A Situation Assessment Method in Conditional Evidential Networks based on DSm-PCR5

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Abstract

Aiming to solving the problem that the evidence information based on Dezert-Smarandache (DSm) model cannot be effectively conditionally reasoned in multi-source heterogeneous network which leads to the low rate of situation assessment, a situation assessment method in Conditional Evidential Network based on DSm-Proportional Conflict Redistribution No.5 (PCR5) is proposed. First, the conditional reasoning formula in Conditional Evidential Network based on DSm model is given. Then, the Disjunctive Rule of Combination(DRC) based on DSm-PCR5 is proposed and the Generalized Bayesian Theorem (GBT) for multiple intersection sets of focal elements can be obtained in the premise that the conditional mass assignments functions of focal elements in refinement of hyper-power set is known. Finally, through the simulation experiments results of situation assessment, the effectiveness of the proposed method is verified.

Keywords: Information fusion; Conditional Evidential Network; Dezert-Smarandache (DSm) model; Smarandache Codification; Proportional Conflict Redistribution No.5 (PCR5); Situation assessment

1. Introduction[1]

In the current research fields of information fusion, multi-source fusion problems in the same discernment frame have been widely studied. Most research focus on how to carry on effective fusion for highly conflict evidence [1-3]. However, due to the factors that information environment becomes more and more complex and technology of diversiform sensors develops rapidly, multi-source information fusion only in the same discernment frame cannot meet the requirements of intelligent information and decision systems and it also cannot provide favorable support to situation assessment of information fusion in high levels. Therefore, how to solve the evidence information fusion problems in different discernment frames of multi heterogeneous sensors, is gradually paid attention of scholars [4-8].

For solving the uncertain evidence fusion problems in different frames, Shenoy proposed Value Network (VN) theory[9,10]. VN transforms a prior knowledge of complex questions to the hierarchical network structure which consist of multiple variable nodes and relation nodes. Each variable node represents belief measures of the focal elements in each discernment frame, and relation nodes represent joint belief functions of relevant focal elements in different discernment frames. Evidence information in different discernment frames is transformed to the evidence in the same discernment frame by marginalization and extension. However, knowledge representation and reasoning by joint belief functions causes large storage space and multiplications. Aiming to solve this problem, Xu and Smets[4,5] proposed ENC which replaces the joint belief functions by conditional belief functions. In ENCs, any computations involving two connected variables X and Y are processed on the space Θ_X or Θ_Y, while in the network with joint beliefs, such computations are always done on the product space $\Theta_X \times \Theta_Y$. Thus the computations in an ENC needs fewer set-comparisons and multiplications than networks with joint belief functions.

ENC is based on Smets model or DS model, which requires power sets of exhaustive and exclusive hypotheses. However, for a wide class of fusion problems the intrinsic nature of hypotheses can be only vague and imprecise in such a way that precise refinement is just impossible to obtain in reality so that the exclusive elements θ_i cannot be properly identified and precisely separated. For resolving this problem, DSm model [11], proposed by Dezert and Smarandache, considers Θ only as a frame of exhaustive elements $\theta_i, i = 1, \cdots, n$ which can potentially overlap and a hybrid DSm model is also constructed when the integrity constraints are explicitly and formally introduced into the free DSm mode. The conditional reasoning method based on DSm model in the same discernment frame are also been proposed and widely studied in recent years [12-15]. However, ENC is not fit for fusion problems of heterogeneous evidence information in DSm model. Aiming at realizing effective information fusion and conditional reasoning in DSm model, a Situation assessment method in Conditional Evidential Network based on DSm-PCR5 is proposed.

In section 2, belief function theory, ENC, DSm model and PCR5 rules are introduced briefly. In section 3, firstly, the conditional reasoning formulas in Conditional Evidential Network based on DSm model is given and the reasoning algorithm is proposed; secondly, the method of calculating conditional mass assignments functions of focal elements in DSm model is given. In section 4, Monte Carlo simulation experiments of situation assessment are presented.

2. Basic theory

2.1. belief function theory

Let Θ be a finite nonempty set called the discernment frame[16,17]. The mapping $bel : 2^\Theta \to [0,1]$ is an belief function if and only if there exists a basic belief assignment (bba) $m : 2^\Theta \to [0,1]$ such that:

$$\sum_{A \subseteq \Theta} m(A) = 1 \tag{1}$$

$$bel(A) = \sum_{B \subseteq A, B \neq \varnothing} m(B), \, bel(\varnothing) = 0 \tag{2}$$

Those subsets A such that $m(A) > 0$ are called the focal elements. The value $bel(A)$ quantifies the strength of the belief that the event A occurs. The value $m(A)$ represents the part of belief that supports the fact that A occurs and can't support any more specific event.

