A Suggested Boundary for Heisenberg's Uncertainty Principle

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Abstract

In this paper we are combining Heisenberg's uncertainty principle with Haug's suggested maximum velocity for anything with rest-mass; see [1, 2, 3, 4]. This leads to a suggested exact boundary condition on Heisenberg's uncertainty principle. The uncertainty in position at the potential maximum momentum for subatomic particles as derived from the maximum velocity is half of the Planck length.

Perhaps Einstein was right after all when he stated, "God does not play dice." Or at least the dice may have a stricter boundary on possible outcomes than we have previously thought.

We also show how this suggested boundary condition seems to make big G consistent with Heisenberg's uncertainty principle. We obtain a mathematical expression for big G that is fully in line with empirical observations.

Hopefully our analysis can be a small step in better understanding Heisenberg's uncertainty principle and its interpretations and, by extension, the broader implications for the quantum world.

Key words: Heisenberg's uncertainty principle, maximum velocity of matter, point particle, boundary condition, big G, Planck mass particle, Planck length, reduced Compton wavelength.

1 Introduction

Haug [1, 2, 3, 4] has recently introduced a new maximum velocity for subatomic particles (anything with mass) that is just below the speed of light. The formula is given by

$$v_{max} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}} \tag{1}$$

where $\bar{\lambda}$ is the reduced Compton wavelength of the particle we are trying to accelerate and l_p is the Planck length [5]. This formula can be derived from special relativity by simply assuming the maximum frequency one can have is the Planck frequency $\frac{c}{l_p}$, or that the shortest wavelength possible is the Planck length. We will also get the same formula if we assume that the ultimate fundamental particle has a spatial dimension equal to l_p and is always traveling at the speed of light, a model outlined by [6, 1].

This maximum velocity puts an upper boundary condition on the kinetic energy, the momentum, and the relativistic mass, as well as on the relativistic Doppler shift in relation to subatomic particles. Basically, no fundamental particle can attain a relativistic mass higher than the Planck mass, and the shortest reduced Compton wavelength we can observe from length contraction is the Planck length. In addition, the maximum frequency is limited to the Planck frequency. Here we will combine this equation with Heisenberg's uncertainty principle.

2 Heisenberg's Uncertainty Principle in Relation to Maximum Momentum

Heisenberg's uncertainty principle [8] is given by[1]

$$\sigma_x \sigma_p \geq \frac{\hbar}{2} \tag{2}$$

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[1] See also Kennard [9], who was the first to "prove" this modern inequality based on the work of Heisenberg.

where σ_x is considered to be the uncertainty in the position, σ_p is the uncertainty in the momentum, and $\hbar$ is the reduced Planck constant.

Haug [1] has shown that the maximum momentum for a fundamental particle likely is given by

$$\begin{aligned}
p_{max} &= \frac{mv_{max}}{\sqrt{1-\frac{v_{max}^2}{c^2}}} \\
p_{max} &= \frac{mv_{max}}{\sqrt{1-\frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} \\
p_{max} &= \frac{mv_{max}}{\sqrt{1-\frac{\left(c^2-c^2\frac{l_p^2}{\bar{\lambda}^2}\right)}{c^2}}} \\
p_{max} &= \frac{mc\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}{\frac{l_p}{\bar{\lambda}}} \\
p_{max} &= m_p c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}
\end{aligned} \tag{3}$$

Based on this we can find a lower boundary in the uncertainty of the position, σ_x, for of any fundamental particle when assuming that σ_p is limited to the maximum momentum for the subatomic particle in question. From this we get

$$\begin{aligned}
\sigma_x\sigma_p &\geq \frac{\hbar}{2} \\
\sigma_x\frac{mv_{max}}{\sqrt{1-\frac{v_{max}^2}{c^2}}} &\geq \frac{\hbar}{2} \\
\sigma_x m_p v_{max} &\geq \frac{\hbar}{2} \\
\sigma_x m_p c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} &\geq \frac{\hbar}{2} \\
\sigma_x &\geq \frac{\hbar}{2m_p c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}
\end{aligned} \tag{4}$$

and since the Planck mass can be written as $m_p = \frac{\hbar}{l_p}\frac{1}{c}$, we can rewrite this as

$$\begin{aligned}
\sigma_x &\geq \frac{\hbar}{2\frac{\hbar}{l_p}\frac{1}{c}c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}} \\
\sigma_x &\geq \frac{l_p}{2\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}
\end{aligned} \tag{5}$$

