\lambda , \tag{6.1}$$

is found to be more adequate than the canonical spin tensor. However, spin tensor (6.1) was derived also by an application of a formal scheme to the canonical tensors. [3,17,18].

The use of the massless vetctor field Lagrangian instead of the canonical Lagrangian $\mathsf{L} = -F_{\mu\nu}F^{\mu\nu}/4$ for deriving a spin tensor must not put us on guard. In any case, the Lagrange formalism does not guarantee quality of energy-momentum and spin tensors no matter what Lagrangian is used. Hereafter we will use the spin tensor (6.1) and will calculate the angular momentum transfer to the mirror.

7. Spin angular momentum flux density transferred to the mirror

In accordance with Fig. 1b, the S^{yz} component of the spin is transferred to the mirror. The flux density of this spin component on the mirror is given by the component

$$\Upsilon^{yzz} = A^y\partial^z A^z - A^z\partial^z A^y \tag{7.1}$$

of the spin tensor, and, in the absence of interference, it is possible to calculate this component only for the incident wave and to double it. From the formula $\mathbf{A} = -\int \mathbf{E}dt$ we obtain the magnetic vector potentials in the incident wave:

$$A_1^y = \cos(x\sin\varphi + z\cos\varphi - t), \quad A_1^z = -\sin\varphi\sin(x\sin\varphi + z\cos\varphi - t)\,. \tag{7.2}$$

Thus, given that $\partial^z = -\partial_z$ due to the metrics signature $(+---)$, the spin flux density on the mirror is equal to the expression

$$\Upsilon^{yzz} = 2(A_1^y\partial^z A_1^z - A_1^z\partial^z A_1^y) = \sin(2\varphi) \tag{7.3}$$

On the other hand, this quantity is the result of the distributed torque on the currents induced in the mirror. Indeed, we can express the torque $d\tau^{yz}$ acting on the area da_z of the mirror surface through the divergence of the spin tensor component (see Fig. 2)

$$d\tau^{yz} = \Upsilon^{yzz} da_z = -\oint_{\partial dV} \Upsilon^{yzi} da_i = -\partial_i \Upsilon^{yzi} dV\,. \tag{7.4}$$

Here we assume integration over the boundary of volume dV, which is obtained by closing the area da_z inside the mirror material with changing the external orientation to the opposite one. Since

$$-\partial_\nu \Upsilon^{\lambda\mu\nu} = -2\partial_\nu(A^{[\lambda}\partial^{|\nu|}A^{\mu]}) = 2j^{[\lambda}A^{\mu]}, \tag{7.5}$$

and since the electromagnetic spin does not accumulate in the mirror, $\partial_t \Upsilon^{yzt} = 0$, the divergence of the spin tensor component is expressed in terms of the *torque density* $[\mathbf{jA}]$, which is an analogue of the Lorentz force density

$$-\partial_i \Upsilon^{yzi} = 2j^{[y}A^{z]} = [\mathbf{jA}]^x, \quad d\tau^{yz} = [\mathbf{jA}]^x dV\,. \tag{7.6}$$

8. Conclusions

The given calculations show that spin is a natural property of a plane electromagnetic wave, similar to energy and momentum. If we recognize the existence of photons with momentum, energy and spin in a plane electromagnetic wave, it is strange to deny the existence of the spin of the wave, which unfortunately is the case in the modern electrodynamics.

We are eternally grateful to Professor Robert Romer, having courageously published the question: "Does a plane wave really not carry spin?" [30].

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29. Khrapko R.I. "Spin transferred to a mirror reflecting light" (2005) http://www.mai.ru/science/trudy/published.php?ID=34126 in Russian.
30 Khrapko R I "Does plane wave not carry a spin?" *Amer. J. Phys*. 69, 405 (2001)

Reviewers’ comments and Author’s reply

> **Reviewer 1**: I have major objections are to some of the material in the new secs. 2 and 6. The author’s statements after eq. 2.5 are just wrong. First, in contrast to his statement, adding the term he mentions in eq. 2.3 to eq. 2.1 does yield the correct symmetrized Maxwell stress-energy-momentum tensor, the same one that follows from the variation of the Maxwell Lagrangian w.r. to a curvilinear metric later taken to be a Lorentz metric, as shown by Landau and Lifshitz, the author’s ref 22.

