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# On Soft Mixed Neutrosophic N-Algebraic Structures

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**Abstract**: Soft set theory is a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. In this paper we introduced soft mixed neutrosophic N-algebraic with the discussion of some of their characteristics. We also introduced soft mixed dual neutrosophic N-algebraic structures, soft weak mixed neutrosophic N-algebraic structures, soft Lagrange mixed neutrosophic N-algebraic structures, soft weak Lagrange mixed neutrosophic and soft Lagrange free mixed neutosophic N-algebraic structures. the so called soft strong neutrosophic loop which is of pure neutrosophic character. We also introduced some of new notions and some basic properties of this newly born soft mixed neutrosophic N-structures related to neutrosophic theory.

**Key Words**: Neutrosophic mixed N-algebraic structure, soft set, soft neutrosophic mixed neutrosophic N-algebraic structure.

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## §1. Introduction

Smarandache proposed the concept of neutrosophy in 1980, which is basically a new branch of philosophy that actually deals the origion, nature, and scope of neutralities. He also introduced the neutrosophic logic due to neutrosophy. In neutrosophic logic each proposition is approximated to have the percentage of truth in a subset $T$, the percentage of indeterminacy in a subset $I$ and the percentage of falsity in a subset $F$. Basically, a neutrosophic logic is an extension of fuzzy logic. In fact the neutrosophic set is the generalization of classical set, fuzzy conventional set, intuitionistic fuzzy set etc. Neutrosophic logic is used to overcome the problems of imperciseness, indeterminate and inconsistentness of the data. The theoy of neutrosophy is also applicable in the field of algebra. For example, Kandasamy and Smarandache introduced neutrosophic fields, neutrosophic rings, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups and neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemi-

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groups, and neutrsosophic N-semigroups, neutrosophic loops, nuetrosophic biloops, and neutrosophic N-loops, and so on. Mumtaz ali et.al. introduced nuetosophic LA-semigoups and also give their generalization.

Molodtsov intorduced the theory of soft set. This mathematical tool is free from parameterization inadequacy, syndrome of fuzzy set theory, rough set theory, probability theory and so on. This theory has been applied successfully in many fields such as smoothness of functions, game theory, operation research, Riemann integration, Perron integration, and probability. Recently soft set theory attained much attention of the researchers since its appearance and the work based on several operations of soft set introduced in [5, 6, 7, 8, 10]. Some properties and algebra may be found in [11]. Feng et.al. introduced soft semirings in [9].

In this paper we introduced soft mixed nuetrosophic N-algebraic structures. The organization of this paper is follows. In section one we put the basic concepts about mixed neutrosophic N-algebraic structures and soft sets with some of their operations. In the next sections we introduce soft mixed neutrosophic N-algebraic structures with the construction of some their related theory. At the end we concluded the paper.

## §2. Basic Concepts

### 2.1 Mixed Neutrosophic $N$-Algebraic Structures

**Definition** 2.1 *Let $\{\langle M\cup I\rangle = (M_1\cup M_2\cup\cdots\cup M_N, *_1, *_2, \cdots, *_N)\}$ such that $N\geq 5$. Then $\langle M\cup I\rangle$ is called a mixed neutrosophic $N$-algebraic structure if*

(1) *$\langle M\cup I\rangle = M_1\cup M_2\cup\cdots\cup M_N$, where each $M_i$ is a proper subset of $\langle M\cup I\rangle$ for all $i$;*

(2) *some of $(M_i, *_i)$ are neutrosophic groups;*

(3) *some of $(M_j, *_j)$ are neutrosophic loops;*

(4) *some of $(M_k, *_k)$ are neutrosophic groupoids;*

(5) *some of $(M_r, *_r)$ are neutrosophic semigroups.*

(6) *the rest of $(M_t, *t)$ can be loops or groups or semigroups or groupoids.* (*'or' not used in the mutually exclusive sense*).