For any known fundamental particle, $\bar{\lambda} >> l_p$ so we can use the first term of a series expansion: $\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} \approx 1-\frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2}$. This gives us

$$\begin{aligned}
\sigma_x &\geq \frac{l_p}{2\left(1-\frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2}\right)} \\
\sigma_x &\geq \frac{l_p}{2-\frac{l_p^2}{\bar{\lambda}^2}}
\end{aligned} \tag{6}$$

and when $\bar{\lambda} >> l_p$ we have a very good approximation by

$$\sigma_x \geq \frac{l_p}{2} \tag{7}$$

In other words, the maximum uncertainty in the position of any fundamental subatomic particle (when assuming σ_p is equal to the maximum momentum of the particle) is half the Planck length. This lies in strong contrast to standard physics, where there is basically no boundary on the maximum momentum a fundamental particle can achieve as long as it is below infinity. Therefore, in the standard theory there is no limit on how close σ_x can be, relative to zero. As [10] recently has shown, this leads to absurd possibilities for relativistic mass, kinetic energy, and momentum. Under the standard theory, an electron could attain a relativistic mass equal to the rest-mass of the Moon, the Earth, the Sun, and even the entire observable universe while still traveling below the speed of light.

In the new theory presented by Haug no fundamental particle can attain a relativistic mass larger than the Planck mass. Under our new interpretation of Heisenberg's principle there is an exact upper limit on the momentum equal to the Planck momentum, and it is identical for all subatomic fundamental particles. Naturally this will only hold true because their maximum velocities are not the same and are dependent on their reduced Compton wavelengths. Our theory gives an exact limit on how close v can get to c. For example, for an electron this maximum velocity is

$$\Delta v_{max} = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}_e^2}} \approx c \times 0.999999999999999999999999999999999999999999999124 \qquad (8)$$

This is the same maximum velocity as given by [1, 2]. These calculations require very high precision and were calculated in Mathematica.[2]

In our view, one possible interpretation is that the reduced Compton wavelength of the electron is contracted down to the Planck length at this maximum velocity, as discussed by [7]. In this case, we cannot claim that the electron is at an exact point location $\sigma_x \approx 0$, simply because it is not a point particle. The reduced Compton wavelength is, in our view, the distance from center to center between two indivisible particles that make up the electron, traveling back and forth counter-striking. When they are ultimately compressed (due to length contraction of the void in between the indivisibles making up the fundamental particle), the particles must lie side by side. The reduced Compton wavelength is now l_p. And our best estimate of where the electron is now would be half the Planck length, that is to say, in the middle of its contracted reduced Compton wavelength. Heisenberg's uncertainty principle combined with our maximum velocity formula possibly indicates that there can be no point particles. Alternatively, one can just interpret this as if there is a known maximum momentum for a fundamental particle, then this must be the maximum uncertainty in momentum and then there must be a limitation on how low the uncertainty in location can be, taking the Heisenberg principle into account.

Based on this maximum velocity Haug claims that the Planck-mass particle and the Planck length are the same and is invariant as seen from any reference frame. This can only hold true if the Planck mass only lasts for an instant. The Planck mass can be seen as the collision of two light particles, and therefore constitutes the turning point of light. When a photon changes direction by 180 degrees (backscattering) does it not, at the very turning point, stand still for an instant? The concept of the collision of two photons creating matter was first suggested by Breit and Wheeler 1934, see [11]. Implications from light colliding with light have recently received increased attention, see for example [12, 13, 14].