I am forced to present the detailed calculation which shows that the Belinfante-Rosenfeld term

$$\partial_\alpha (A^\mu F^{\nu\alpha}), \tag{2.3}$$

in reality, does NOT yield the Maxwell tensor and even does NOT symmetrize the canonical tensor $\underset{c}{T}^{\mu\nu}$ (2.1) that follows from Noether’s theorem. The Belinfante-Rosenfeld term yields the “standard” energy-momentum tensor. Please see

$$\begin{aligned}\underset{st}{T}^{\mu\nu} = \underset{c}{T}^{\mu\nu} + \partial_\alpha (A^\mu F^{\nu\alpha}) &= -\partial^\mu A_\alpha F^{\nu\alpha} + g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}/4 + \partial_\alpha (A^\mu F^{\nu\alpha}) \\ &= -\partial^\mu A_\alpha F^{\nu\alpha} + g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}/4 + \partial_\alpha A^\mu F^{\nu\alpha} + A^\mu \partial_\alpha F^{\nu\alpha} \\ &= g^{\mu\lambda}(-\partial_\lambda A_\alpha F^{\nu\alpha} + \partial_\alpha A_\lambda F^{\nu\alpha}) + g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}/4 + A^\mu \partial_\alpha F^{\nu\alpha} \\ &= -g^{\mu\lambda} F_{\lambda\alpha} F^{\nu\alpha} + g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}/4 + A^\mu \partial_\alpha F^{\nu\alpha}\end{aligned} \tag{2.4}$$

The Maxwell tensor is $T^{\mu\nu} = -g^{\mu\lambda} F_{\lambda\alpha} F^{\nu\alpha} + g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}/4$. The Maxwell tensor cannot be obtain by any variations. So, **I repeat**, Belinfante's adding does NOT yield the Maxwell tensor and even does not symmetrize the canonical tensor (2.1). Everybody sees it in (2.4). The term $A^{\mu}\partial_{\alpha}F^{\nu\alpha}$ vanishes only when interactions are absent, but this case has no sense.

> **Reviewer 1**:Second, the total angular momentum tensor that the author calls "standard" is not the correct total angular momentum tensor that follows from Noether's theorem, which is simply J = J_orb + Y.

Yes! The "standard" total angular momentum tensor (2.5) does not follow from Noether's theorem. This "standard" tensor is obtained by Belinfante & Rosenfeld by adding the term

$$\mathrm{D} = 2\partial_{\alpha}(x^{[\lambda}A^{\mu]}F^{\nu\alpha}) \qquad (2.3)$$

to the canonical total angular momentum tensor $\underset{c}{J}^{\lambda\mu\nu}$ (2.2) that follows from Noether's theorem: $\underset{st}{J}^{\lambda\mu\nu} = \underset{c}{J}^{\lambda\mu\nu} + 2\partial_{\alpha}(x^{[\lambda}A^{\mu]}F^{\nu\alpha})$. But neither "standard" nor canonical tensor is correct.

> **Reviewer 1:** Third, the so-called "standard" total ang mom tensor J_st is related to J by an identity, J = J_st – D, where D is the total derivative defined in the second term in eq. 2.3. Since the relation is an identity, you must obtain the same results for J for all emag problems whether you apply the orbital plus spin sum or J = J_st – D. The trouble with applying the latter is that you don't know how to do it for infinite plane waves and an infinite mirror. In applying the orbital plus spin sum, the orbital part must be evaluated as well as the spin part, and again we don't know how to do that for infinite plane waves and an infinite or finite mirror

The tensors $\underset{c}{J}^{\lambda\mu\nu}$ or $\underset{st}{J}^{\lambda\mu\nu} = \underset{c}{J}^{\lambda\mu\nu} + D$ can be applied for infinite plane waves according to the definition (2.2) or (2.5). There is no trouble. But neither "standard" nor canonical tensor is correct.

> **Reviewer 1:** Fourth, the author's new statements in sec. 6 are also wrong. The definitions of the Noether stress-energy-momentum tensor and angular momentum tensor follow from Noether's theorem applied to translational and Lorentz rotational invariance of the Lagrangian, for all kinds of fields, not just electromagnetic. But the author never mentions this most powerful theorem. Belinfante's generalized procedure to obtain the symmetrized stress-energy-momentum tensor is straightforward and manifestly correct, in no way a delusion.
> My greatest objection to this work remains the author's switch from the correct Maxwell Lagrangian to a massless vector field Lagrangian, and his statement that this switch should not be cause for concern. On the contrary, at the very least the author must show in great detail exactly how the conventional Maxwell equations with currents follow from the vector field Lagrangian. For example, how is the electric field defined? How do you get electrostatics? Are all radiation phenomena unchanged? How does the lack of gauge invariance impact classical electromagnetic theory and the coupled Dirac-electromagnetic field equations and results in QED? All of these questions and more must be resolved before one can justify the switch from a Maxwell to a massless vector field Lagrangian, and a proper resolution would require many papers and much discussion. Surely this has been considered in the past?