**Definition** 2.2 *Let $\{\langle D\cup I\rangle = (D_1\cup D_2\cup\cdots\cup D_N, *_1, *_2, \cdots, *_N)\}$. Then $\langle D\cup I\rangle$ is called a mixed dual neutrosophic $N$-algebraic structure if*

(1) *$\langle D\cup I\rangle = D_1\cup D_2\cup\cdots\cup D_N$, where each $D_i$ is a proper subset of $\langle D\cup I\rangle$ for all $i$;*

(2) *some of $(D_i, *_i)$ are neutrosophic groups;*

(3) *some of $(D_j, *_j)$ are neutrosophic loops;*

(4) *some of $(D_k, *_k)$ are neutrosophic groupoids;*

(5) *some of $(D_r, *_r)$ are neutrosophic semigroups;*

(6) *the rest of $(D_t, *t)$ can be loops or groups or semigroups or groupoids.* (*'or' not used in the mutually exclusive sense*).

**Definition** 2.3 *Let $\{\langle W\cup I\rangle = (W_1\cup W_2\cup\cdots\cup W_N, *_1, *_2, \cdots, *_N)\}$. Then $\langle W\cup I\rangle$ is called a weak mixed neutrosophic $N$-algebraic structure if*

*(1) $\langle W \cup I \rangle = W_1 \cup W_2 \cup ... \cup W_N$, where each $W_i$ is a proper subset of $\langle W \cup I \rangle$ for all $i$;*

*(2) some of $(W_i, *_i)$ are neutrosophic groups or neutrosophic loops;*

*(3) some of $(W_k, *_k)$ are neutrosophic groupoids or neutrosophic semigroups;*

*(4) the Rest of $(W_t, *t)$ can be loops or groups or semigroups or groupoids. i.e in the collection , all the algebraic neutrosophic structures may not be present.*

At most 3-algebraic neutrosophic structures are present and at least 2-algebraic neutrosophic structures are present. Rest being non-neutrosophic algebraic structures.

**Definition** 2.4 *Let $\{\langle V \cup I \rangle = (V_1 \cup V_2 \cup \cdots \cup V_N, *_1, *_2, \cdots, *_N)\}$. Then $\langle V \cup I \rangle$ is called a weak mixed dual neutrosophic $N$-algebraic structure if*

*(1) $\langle V \cup I \rangle = V_1 \cup V_2 \cup \cdots \cup V_N$, where each $W_i$ is a proper subset of $\langle V \cup I \rangle$ for all $i$;*

*(2) some of $(V_i, *_i)$ are neutrosophic groups or neutrosophic loops;*

*(3) some of $(V_k, *_k)$ are neutrosophic groupoids or neutrosophic semigroups;*

*(4) the rest of $(V_t, *t)$ can be loops or groups or semigroups or groupoids.*

**Definition** 2.5 *Let $\{\langle M \cup I \rangle = (M_1 \cup M_2 \cup \cdots \cup M_N, *_1, *_2, \cdots, *_N)\}$ be a neutrosophic $N$-algebraic structure. A proper subset $\{\langle P \cup I \rangle = (P_1 \cup P_2 \cup \cdots \cup P_N, *_1, *_2, \cdots, *_N)\}$is called a mixed neutrosophic sub $N$-structure if $\langle P \cup I \rangle$ is a mixed neutrosophic $N$-structure under the operation of $\langle M \cup I \rangle$.*

**Definition** 2.6 *Let $\{\langle W \cup I \rangle = (W_1 \cup W_2 \cup \cdots \cup W_N, *_1, *_2, \cdots, *_N)\}$ be a mixed neutrosophic $N$-algebraic structure. We call a finite non-empty subset $P$ of $\langle W \cup I \rangle$ to be a weak mixed deficit neutrosophic sub $N$-algebraic structure if $\{\langle P \cup I \rangle = (P_1 \cup P_2 \cup \cdots \cup P_t, *_1, *_2, \cdots, *_t)\}$, $1 < t < N$ with $P_i = P \cap L_k$, $1 \leq i \leq t$, and $1 \leq k \leq N$ and some $P_i's$ are neutrosophic groups or neutrosophic loops, some $P_j's$ are neutrosophic groupoids or neutrosophic semigroups and the rest of $P_k's$ are groups or loops or groupoids or semigroups.*