The shortest σ_x we can have in relation to a given momentum is $\frac{1}{2}l_p$, which again can be used to find the maximum velocity for any subatomic particle.

[2] We used several different set-ups in Mathematica; here is one of them: N[Sqrt[1 − (1616199 ∗ 10^(−41))^2/(3861593 ∗ 10^(−19))^2], 50], where 1616199 ∗ 10^(−41) is the Planck length and 3861593 ∗ 10^(−19) is the reduced Compton wavelength of the electron. An alternative way to write it is: N[Sqrt[1 − (SetPrecision[1.616199 ∗ 10^(−35))^2, 50]/(SetPrecision[3.861593 ∗ 10^(−13))^2, 50]], 50].

$$
\begin{aligned}
\sigma_x \sigma_p &\geq \frac{\hbar}{2} \\
\sigma_p &\geq \frac{\hbar}{2\sigma_x} \\
\sigma_p &\geq \frac{\hbar}{2\frac{1}{2}l_p} \\
\frac{mv}{\sqrt{1-\frac{\Delta v^2}{c^2}}} &\geq \frac{\hbar}{l_p} \\
\frac{v}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \frac{\hbar}{l_p m} \\
\frac{v^2}{1-\frac{v^2}{c^2}} &\geq \frac{\hbar^2}{l_p^2 m^2} \\
v^2 &\leq \frac{\hbar^2}{l_p^2 \frac{\hbar^2}{\bar{\lambda}^2}\frac{1}{c^2}}\left(1-\frac{v^2}{c^2}\right) \\
v^2 &\leq \frac{\bar{\lambda}^2 c^2}{l_p^2}\left(1-\frac{v^2}{c^2}\right) \\
v^2 &\leq \frac{\bar{\lambda}^2 c^2}{l_p^2} - \frac{\bar{\lambda}^2}{l_p^2} v^2 \\
v^2\left(1+\frac{\bar{\lambda}^2}{l_p^2}\right) &\leq \frac{\bar{\lambda}^2 c^2}{l_p^2} \\
v^2 &\leq \frac{\frac{\bar{\lambda}^2 c^2}{l_p^2}}{\left(1+\frac{\bar{\lambda}^2}{l_p^2}\right)} \\
v &\leq \frac{c}{\sqrt{1+\frac{l_p^2}{\bar{\lambda}^2}}}
\end{aligned} \tag{9}
$$

This is the maximum uncertainty in velocity for a subatomic particle with known mass or known reduced Compton wavelength. A Taylor series expansion gives

$$
v \leq \frac{c}{\sqrt{1+\frac{l_p^2}{\bar{\lambda}^2}}} \approx c\left(1-\frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2}+\frac{3}{8}\frac{l_p^4}{\bar{\lambda}^4}-\frac{5}{16}\frac{l_p^6}{\bar{\lambda}^6}...\right) \tag{10}
$$

and the maximum velocity formula suggested by Haug is

$$
v_{max} = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} \approx c\left(1-\frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2}+\frac{1}{8}\frac{l_p^4}{\bar{\lambda}^4}-\frac{1}{16}\frac{l_p^6}{\bar{\lambda}^6}...\right) \tag{11}
$$

In both formulas we get a highly accurate result by using the first term of the Taylor expansion and we see they are the same.

We are not the only ones to suggest an absolute minimum uncertainty in the position of any particle, such as an electron. Adler and Santiago [15] have, based on assumed gravitational interaction of the photon and the particle being observed, modified the uncertainty principle with an additional term. By doing this they find a minimum uncertainty in the position that is not far from our prediction. The strength in our result is that no additional terms in the Heisenberg principle are needed to get a minimum uncertainty in the position of any particle, and thereby also a maximum limit in the uncertainty of the momentum.