Yes, energy-momentum, total angular momentum, and spin tensors follow from Noether's theorem. But the tensors, as well as field equations, depend on the Lagrangian in use. Pleas see:

Barut A. O. (Macmillan, New York, 1964). **Table I**

Lagrangians and Equations of Motion for the Most Common Fields

Field	*Lagrongian*	*Field Equations*
Linear Chain	$\frac{1}{2}(\rho\dot{\psi}^2 - Y\psi^2{}_{,x})$	$\ddot{\psi} = \frac{Y}{\rho}\frac{\partial^2\psi}{\partial x^2}$
Real Scalar Field	$\frac{1}{2}(g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - m^2\phi^2)$	$(\Box^2 + m^2)\phi = 0$
Complex Scalar Field	$\frac{1}{2}(\phi^{,\mu}\phi^*{}_{,\mu} - m^2\phi\phi^*)$	$(\Box^2 + m^2)\phi = 0; \quad (\Box^2 + m^2)\phi^* = 0$
Free Electromagnetic Field	$L_{\mathrm{I}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(E^2 - B^2)$ $L_{\mathrm{II}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(A^\mu{}_{,\mu})^2$ $L_{\mathrm{III}} = -\frac{1}{2}A^\mu{}_{,\nu}A_\mu{}^{,\nu}$ $L_{\mathrm{IV}} = \frac{1}{2}[A_\nu F^{\mu\nu}{}_{,\mu} - A_{\nu,\mu}F^{\mu\nu}] + \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$	$F^{\mu\nu}{}_{,\nu} = 0$ $\Box^2 A_\mu = 0$ $\Box^2 A_\mu = 0$ $\Box^2 A_\mu = 0$
Electromagnetic Field with an External Current	$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{c}A_\mu j^\mu$	$F^{\mu\nu}{}_{,\nu} = -\frac{1}{c}j^\mu$
Dirac Field	$L_{\mathrm{I}} = i\bar{\psi}\gamma^\mu\partial_\mu\psi + m\bar{\psi}\psi$ $L_{\mathrm{II}} = \frac{1}{2}\bar{\psi}(i\gamma^\mu\overrightarrow{\partial_\mu} + m)\psi + \frac{1}{2}\bar{\psi}(-i\gamma^\mu\overleftarrow{\partial_\mu} + m)\psi$	$(i\gamma^\mu\partial_\mu + m)\psi(x) = 0; \quad i\bar{\psi}_{,\mu}\gamma^\mu - m\bar{\psi} = 0$ $(i\gamma^\mu\partial_\mu + m)\psi(x) = 0; \quad i\bar{\psi}_{,\mu}\gamma^\mu - m\bar{\psi} = 0$
Interacting Dirac and Maxwell Fields	$L_{\mathrm{Dirac}} + L_{\mathrm{Maxwell}} - \frac{e}{c}A_\mu(x)\bar{\psi}\gamma^\mu\psi$	$\Box^2 A_\mu(x) = e\bar{\psi}\gamma_\mu\psi$ $\left[\gamma^\mu\left(i\partial_\mu - \frac{e}{c}A_\mu(x)\right) + m\right]\psi(x) = 0$

However, A. Barut did not show energy-momentum and spin tensors corresponding to these Lagrangians. So, we add Table 2.

Table 2.

Electrodynamics' Lagrangians, Energy-Momentum Tensors, and Spin Tensors

Lagrangian	Energy-momentum tensor	Spin tensor
$L_I = \underset{c}{L} = -F_{\mu\nu}F^{\mu\nu}/4$	$T_I^{\lambda\mu} = \underset{c}{T}{}^{\lambda\mu} = -A_\nu{}^{,\lambda}F^{\mu\nu} + g^{\lambda\mu}F_{\sigma\nu}F^{\sigma\nu}/4$	$\mathrm{Y}_I^{\lambda\mu\nu} = \underset{c}{\mathrm{Y}}{}^{\lambda\mu\nu} = -2A^{[\lambda}F^{\mu]\nu}$
$L_{II} = -F_{\mu\nu}F^{\mu\nu}/4 - (A^\mu{}_{,\mu})^2/2$	$T_{II}^{\lambda\mu} = T_I^{\lambda\mu} - A^{\mu,\lambda}A^\sigma{}_{,\sigma} + g^{\lambda\mu}(A^\sigma{}_{,\sigma})^2/2$	$\mathrm{Y}_{II}^{\lambda\mu\nu} = \mathrm{Y}_I^{\lambda\mu\nu} + 2A^{[\lambda}g^{\mu]\nu}A^\sigma{}_{,\sigma}$
$L_{III} = -A^\mu{}_{,\nu}A_\mu{}^{,\nu}/2$	$T_{III}^{\lambda\mu} = -A_\sigma{}^{,\lambda}A^{\sigma,\mu} + g^{\lambda\mu}A_{\sigma,\rho}A^{\sigma,\rho}$	$\mathrm{Y}_{III}^{\lambda\mu\nu} = 2A^{[\lambda}A^{\mu],\nu}$
$L_V = -F_{\mu\nu}F^{\mu\nu}/4 - A_\sigma j^\sigma$	$T_V^{\lambda\mu} = T_I^{\lambda\mu} + g^{\lambda\mu}A_\sigma j^\sigma$	$\mathrm{Y}_V^{\lambda\mu\nu} = \mathrm{Y}_I^{\lambda\mu\nu}$