**Definition** 2.7 *Let $\{\langle M \cup I \rangle = (M_1 \cup M_2 \cup \cdots \cup M_N, *_1, *_2, \cdots, *_N)\}$ be a mixed neutrosophic $N$-algebraic structure of finite order. A proper mixed neutrosophic sub $N$-structure $P$ of $\langle M \cup I \rangle$ is called Lagrange mixed neutrosophic sub $N$-structure if $o(P)\diagup o\langle M \cup I \rangle$.*

If every mixed neutrosophic sub $N$-structure of $\langle M \cup I \rangle$ is a Lagrange mixed neutrosophic sub $N$-structures. Then $\langle M \cup I \rangle$ is said to be a Lagrange mixed neutrosophic $N$-structure.

If some mixed neutrosophic sub $N$-structure of $\langle M \cup I \rangle$ are Lagrange mixed neutrosophic sub $N$-structures. Then $\langle M \cup I \rangle$ is said to be a weak Lagrange mixed neutrosophic $N$-structure.

If every mixed neutrosophic sub $N$-structure of $\langle M \cup I \rangle$ is not a Lagrange mixed neutrosophic sub $N$-structures. Then $\langle M \cup I \rangle$ is said to be a Lagrange free mixed neutrosophic $N$-structure.

### 2.2 Soft Sets

Throughout this subsection $U$ refers to an initial universe, $E$ is a set of parameters, $P(U)$ is the power set of $U$, and $A \subset E$. Molodtsov [12] defined the soft set in the following manner.

**Definition** 2.8 *A pair $(F, A)$ is called a soft set over $U$ where $F$ is a mapping given by $F : A \longrightarrow P(U)$.*

In other words, a soft set over $U$ is a parameterized family of subsets of the universe $U$. For $e \in A$, $F(a)$ may be considered as the set of $a$-elements of the soft set $(F, A)$, or as the set of e-approximate elements of the soft set.

**Definition** 2.9 *For two soft sets $(F, A)$ and $(H, B)$ over $U$, $(F, A)$ is called a soft subset of $(H, B)$ if*

(1) $A \subseteq B$ *and*
(2) $F(a) \subseteq G(a)$, *for all* $a \in A$.

This relationship is denoted by $(F, A) \overset{\sim}{\subset} (H, B)$. Similarly $(F, A)$ is called a soft superset of $(H, B)$ if $(H, B)$ is a soft subset of $(F, A)$ which is denoted by $(F, A) \overset{\sim}{\supset} (H, B)$.

**Definition** 2.10 *Two soft sets $(F, A)$ and $(H, B)$ over $U$ are called soft equal if $(F, A)$ is a soft subset of $(H, B)$ and $(H, B)$ is a soft subset of $(F, A)$.*

**Definition** 2.11 *$(F, A)$ over $U$ is called an absolute soft set if $F(a) = U$ for all $a \in A$ and we denote it by $\mathcal{F}_U$.*

**Definition** 2.12 *Let $(F, A)$ and $(G, B)$ be two soft sets over a common universe $U$ such that $A \cap B \neq \phi$. Then their restricted intersection is denoted by$(F, A) \cap_R (G, B) = (H, C)$ where $(H, C)$ is defined as $H(c) = F(a) \cap G(a)$ for all $a \in C = A \cap B$.*

**Definition** 2.13 *The extended intersection of two soft sets $(F, A)$ and $(G, B)$ over a common universe $U$ is the soft set $(H, C)$, where $C = A \cup B$, and for all $a \in C$, $H(a)$ is defined as*

$$H(a) = \begin{cases} F(a) & \text{if } a \in A - B \\ G(a) & \text{if } a \in B - A \\ F(a) \cap G(a) & \text{if } a \in A \cap B. \end{cases}$$

We write $(F, A) \cap_\varepsilon (G, B) = (H, C)$.