3 Time and Energy

In terms of time and energy, Heisenberg's uncertainty principle can be written as

$$
\sigma_t \sigma_E \geq \frac{\hbar}{2} \tag{12}
$$

Haug [1] has shown that the maximum kinetic energy of a fundamental particle with reduced Compton wavelength of $\bar{\lambda}$ is given by

$$
\begin{aligned}
E_{k,max} &= \frac{mc^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}} - mc^2 \\
E_{k,max} &= \frac{mc^2}{\sqrt{1-\frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} - mc^2 \\
E_{k,max} &= \frac{mc^2}{\sqrt{1-\frac{c^2\left(1-\frac{l_p^2}{\bar{\lambda}^2}\right)}{c^2}}} - mc^2 \\
E_{k,max} &= \frac{mc^2}{\frac{l_p}{\bar{\lambda}}} - mc^2 \\
E_{k,max} &= \frac{\bar{\lambda}}{l_p}mc^2 - mc^2 \\
E_{k,max} &= \frac{\bar{\lambda}}{l_p}\frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2 - \frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2 \\
E_{k,max} &= \frac{\hbar}{l_p}c - \frac{\hbar}{\bar{\lambda}}c \\
E_{k,max} &= \hbar c\left(\frac{1}{l_p} - \frac{1}{\bar{\lambda}}\right) \qquad (13)
\end{aligned}
$$

We can use this result in Heisenberg's time energy uncertainty inequality equation

$$
\begin{aligned}
\sigma_t\sigma_E &\geq \frac{\hbar}{2} \\
\sigma_t\hbar c\left(\frac{1}{l_p} - \frac{1}{\bar{\lambda}}\right) &\geq \frac{\hbar}{2} \\
\sigma_t &\geq \frac{\hbar}{2\hbar c\left(\frac{1}{l_p} - \frac{1}{\bar{\lambda}}\right)} \\
\sigma_t &\geq \frac{1}{2c\left(\frac{1}{l_p} - \frac{1}{\bar{\lambda}}\right)} \qquad (14)
\end{aligned}
$$

and when $\bar{\lambda} >> l_p$, we have a very good approximation by

$$
\sigma_t \geq \frac{1}{2}\frac{l_p}{c} \qquad (15)
$$

Which is half a Planck second. It is worth mentioning that the half Planck second and half Planck length found as boundary conditions here are exactly the same as the results we obtained when looking at the Lorentz transformation in the limit of the maximum velocity of mass [16].

4 Big G and Heisenberg's Uncertainty Principle

As shown in [3], the maximum velocity can also be written as

$$
v_{max} = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} = c\sqrt{1-\frac{Gm^2}{\hbar c}} \qquad (16)
$$

where G is Newton's gravitational constant [17] and m is the mass of a fundamental particle. It is important to understand m in this context is not just any mass; this mass must have a reduced Compton wavelength. In other words, it is the mass of fundamental particles. Based on this observation, we can assess whether or not we can use this in combination with Heisenberg's uncertainty principle to derive a theoretical value of big G. We are not the first to suggest that Heisenberg's uncertainty principle could be related to Newtonian gravity. McCulloch [18] has shown that Newton's gravity formula basically can be derived from Heisenberg's uncertainty principle. However, he has not shown how big G also can be derived from it.

We could also say that this is just another way to show the maximum velocity for matter may be consistent with Heisenberg's uncertainty principle, although this should not be considered as evidence that we will get big G from Heisenberg's uncertainty principle. We have