> **Reviewer 1:** On the other hand, the weird angular momentum fluxes that occur for the Maxwell Lagrangian don't worry me much, because they integrate to zero for the infinite case, which I don't trust, and we can't evaluate their integrals for any finite case because we then don't have just one or two infinite plane waves.

The "weird" canonical angular momentum fluxes density that occur for the canonical Lagrangian completely discredits the variation method at all.

> **Reviewer 1:** Overall, I feel that the author has just made too many errors, as discussed above, and also that he is unable to account for edge effects in finite systems such as a finite beam of light incident on a finite plane mirror. One needs to be able to solve such realistic systems because of the above-mentioned derivative term in the total angular momentum tensor, and also because of the unboundedness of the orbital and the

"standard" total angulr momentum tensors at spatial and temporal infinity. So I recommend rejection of this ms.

The edge effect in light beam was accouted in
Khrapko R.I. "Mechanical stresses produced by a light beam" *J. Modern Optics*, **55**, 1487-1500 (2008) http://khrapkori.wmsite.ru/ftpgetfile.php?id=9&module=files

Reviewer 2: This is not the first time I see a paper that:
(i) starts with citations of some authoritative sources declaring that the spin density of a homogeneous plane wave is zero (here, the statements of Heitler [18], Simmonds & Guttmann [19] and Allen & Padgett [20] are reproduced), and which are summarized by the author as the "widespread opinion that an electromagnetic wave has no spin density everywhere where there is no intensity gradient";
(ii) in the main text the author analyzes some physical phenomena or discusses gedanken experiments where the spin of a circularly polarized light wave plays an important role;
(iii) finally, the author puts forward the conclusion "that spin is a natural property of a plane electromagnetic wave, similar to energy and momentum… it is strange to deny the existence of the spin of the wave, which unfortunately is the case in the modern electrodynamics."
This looks as if the modern scientific community is disastrously deluded, and only due to the present paper, it finds a chance to remove the veil from its eyes. Namely, that everybody is confused, and this manuscript will show the light to the confused masses. And due to this delusion, the author hopes that the editor should excuse plentiful mistakes, confusions, inaccuracies, chaotic style of presentation, etc., and … immediately publish the submitted paper.
It does not seem to be reasonable to publish a paper with mistakes, confusions, inaccuracies, chaotic style of presentation, etc.

Let's ignore these insignificant words.

Reviewer 2: First, I do not agree with the author's reproaches to the physics community for denying the spin density in a circularly polarized plane wave. There are many quite authoritative books and articles that, oppositely to the author's judgment (iii), by no mean "deny the existence of the spin of the plane wave" but operate with this notion as with a very common physical instrument. See, for example the very basic undergraduate textbook [1*, p. 365]:
"…A circularly polarized travelling plane wave carries angular momentum".
There are many other sources making similar claims. It is puzzling why the author would ignore those sources, and present a one-sided view of this field.
In p. 5, the author reproduces the expression for the plane-wave spin density from the book by Soper [8]. **The notion of the spin density of a plane wave is widely used in the current literature**, for example in Refs. [2*,3*].

The Reviewer contradicts himself. I expose the two mutual excluding views that the Reviewer represents: **"The notion of the spin density of a plane wave is widely used in the current literature"** & **"the spin density is not well defined for a plane wave".**
The Reviewer contradicts Allen & Padgett: **"For a plane wave there is no gradient and the spin density is zero".**

Reviewer 2: The seemingly contradictory statements in (i) are related not with a misunderstanding of the physical nature of the light-wave spin, but rather with the limited validity of the 'plane wave' as a model of real physical objects.
The 'plane wave' model is very suitable for many purposes, but some aspects of electromagnetic fields cannot be properly described nor even understood in its frame. This can be seen even in the author's statement cited in (iii):
"spin is a natural property of a plane electromagnetic wave, similar to energy and momentum".