**Definition** 2.14 *The restricted union of two soft sets $(F, A)$ and $(G, B)$ over a common universe $U$ is the soft set $(H, C)$, where $C = A \cup B$, and for all $a \in C$, $H(a)$ is defined as the soft set $(H, C) = (F, A) \cup_R (G, B)$ where $C = A \cap B$ and $H(a) = F(a) \cup G(a)$ for all $a \in C$.*

**Definition** 2.15 *The extended union of two soft sets $(F, A)$ and $(G, B)$ over a common universe $U$ is the soft set $(H, C)$, where $C = A \cup B$, and for all $a \in C$, $H(a)$ is defined as*

$$H(a) = \begin{cases} F(a) & \text{if } a \in A - B \\ G(a) & \text{if } a \in B - A \\ F(a) \cup G(a) & \text{if } a \in A \cap B. \end{cases}$$

We write $(F, A) \cup_\varepsilon (G, B) = (H, C)$.

## §3. Soft Mixed Neutrosophic N-Algebraic Structures

**Definition** 3.1 *Let $\langle M \cup I\rangle$ be a mixed neutrosophic $N$-algebraic structure and let $(F, A)$ be a soft set over $\langle M \cup I\rangle$. Then $(F, A)$ is called a soft mixed neutrosophic $N$-algebraic structure if and only if $F(a)$ is a mixed neutrosophic sub $N$-algebraic structure of $\langle M \cup I\rangle$ for all $a \in A$.*

**Example** 3.1 Let $\{\langle M \cup I\rangle = M_1 \cup M_2 \cup M_3 \cup M_4 \cup M_5, *_1, *_2, \cdots, *_5\}$ be a mixed neutosophic 5-structure, where

$M_1 = \langle \mathbb{Z}_3 \cup I\rangle$, a neutrosophic group under multiplication mod3,
$M_2 = \langle \mathbb{Z}_6 \cup I\rangle$, a neutrosophic semigroup under multiplication mod6,
$M_3 = \{0, 1, 2, 3, I, 2I, 3I\}$, a neutrosophic groupoid under multiplication mod4,
$M_4 = S_3$, and
$M_5 = \{\mathbb{Z}_{10}$, a semigroup under multiplication mod10$\}$.

Let $A = \{a_1, a_2, a_3\} \subset E$ be a set of parameters and let $(F, A)$ be a soft set over $\langle M \cup I\rangle$, where

$$
\begin{aligned}
F(a_1) &= \{1, I\} \cup \{0, 3, 3I\} \cup \{0, 2, 2I\} \cup A_3 \cup \{0, 2, 4, 6, 8\}, \\
F(a_2) &= \{2, I\} \cup \{0, 2, 4, 2I, 4I\} \cup \{0, 2, 2I\} \cup A_3 \cup \{0, 5\}, \\
F(a_3) &= \{1, 2\} \cup \{0, 3\} \cup \{0, 2\} \cup A_3 \cup \{0, 2, 4, 6, 8\}.
\end{aligned}
$$

Clearly $(F, A)$ is a soft mixed neutrosophic 5-algebraic structure over $\langle M \cup I\rangle$.

**Proposition** 3.1 *Let $(F, A)$ and $(H, A)$ be two soft mixed neutrosophic $N$-algebraic structures over $\langle M \cup I\rangle$. Then their intersection is again a soft mixed neutrosophic $N$-algebraic structure over $\langle M \cup I\rangle$.*

*Proof* The proof is straightforward. □

**Proposition** 3.2 *Let $(F, A)$ and $(H, B)$ be two soft mixed neutrosophic $N$-algebraic structures over $\langle M \cup I\rangle$. If $A \cap B = \phi$, then $(F, A) \cap (H, B)$ is a soft mixed neutrosophic $N$-algebraic structure over $\langle M \cup I\rangle$.*

*Proof* The proof is straightforward. □

**Proposition** 3.3 *Let $(F, A)$ and $(H, B)$ be two soft mixed neutrosophic $N$-algebraic structure over $\langle M \cup I\rangle$. Then*

(1) *their extended intersection is a soft mixed neutrosophic $N$-algebraic structure over $\langle M \cup I\rangle$;*

(2) *their restricted intersection is a soft mixed neutrosophic $N$-algebraic structure over $\langle M \cup I\rangle$;*

(3) *their $AND$ operation is a soft mixed neutrosophic $N$-algebraic structure over $\langle M \cup I\rangle$.*

**Remark** 3.1 Let $(F, A)$ and $(H, B)$ be two soft mixed neutrosophic $N$-algebraic structure over

$\langle M \cup I \rangle$. Then

(1) their restricted union may not be a soft mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$.