$$\begin{aligned}
\sigma_x\sigma_p &\geq \frac{\hbar}{2} \\
\sigma_x\frac{mv_{max}}{\sqrt{1-\frac{v_{max}^2}{c^2}}} &\geq \frac{\hbar}{2} \\
\sigma_x m_p v_{max} &\geq \frac{\hbar}{2} \\
\sigma_x m_p c\sqrt{1-\frac{Gm^2}{\hbar c}} &\geq \frac{\hbar}{2} \\
\sqrt{1-\frac{Gm^2}{\hbar c}} &\geq \frac{\hbar}{2\sigma_x m_p c} \\
1-\frac{Gm^2}{\hbar c} &\geq \frac{\hbar^2}{4\sigma_x^2 m_p^2 c^2} \\
G &\geq \frac{\hbar c}{m^2}-\frac{\hbar^2\hbar c}{4\sigma_x^2 m_p^2 c^2 m^2} \\
G &\geq \frac{\hbar c}{\frac{\hbar^2}{\bar{\lambda}^2}\frac{1}{c^2}}-\frac{\hbar^2\hbar c}{4\frac{\hbar^2}{4m_p^2 v_{max}^2}m_p^2c^2m^2} \\
G &\geq \frac{\bar{\lambda}^2c^3}{\hbar}-\frac{\hbar c v_{max}^2}{m^2c^2} \\
G &\geq \frac{\bar{\lambda}^2c^3}{\hbar}-\frac{\hbar c c^2\left(1-\frac{l_p^2}{\bar{\lambda}^2}\right)}{m^2c^2} \\
G &\geq \frac{\bar{\lambda}^2c^3}{\hbar}-\frac{\hbar c\left(1-\frac{l_p^2}{\bar{\lambda}^2}\right)}{\frac{\hbar^2}{\bar{\lambda}^2}\frac{1}{c^2}} \\
G &\geq \frac{\bar{\lambda}^2c^3}{\hbar}-\frac{\bar{\lambda}^2c^3\left(1-\frac{l_p^2}{\bar{\lambda}^2}\right)}{\hbar} \\
G &\geq \frac{l_p^2c^3}{\hbar}\approx 6.67384\times 10^{-11} \qquad (17)
\end{aligned}$$

To write the gravitational constant as $G=\frac{l_p^2c^3}{\hbar}$ has already been suggested by Haug [19, 20] in order to simplify a series of expressions in Newtonian and Einsteinian gravity end results. It has also been derived by dimensional analysis [3] and used to simplify the equation form of the Planck units. Further, Haug has suggested that the Planck length (at least in a thought experiment) can be found independent of G based on the maximum velocity formula.

Since v_{max} here is a function of the universal constants G, $\hbar$ and c one could try to argue that this is a evidence l_p must be a function of G and $\hbar$ and c and not that G is a function of l_p. In other words, that G must be a universal constant and l_p is just a derived constant. However, the beauty of the maximum velocity formula is that G and $\hbar$ cancel out and we are left with that v_{max} only is a function of c, l_p and the reduced Compton wavelength of the particle in question, $\bar{\lambda}$, and not of G and $\hbar$. It is worth pointing out that the reduced Compton wavelength of an electron can be found experimentally, completely independent of any knowledge of G, see [21]. To find l_p one needs the reduced Compton wavelength that can be found totally independent on G as well at the maximum velocity for an electron, v_{max}. This maximum velocity has to be found experimentally. This maximum velocity for an electron is very close to c, but still higher than the velocities one operates with at the Large Hadron Collider. However, the fact that something is predicted but not yet found, is not a sufficient argument for rejecting a theory outright.

Our formula for big G gives the same value as the gravitational constant, as is known from experiments, it can actually be calibrated to the experiments. There is still considerable uncertainty about the exact measurement of the gravitational constant. Experimentally, substantial progress has been made in recent years based on various methods. See, for example, [22, 23, 24, 25, 26]. In the formula presented here, the uncertainty lies in the exact value of the Planck length, as well as in $\hbar$; the speed of light $c = 299792458$

is exact per definition. At the moment, the Planck length can only be found from G, but if we had access to much more advanced particle accelerators than the Large Hadron Collider, we could expect to detect v_{max} and then back the Planck length out from there. We claim that big G is indeed a universal constant, but it is a composite constant that is dependent on three even more fundamental constants, namely $\hbar$, l_p, and c.

5 Conclusion

By combining Heisenberg's uncertainty principle with the newly introduced maximum velocity on mass, we have shown that the smallest location uncertainty of a fundamental particle is related to half the Planck length, and that the shortest time interval is related to half the Planck time. This is the "same" finding as the one we obtained when we combined this maximum velocity with the Lorentz transformation [16].

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