In reality, the 'plane wave' model is very suitable to describe spin as a natural property of a plane electromagnetic wave, similar to energy and momentum.

Reviewer 2: Speaking rigorously, the energy and momentum of a plane wave are not well-defined quantities: the total energy and total momentum of a plane wave are infinite.

This words are obviously insignificant because the energy-momentum *density* of plane waves are considered always, while "the total energy and momentum of a plane wave" are never considered

Reviewer 2: Similarly, **the spin density is not well defined for a plane wave**, and it should be used with precautions; that is the meaning of the authoritative statements mentioned in (i). These are just precautionary comments, and the author should have known this.

The Reviewr should have known that the spin density is very well defined for a plane wave by a spin tensor.

Reviewer 2: Actually, the doubtless fact that the spin density of a perfect plane wave vanishes, only means that the perfect plane wave is a very idealized limit. A theoretical construct, a certain idealization of real objects, and its validity is limited. There is no perfect infinite plane wave in reality, and any "physical" plane wave does carry spin.

The Reviewer must explain the difference between the "perfect plane wave" and "plane wave" mentioned above.

Reviewer 2: There are different ways to reconcile the 'plane-wave' idealized concept with more realistic situations, and one of them is to take into account that any observation of the plane-wave field, and any of its interactions, even with a single atom, inevitably destroys its "ideal" character and "selects" certain finite fragment of its infinite cross section.

In reality, there is no need "to reconcile the 'plane-wave' idealized concept with more realistic situations" because "the notion of the spin density of *a plane wave* is widely used in the current literature"

Reviewer 2: Despite the fact that the ideal, platonic, theoretical plane wave 'per se' carries no angular momentum density, the rigorously calculated angular momentum of this transverse fragment exactly equals to what is dictated by the homogeneous distribution of the constant spin density across the plane wave. This was reported several times in different forms; For example in Ref. [4*]. Afterwards, this approach was described in several reviews [5*,6*].

In reality, the rigorously calculated angular momentum equals to what is dictated by the distribution of the spin density because of the fact that the ideal, platonic, theoretical plane wave 'per se' carries spin density

> **Reviewer 2:** Thus, the **vanishing** spin density of an ideal circularly polarized plane wave is completely compatible with its ability to carry angular momentum and transmit it to material objects. References [18–20], which seem to have motivated the author's efforts, are not misleading, and there is no necessity to prove again the well established fact that a circularly polarized wave **contains** angular momentum. This is well known to very many students.

Please see an absurdity: "the vanishing spin density of an ideal circularly polarized plane wave is completely compatible with the fact that a circularly polarized wave contains angular momentum".

> **Reviewer 2:** On the other hand, the author's calculations of the angular momentum exchange in the process of light reflection can be interesting per se but mostly if these were presented in a more pedagogical aspect. Regrettably, the entire analysis is presented in a rather careless and confusing form. **First, the use of** Fνμ = k = c = ε0 = μ0 = 1 provides "simplicity" for the author, but not for the readers; as for me, this admission prevents from tracing the transformations and understanding the final results**. Also, notations** … νμ, FνμF (p. 2) are never defined nor explained. There are also many other examples of confusing presentation in the text.

We cannot read the Reviwer's symbols. But, probably, the Reviewer does not know that $k = c = \varepsilon_0 = \mu_0 = 1$ is used even by Soper, and the electromagnetic field tensor is denoted by $F_{\mu\nu}$ even by Landau & Lifshitz and Jackson.

> **Reviewer 2:** The examples of the spin and momentum transfer considered in the paper (Figs. 1 and 2) are suitable for analysis by elementary means, and the reader expects their clear physical interpretation, which is absent in this manuscript.

Well, we explain these elementary means:
$\mathbf{P_1}$ is the momentum of the incident photons
$\mathbf{P_2}$ is the momentum of the reflected photons
$\mathbf{P_1 - P_2}$ is the moment transferred to the mirror
$\mathbf{S_1}$ is the spin of the incident photons
$\mathbf{S_2}$ is the spin of the reflected photons
$\mathbf{S_1 - S_2}$ is the spin transferred to the mirror

> **Reviewer 2:** Regarding the strangely-looking formula (5.2) for the momentum transferred to the mirror: elementary considerations suggest that it should be proportional to cos not cos2 . After this simple result for the momentum transfer, one expects something similar for the spin transfer in the situation of Fig. 2. However, no simple expression for the spin exchange is presented. Only the spin flux density (7.3) is given, which needs to be commented: why this expression has a maximum for φ, not cos2φ