(2) their extended union may not be a soft mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$.

(3) their $OR$ operation may not be a soft mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$.

To establish the above remark, see the following example.

**Example** 3.2 Let $\{\langle M \cup I \rangle = M_1 \cup M_2 \cup M_3 \cup M_4 \cup M_5, *_1, *_2, \cdots, *_5\}$ be a mixed neutosophic 5-structure, where

$M_1 = \langle \mathbb{Z}_3 \cup I \rangle$, a neutrosophic group under multiplication mod3,

$M_2 = \langle \mathbb{Z}_6 \cup I \rangle$, a neutrosophic semigroup under multiplication mod6,

$M_3 = \{0, 1, 2, 3, I, 2I, 3I\}$, a neutrosophic groupoid under multiplication mod4,

$M_4 = S_3$, and

$M_5 = \{\mathbb{Z}_{10}$, a semigroup under multiplication mod10$\}$.

Let $A = \{a_1, a_2, a_3\} \subset E$ be a set of parameters and let $(F, A)$ be a soft set over $\langle M \cup I \rangle$, where

$$\begin{aligned} F(a_1) &= \{1, I\} \cup \{0, 3, 3I\} \cup \{0, 2, 2I\} \cup A_3 \cup \{0, 2, 4, 6, 8\}, \\ F(a_2) &= \{2, I\} \cup \{0, 2, 4, 2I, 4I\} \cup \{0, 2, 2I\} \cup A_3 \cup \{0, 5\}, \\ F(a_3) &= \{1, 2\} \cup \{0, 3\} \cup \{0, 2\} \cup A_3 \cup \{0, 2, 4, 6, 8\}. \end{aligned}$$

Let $B = \{a_1, a_4\}$ be a another set of parameters and let $(H, B)$ be a another soft mixed neutrosophic 5-algebraic structure over $\langle M \cup I \rangle$, where

$$\begin{aligned} H(a_1) &= \{1, I\} \cup \{0, 3I\} \cup \{0, 2, 2I\} \cup A_3 \cup \{0, 2, 4, 6, 8\}, \\ H(a_4) &= \{1, 2\} \cup \{0, 3I\} \cup \{0, 2I\} \cup A_3 \cup \{0, 5\}. \end{aligned}$$

Let $C = A \cap B = \{a_1\}$. The restricted union $(F, A) \cup_R (H, B) = (K, C)$, where

$$K(a_1) = F(a_1) \cup H(a_1) = \{1, I, 2\} \cup \{0, 3I\} \cup \{0, 2, 2I\} \cup A_3 \cup \{0, 2, 4, 5, 6, 8\}$$

Thus clearly $\{1, 2, I\}$ and $\{0, 2, 4, 5, 6, 8\}$ in $H(a_1)$ are not subgroups. This shows that $(K, C)$ is not a soft mixed neutrosophic 5-algebraic structure over $\langle M \cup I \rangle$. Similarly one can easily show 2 and 3 by the help of examples.

**Definition** 3.2 *Let $\langle D \cup I \rangle$ be a mixed dual neutrosophic $N$-algebraic structure and let $(F, A)$ soft set over $\langle D \cup I \rangle$. Then $(F, A)$ is called a soft mixed dual neutrosophic $N$-algebraic structure if and only if $F(a)$ is a mixed dual neutrosophic sub $N$-algebraic structure $\langle D \cup I \rangle$ of for all $a \in A$.*

**Example** 3.3 Let $\{\langle D\cup I\rangle = D_1\cup D_2\cup D_3\cup D_4\cup D_5, *_1, *_2, \cdots, *_5\}$ be a mixed dual neutosophic 5-algebraic structure, where

$D_1 = L_7(4)$, $D_2 = S_4$, $D_3 = \{\mathbb{Z}_{10}$, a semigroup under multiplication modulo 10$\}$,
$D_4 = \{0, 1, 2, 3,$ a groupoid under multiplication modulo 4$\}$,
$D_5 = \langle L_7(4)\cup I\rangle$.

Let $A = \{a_1, a_2\}$ be a set of parameters and let $(F, A)$ be a soft set over $\langle D\cup I\rangle$, where

$$F(a_1) = \{e, 2\}\cup A_4\cup\{0, 2, 4, 6, 8\}\cup\{0, 2\}\cup\{e, eI, 2, 2I\},$$
$$F(a_2) = \{e, 3\}\cup S_4\cup\{0, 5\}\cup\{0, 2\}\cup\{e, eI, 3, 3I\}$$

Clearly $(F, A)$ is a soft mixed dual neutrosophic -structure over $\langle D\cup I\rangle$.

**Theorem** 3.1 *If $\langle D\cup I\rangle$ is a mixed dual neutrosophic $N$-algebraic structure. Then $(F, A)$ over $\langle D\cup I\rangle$ is also a soft mixed dual neutrosophic $N$-algebraic structure.*

**Proposition** 3.4 *Let $(F, A)$ and $(H, B)$ be two soft mixed dual neutrosophic $N$-algebraic structures over $\langle D\cup I\rangle$. Then their intersection is again a soft mixed dual neutrosophic $N$-algebraic structure over $\langle D\cup I\rangle$.*

*Proof* The proof is straightforward. □

**Proposition** 3.5 *Let $(F, A)$ and $(H, B)$ be two soft mixed dual neutrosophic $N$-algebraic structures over $\langle D\cup I\rangle$. If $A\cap B = \phi$, then $(F, A)\cap(H, B)$ is a soft mixed dual neutrosophic $N$-algebraic structure over $\langle D\cup I\rangle$.*

*Proof* The proof is straightforward. □

**Proposition** 3.6 *Let $(F, A)$ and $(H, B)$ be two soft mixed dual neutrosophic $N$-algebraic structures over $\langle D\cup I\rangle$. Then*

(1) *their extended intersection is a soft mixed dual neutrosophic $N$-algebraic structure over $\langle D\cup I\rangle$;*

(2) *their restricted intersection is a soft mixed dual neutrosophic $N$-algebraic structure over $\langle D\cup I\rangle$;*

(3) *their $AND$ operation is a soft mixed dual neutrosophic $N$-algebraic structure over $\langle D\cup I\rangle$.*

**Remark** 3.2 Let $(F, A)$ and $(H, B)$ be two soft mixed Dual neutrosophic $N$-algebraic structures over $\langle D\cup I\rangle$. Then

(1) their restricted union may not be a soft mixed dual neutrosophic $N$-algebraic structure over $\langle D\cup I\rangle$;

(2) their extended union may not be a soft mixed dual neutrosophic $N$-algebraic structure over $\langle D\cup I\rangle$;

(3) their $OR$ operation may not be a soft mixed dual neutrosophic $N$-algebraic structure over $\langle D \cup I \rangle$.

One can easily establish the above remarks by the help of examples.

**Definition** 3.3 *Let $\langle W \cup I \rangle$ be a weak mixed neutrosophic $N$-algebraic structure and let $(F, A)$ soft set over $\langle W \cup I \rangle$. Then $(F, A)$ is called a soft weak mixed neutrosophic $N$-algebraic structure if and only if $F(a)$ is a weak mixed neutrosophic sub $N$-structure of $\langle W \cup I \rangle$ for all $a \in A$.*

**Theorem** 3.2 *If $\langle W \cup I \rangle$ is a weak mixed neutrosophic $N$-algebraic structure. Then $(F, A)$ over $\langle W \cup I \rangle$ is also a soft weak mixed neutrosophic $N$-algebraic structure.*

The restricted intersection, extended intersection and the $AND$ operation of two soft weak mixed neutrosophic $N$-algebraic structures is again soft weak mixed neutrosophic $N$-algebraic structures.