Formula (5.1) is the formula from Landau & Lifshitz § 33. Let us substitute expressions for the incident wave (4.3) – (4.5) into formula (5.1) and double the result. Thus, we obtain (5.2):

$$\mathscr{P} = -\sin^2\varphi\cos^2\alpha + \cos^2\varphi\cos^2\alpha + \sin^2\alpha - \sin^2\varphi\sin^2\alpha + \cos^2\varphi\sin^2\alpha + \cos^2\alpha = 2\cos^2\varphi$$

(the second cos is appeared because of increasing of the illuminated surface).

In accordance with Fig. 1b, the S^{yz}-component of the spin is transferred to the mirror. The flux density of this spin component on the mirror is given by the component $\Upsilon^{yzz} = A^y\partial^z A^z - A^z\partial^z A^y$ (7.1). Substituting of A^y, A^z and doubling yields $\sin(2\varphi)$ (7.3). The spin flux density on the mirror is zero if $\varphi = 0$ or $\varphi = \pi/2$.

> **Reviewer 2:** As a result, in its present form, the paper has several significant problems, it is confusing, and it conveys no useful information. It is very unlikely that it could be upgraded to a publishable condition.

Let's ignore these insignificant words.

> **Reviewer 2:** The first 12 referees are mostly very old. As if almost nobody has worked on this problem for a long time. After these 12 old references, three self-citations of Khrapko (two unpublished), followed by very old references from 1939, 1940, 1954, 1970, and only one somewhat recent (2009). Then afterwards five more references to Khrapko, plus standard textbooks.

The Reviewer of the paper
Khrapko R.I. "Mechanical stresses produced by a light beam" *J. Modern Optics*, **55**, 1487-1500 (2008) http://khrapkori.wmsite.ru/ftpgetfile.php?id=9&module=files
wrote: "The paper attempts to clarify and correct some questions in one of the 4 or so century-old controversies in classical electrodynamics, perhaps the major one of interest in modern optics. The paper is on a topic where the literature is literately riddled with error, confusion, and dispute. The topic is of interest in practical issues in optical micromanipulation and of theoretical interest in the foundations of field theory and classical electrodynamics".

> **Reviewer 2:** Why not spend more time studying carefully recent works on this topic? Why so focused on several old papers and the works by just one person?

There is no progress from 1936 to now, including recent works on this topic.

Heitler W *The Quantum Theory of Radiation*

FIRST EDITION 1936
Second edition printed lithographically in Great Britain
at the UNIVERSITY PRESS, OXFORD, 1944
from corrected sheets of the first edition
Reprinted 1947, 1949, 1950

1. The angular momentum of light (§ 7)

IN Maxwell's theory the Poynting vector **S** (divided by c^2) is interpreted as the density of momentum of the field. We can then also define an angular momentum relative to a given point O or to a given axis,

$$\mathbf{M} = \frac{1}{c^2}\int [\mathbf{rS}]\, d\tau, \qquad (1)$$

where **r** is the distance from O.

A plane wave travelling in the z-direction and with infinite extension in the xy-directions can have no angular momentum about the z-axis, because **S** is in the z-direction and $[\mathbf{rS}]_z = 0$. However, this is no longer the case for a wave

Andrews D.L., M. Babiker (Editors) *The angular momentum of light* (Cambrige 2013)

$$\mathbf{J} = \int dV \epsilon_0\, \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) = \mathbf{L} + \mathbf{S}, \qquad (1.20)$$

The article under consideration shows that, in reality, spin is not part of moment of a linear momentum as physicists believe; the angular momentum of light equals the sum of moment of a linear momentum and spin:

$$J^{\lambda\mu} = \int (2x^{[\lambda}T^{\mu]\nu} + \Upsilon^{\lambda\mu\nu})dV_\nu .$$

The reviewer is angry, because the author's article refutes the works of Bekshaev & Bliokh:

2*. K. Y. Bliokh, A. Y. Bekshaev, and F. Nori, "Extraordinary momentum and spin in evanescent waves", Nature Commun. 5, 3300 (2014);
3*. K. Y. Bliokh, F. Nori, "Transverse and longitudinal angular momenta of light", Physics Reports, 592, 1-38 (2015).
4*. A. Y. Bekshaev, Spin angular momentum of inhomogeneous and transversely limited light beams Proc. SPIE 6254 56–63 (2006).
5*. A. Bekshaev, M. Soskin and M. Vasnetsov, Paraxial Light Beams with Angular Momentum (New York: Nova Science Publishers, 2008) (see also arXiv:0801.2309);
6*. A. Bekshaev, K. Bliokh, M. Soskin, Internal flows and energy circulation in light beams. J. Opt. 13, 053001 (2011).

The Reviewer 1 answer

The resubmitted version is a bit improved. But just a bit. This is very unfortunate, because the author could have taken advantage of the extensive feedback to significantly change the paper and improve it enormously. I feel that the time spent providing feedback was deliberately ignored by the author.

Unfortunately, the main message has not changed. The physics could have been significantly improved, and the author decided not to improve it. This seems to be very stubborn and obstinate.

Even if the contents had been satisfactory, the style of presentation is far beyond the usual admissible frames.

It seems pointless to provide more detailed feedback, which will be ignored again. It is awfully tedious to anatomize his arguments and, after all, the author does not seem interested in learning from others, but just repeating his point of view.

If I oppose the publication of this paper, the author will send it to another journal, taking time and energies from other editors and referees (and the author will keep ignoring feedback). This process will be repeated over and over, taking lots of valuable time from many people. Thus, it seems better to just let him publish this. I will not oppose the publication of this paper. Still, before publication, it might be desirable to allow the author one last chance to significantly improve this manuscript, incorporating the feedback provided before and making profound changes to the manuscript, not minor changes here and there.

Some authors are stubborn and obstinate. We should be happy that he is interested in physics and not in world policy, or commanding armies.

The Reviewer 2 answer

The other referee found many serious problems. But the author ignored substantial parts of it.
My report listed many serious problems. But the author ignored substantial parts of it.
The author writes Let's ignore these insignificant words. several times. I have never seen such disrespectful reply in a reply to a referee report.
The author's concluding remark is this:
"There is no progress from 1936 to now, including recent works on this topic."

This implies that the work of the author has provided no progress on this topic.
The author is correct about this, his work has provided no progress on this topic,
and therefore there is no need to publish it.
The author is convinced to be right, and that others are wrong, so this settles the issue. He is right and his work has provided no progress from 1936.

Correspondence with The Physical Review

Editorial Acknowledgment AL11520 Khrapko 13 November 2017
Spin transferred to a mirror reflecting light

Your_manuscript AL11520 Khrapko 21 November 2017
Dear Dr. Khrapko, The Physical Review editors attempt to accept only papers that are scientifically sound, important to the field, and contain significant new results in physics. We judge that these acceptance criteria are not met by your manuscript.
We regret that consequently we cannot accept the paper for publication in the Physical Review.
Yours sincerely, Frank Narducci Associate Editor Physical Review A

Author's reply
Dear Editor,
I appeal your decision to reject the manuscript again
Failure to review an article that proves mistakes of authorities indicates the corruption of the American Physical Society
I ask you to review this paper.

> The paper attempts to clarify and correct some questions in one of the 4 or so century-old controversies in classical electrodynamics, perhaps the major one of interest in modern optics. The paper is on a topic where the literature is literately riddled with error, confusion, and dispute. The topic is of interest in practical issues in optical micromanipulation and of theoretical interest in the foundations of field theory and classical electrodynamics.
> The result, dealing with matters at the heart of the rather confused matter of electromagnetic angular momentum, is interesting

Your_manuscript AL11520 Khrapko 07 December 2017
This is in reference to your appeal on the above-mentioned paper. We append below the report of our Editorial Board member, Dr. Günter Steinmeyer, which sustains the decision to reject.
Under the revised Editorial Policies of the Physical Review (appended further below), this completes the scientific review of your paper.Yours sincerely, Frank Narducci Associate Editor

Report of the Editorial Board Member -- AL11520/Khrapko

I have read the manuscript „Spin transferred to a mirror reflecting light" by Dr. Khrapko. I may not be the foremost expert for reviewing this manuscript, but feel rather confident in providing some comments and observations:
1. According to the letter to the editor, it seems that the author's main point is to disprove the proportionality between spin density and the gradient of the energy density. It seems that the source of this alleged proportionality is a paper by Allen and Padgett "Response to Question #79. Does a plane wave carry spin angular momentum?" Am. J. Phys. 70, 567 (2002), which was written in reply to an earlier paper of the author, i.e., Khrapko, "Question #79 Does plane wave not carry a spin?" Am. J. Phys. 69 405 (2001). There also exists a wrap-up of the Q&A discussion that was published by A M Stewart, "Angular momentum of the electromagnetic field: the plane wave paradox resolved," Eur. J. Phys. 26 635 (2005). In the latter, numerous answers are listed that tried to explain the paradox. It may well be that not all of these answers are correct, but from reading the Stewart paper, I have the impression that there is at least some consensus how to explain the paradox. And this happened more than 10 years ago.