The restricted union, extended union and the $OR$ operation of two soft weak mixed neutrosophic $N$-algebraic structures may not be soft weak mixed neutrosophic $N$-algebraic structures.

**Definition** 3.4 *Let $\langle V \cup I \rangle$ be a weak mixed dual neutrosophic $N$-algebraic structure and let $(F, A)$ soft set over $\langle V \cup I \rangle$. Then $(F, A)$ is called a soft weak mixed dual neutrosophic $N$-algebraic structure if and only if $F(a)$ is a weak mixed dual neutrosophic sub $N$-structure of $\langle V \cup I \rangle$ for all $a \in A$.*

**Theorem** 3.3 *If $\langle V \cup I \rangle$ is a weak mixed dual neutrosophic $N$-algebraic structure. Then $(F, A)$ over $\langle V \cup I \rangle$ is also a soft weak mixed dual neutrosophic $N$-algebraic structure.*

The restricted intersection, extended intersection and the $AND$ operation of two soft weak mixed dual neutrosophic $N$-algebraic structures is again a soft weak mixed dual neutrosophic $N$-algebraic structures.

The restricted union, extended union and the $OR$ operation of two soft weak mixed dual neutrosophic $N$-algebraic structures may not be soft weak mixed dual neutrosophic $N$-algebraic structures.

**Definition** 3.5 *Let $(F, A)$ and $(H, B)$ be two soft mixed neutrosophic $N$-algebraic structures over $\langle M \cup I \rangle$. Then $(H, B)$ is called soft mixed neutrosophic sub $N$-algebraic structure of $(F, A)$, if*

*(1) $B \subseteq A$;*

*(2) $H(a)$ is a mixed neutrosophic sub $N$-structure of $F(a)$ for all $a \in A$.*

It is important to note that a soft mixed neutrosophic $N$-algebraic structure can have soft weak mixed neutrosophic sub $N$-algebraic structure. But a soft weak mixed neutrosophic sub $N$-structure cannot in general have a soft mixed neutrosophic $N$-structure.

**Definition** 3.6 *Let $\langle V \cup I \rangle$ be a weak mixed neutrosophic $N$-algebraic structure and let $(F, A)$ be a soft set over $\langle V \cup I \rangle$. Then $(F, A)$ is called a soft weak mixed deficit neutrosophic $N$-*

*algebraic structure if and only if $F(a)$ is a weak mixed deficit neutrosophic sub $N$-structure of $\langle V \cup I \rangle$ for all $a \in A$.*

**Proposition** 3.7 *Let $(F, A)$ and $(H, B)$ be two soft weak mixed deficit neutrosophic $N$-algebraic structures over $\langle V \cup I \rangle$. Then*

(1) *their extended intersection is a soft weak mixed deficit neutrosophic $N$-algebraic structure over $\langle V \cup I \rangle$;*

(2) *their restricted intersection is a soft weak mixed deficit neutrosophic $N$-algebraic structure over $\langle V \cup I \rangle$;*

(3) *their $AND$ operation is a soft weak mixed deficit neutrosophic $N$-algebraic structure over $\langle V \cup I \rangle$.*

**Remark** 3.3 Let $(F, A)$ and $(H, B)$ be two soft weak mixed deficit neutrosophic $N$-algebraic structures over $\langle V \cup I \rangle$. Then

(1) their restricted union may not be a soft weak mixed deficit neutrosophic $N$-algebraic structure over $\langle V \cup I \rangle$;

(2) their extended union may not be a soft weak mixed deficit neutrosophic $N$-algebraic structure over $\langle V \cup I \rangle$;

(3) their $OR$ operation may not be a soft weak mixed deficit neutrosophic $N$-algebraic structure over $\langle V \cup I \rangle$.

One can easily establish the above remarks by the help of examples.

**Definition** 3.7 *Let $\langle M \cup I \rangle$ be a mixed neutrosophic $N$-algebraic structure and let $(F, A)$ soft set over $\langle M \cup I \rangle$. Then $(F, A)$ is called a soft Lagrange mixed neutrosophic $N$-algebraic structure if and only if $F(a)$ is a Lagrange mixed neutrosophic sub $N$-structure of $\langle M \cup I \rangle$ for all $a \in A$.*

**Theorem** 3.4 *If $\langle M \cup I \rangle$ is a Lagrange mixed neutrosophic $N$-algebraic structure. Then $(F, A)$ over $\langle M \cup I \rangle$ is also a soft Lagrange mixed neutrosophic $N$-algebraic structure.*

**Remark** 3.4 Let $(F, A)$ and $(H, B)$ be two soft Lagrange mixed neutrosophic $N$-algebriac structures over $\langle M \cup I \rangle$. Then

(1) their restricted union may not be a soft Lagrange mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$;

(2) their extended union may not be a soft Lagrange mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$;

(3) their $AND$ operation may not be a soft Lagrange mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$;

(4) their extended intersection may not be a soft Lagrange mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$;

(5) their restricted intersection may not be a soft Lagrange mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$.

(6) their $OR$ operation may not be a soft Lagrange mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$.

One can easily establish the above remarks by the help of examples.

Now on similar lines, we can define soft Lagrange weak deficit mixed neutrosophic $N$-algebraic structures.

**Definition** 3.8 *Let $\langle M \cup I \rangle$ be a mixed neutrosophic $N$-algebraic structure and let $(F, A)$ be a soft set over $\langle M \cup I \rangle$. Then $(F, A)$ is called a soft weak Lagrange mixed neutrosophic $N$-algebraic structure if and only if $F(a)$ is not a Lagrange mixed neutrosophic sub $N$-structure of $\langle M \cup I \rangle$ for some $a \in A$.*

**Remark** 3.5 Let $(F, A)$ and $(H, B)$ be two soft weak Lagrange mixed neutrosophic $N$-algebraic structures over $\langle M \cup I \rangle$. Then

(1) their restricted union may not be a soft weak Lagrange mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$;

(2) their extended union may not be a soft weak Lagrange mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$;

(3) their $AND$ operation may not be a soft weak Lagrange mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$;

(4) their extended intersection may not be a soft weak Lagrange mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$;

(5) their restricted intersection may not be a soft weak Lagrange mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$;

(6) their $OR$ operation may not be a soft weak Lagrange mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$.

One can easily establish the above remarks by the help of examples. Similarly we can define soft weak Lagrange weak deficit mixed neutrosophic $N$-algebraic structures.

**Definition** 3.9 *Let $\langle M \cup I \rangle$ be a mixed neutrosophic $N$-algebraic structure and let $(F, A)$ be a soft set over $\langle M \cup I \rangle$. Then $(F, A)$ is called a soft Lagrange free mixed neutrosophic $N$-algebraic structure if and only if $F(a)$ is not a Lagrange mixed neutrosophic sub $N$-structure of $\langle M \cup I \rangle$ for all $a \in A$.*

**Theorem** 3.5 *If $\langle M \cup I \rangle$ is a Lagrange free mixed neutrosophic $N$-algebraic structure. Then $(F, A)$ over $\langle M \cup I \rangle$ is also a soft Lagrange free mixed neutrosophic $N$-algebraic structure.*

**Remark** 3.6 Let $(F, A)$ and $(H, B)$ be two soft Lagrange free mixed neutrosophic $N$-algebraic structures over $\langle M \cup I \rangle$. Then

(1) their restricted union may not be a soft Lagrange free mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$;

(2) their extended union may not be a soft Lagrange free mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$;

(3) their $AND$ operation may not be a soft Lagrange free mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$;

(4) their extended intersection may not be a soft Lagrange free mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$;

(5) their restricted intersection may not be a soft Lagrange free mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$;

(6) their $OR$ operation may not be a soft Lagrange free mixed neutrosophic $N$-algebraic structure over $\langle M \cup I \rangle$.

One can easily establish the above remarks by the help of examples. Similarly we can define soft Lagrange free weak deficit mixed neutrosophic $N$-algebraic structures.

## §4. Conclusion

This paper is an extension of soft sets to mixed neutrosophic $N$-algebraic structures. Their related properties and results are explained with illustrative examples.

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