2. It may now be that Dr. Khrapko is not happy with the outcome of this discussion and does not accept the general consensus discussed in the review article. Then it would seem appropriate that Dr. Khrapko identifies a weak point in the discussion that is mostly based on the fact that a plane wave with infinite lateral extent does not exist in reality (as it would carry infinite energy), but that in reality there is always a radial gradient of the field. One particular problem seems to be that a hard aperture would locally create an infinitely steep gradient, but I think that the answer to this renewed paradox is rather simple, as one cannot structure a light beam on a scale significantly smaller than the wavelength.
3. In conclusion, I cannot see any useful new contribution to this >15 year old discussion. Half of the current manuscript consists of an unstructured collection of sentences that are isolated out of the context of previous publications. The Stewart paper tries to explain the connections and the physics in the Q&A papers; the current manuscript does not provide any useful new insight. We just learn that Dr. Khrapko is unhappy with the answers that his questions have received, but do not learn what is actually wrong with the apparently widely accepted explanation of the plane wave paradox.
4. Much of the paper digresses on various aspects of electrodynamics, and it is totally unclear what is actually the goal here. Dr. Khrapko would be best advised to concentrate on one point here, rather than trying to illustrate his view of the world of electrodynamics in various chapters. Without the letter to the editor, it would not have even been clear to me where the alleged problem is.
In conclusion, I therefore recommend upholding the decision to reject this manuscript. This is a case closed ten years ago, and I do not see any indication why we should reopen the discussion.
Dr. Günter Steinmeyer Editorial Board Member Physical Review A

Author's reply 9.12.2017
Dear Editor in Chief of the APS, I request that the case be reviewed by the Editor in Chief of the APS, as it is provided with the Editorial Policies and Practices

This is in reference to **unfair double rejections** of the paper AL11520 "Spin transferred to a mirror reflecting light". There was no scientific review of this paper. The Frank Narducci's message of November 21 is a **cliché**. It does not concern this paper at all. In this way, the Editors **conceal the contradiction** between nowadays concept of electrodynamics spin and the Sadowsky and Poynting's theory, which is confirmed experimentally and theoretically and is stated in textbooks.

I contest the Frank Narducci's allegation that the manuscript does not contain significant new results in physics. Contrary, this paper, in particular, refute the **nowadays delusion** that the spin density is proportional to gradient of the energy density.

Dr. Günter Steinmeyer also did not review the paper. Instead, he pointed to the articles, which proclaimed the above delusion and were exposed in the publication J. Modern Optics, **55**, 1487-1500 (2008). Dr. Günter Steinmeyer ignorred articles supporting the Sadowsky and Poynting's theory of spin. ("**Absorption of angular momentum of a plane wave"** Optik 154 (2018) 806–810, "**Reflection of light from a moving mirror"** Optik 136 (2017) 503–506)
I ask you to review the paper AL11520 .

Again, I point out that failure to review an article that proves mistakes of authorities indicates the corruption of the American Physical Society.

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на №

Michael Thoennessen Editor in Chief American Physical Society
1 Research Road Box 9000 Ridge NY 11961-9000 USA

Re: AL11520 “Spin transferred to a mirror reflecting light” by Radi I. Khrapko
http://khrapkori.wmsite.ru/ftpgetfile.php?id=158&module=files.

Dear Michael Thoennessen,
This is in reference to your letter (attached below). I am satisfied that I made you write a lie again in your letter-cliché: “your paper received a fair review” (as to the first lie, see http://khrapkori.wmsite.ru/ftpgetfile.php?id=157&module=files). In reality, the paper received no review because the paper proves the existence of a spin of plane waves. Your letter confirms my diagnosis: the corruption.

Cordially, Prof. Radi Khrapko
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APS physics

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22 December 2017
Dr. Radi I. Khrapko
Re: AL11520 Spin transferred to a mirror reflecting light
by Radi I. Khrapko

Dear Dr. Khrapko,
I have reviewed the file concerning your manuscript which was submitted to Physical Review A. The scientific review of your paper is the responsibility of the editor of Physical Review A, and resulted in the decision to reject your paper. The Editor in Chief must assure that the procedures of our journals have been followed responsibly and fairly in arriving at that decision.

On considering all aspects of this file I have concluded that our procedures have in fact been appropriately followed and that your paper received a fair review. Accordingly, I must uphold the decision of the Editors.

Yours sincerely,

Michael Thoennessen
Editor in Chief
American Physical Society