Given a belief function or bba, a plausibility function $pl : 2^\Theta \to [0,1]$ can be defined as follows:

$$pl(A) = bel(\Theta) - bel(\overline{A}) = \sum_{B \cap A \neq \varnothing} m(B) \text{, and } pl(\varnothing) = 0 \tag{3}$$

Suppose bel quantifies our belief about the discernment frame Θ and we learn that $\overline{A} \subseteq \Theta$ is false. The resulting conditional belief function $bel(. \mid A)$ is obtained through the unnormalized rule of conditioning:

$$m(B|A)=\begin{cases}\sum_{X\subseteq\bar{A}} m(B\cup X) & if\ B\subseteq A\subseteq\Theta\\ 0 & otherwise\end{cases} \tag{4}$$

$$bel(B|A)=bel(B\cup\bar{B}\cup\bar{A})-bel(\bar{A}),\forall B, A\subseteq \tag{5}$$

$$pl(B|A)=pl(A\cap B),\forall B\subseteq\Theta \tag{6}$$

Belief function theory[18] is based on the same discernment frame, but in the reality, complex questions are often based on multi variables in different frames. For dealing with fusion problems with multi variables in different frames, the product spaces of the frames of the variables they include are defined as follow:

Let $\mathrm{U}=\{X,Y,Z,\cdots\}$ be a finite set of variables. $\Theta_X=\{x_1,\cdots,x_n\}$ is a frame of variable X, $\Theta_Y=\{y_1,\cdots,y_n\}$ is a frame of variable Y accordingly. The product space of the frames $\Theta_X\times\Theta_Y$ of the variables X and Y is defined: $\Theta_X\times\Theta_Y=\{(x_i,y_i):x_i\in\Theta_X,y_i\in\Theta_Y\}$

For studying the fusion problem in product space, concepts of projection, extension and marginalization[4,5] are introduced:

Projection of configurations simply means dropping the extra coordinates. If X and Y are sets of variables, $Y\subseteq X$, and x_i is a configuration of Θ_X, then let $x_i^{\downarrow Y}$ denote the projection of x_i on Θ_Y. Then $x^{\downarrow Y}$ is a configuration of Θ_Y. If x is a nonempty subset of Θ_X, then the projection of x on Y, denoted by $x^{\downarrow Y}$, is obtained by $x^{\downarrow Y}=\{x_i^{\downarrow Y}\mid x_i\in x\}$. If y is a subset of Θ_Y, then the cylindrical extension. of y to X, denoted by $y^{\uparrow X}$, is $y\times\Theta_{X-Y}$

Then if X and Y are sets of variables, Θ_X is a frame of variable X, Θ_Y is a frame of variable Y accordingly. Suppose m_X is a bba on X and m_Y is a bba on Y, the conjunctive combination of m_X and m_Y is defined by

$$m_X\oplus m_Y=m_X^{\uparrow X\times Y}\oplus m_Y^{\uparrow X\times Y} \tag{7}$$

Suppose m is a bba on B and $A\subseteq B\subseteq U, A\neq\varnothing$. The marginal of m for A, denoted by $m^{\downarrow A}$, is the bba on A defined by

$$m^{\downarrow A}(a)=\sum_{b\subseteq\Theta_B,B^{\downarrow A}=a} m(b)\qquad for\ all\quad a\subseteq\Theta_A \tag{8}$$

2.2. ENC

2.2.1. Graph structure of ENC

ENC[4] is a directed graph with conditional belief functions for knowledge representation and reasoning, as shown in Figure 1. In ENC, each node represents a variable in the domain of knowledge, and each edge represents a conditional relation between the two nodes it connects. For example, nodes X,Y and Z mean that the knowledge domain is $\mathrm{U}=\{X,Y,Z\}$, and the edges(X,Z) and (Y,Z) mean that we have $\{bel_Z(.|x_i):x_i\in\Theta_X\}$ and $\{bel_Z(.|y_i):y_i\in\Theta_Y\}$

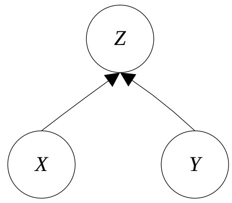

Fig. 1. An example of ENC

2.2.2. The generalized Bayesian theorem (GBT)[5]

Given $bel_X(x \mid y_i)=1, \forall y_i \in y$ and $\forall y_i \in \Theta_y, \forall x_i \in \Theta_x$, then

$$bel_Y(y \mid x) = K \cdot \left(\prod_{y_i \in \bar{y}} bel_X(\bar{x} \mid y_i) - \prod_{y_i \in y} bel_X(\bar{x} \mid y_i)\right) \tag{10}$$

$$pl_Y(y \mid x) = K \cdot \left(1 - \prod_{y_i \in y} (1 - pl_X(x \mid y_i))\right) \tag{11}$$

$$K^{-1} = 1 - \prod_{y_i \in \Theta_y} bel_X(\bar{x} \mid y_i) = 1 - \prod_{y_i \in \Theta_y} (1 - pl_X(x \mid y_i)) \tag{12}$$

2.2.3. The disjunctive rule of combination (DRC)[5]

Given $bel_X(x \mid y_i)=1, \forall y_i \in y$ and $\forall y_i \in \Theta_y, \forall x_i \in \Theta_x$, then

$$bel_X(x \mid y) = \prod_{y_i \in y} bel_X(x \mid y_i) \tag{13}$$

$$pl_X(x \mid y) = 1 - \prod_{y_i \in y} (1 - pl_X(x \mid y_i)) \tag{14}$$

$$m_X(x \mid y) = \sum_{(\cup_{i:y_i \in y} x_i) = x} \prod_{i:y_i \in y} m_X(x_i \mid y_i) \tag{15}$$

2.2.4. Propagating beliefs in an ENC[6]

Inference in ENC in the condition of non-empty a prior mass assignments of evidence is based on the following formulas[19].

Assume $x_i \in \Theta_X$, $y_j \in \Theta_Y$, then $\forall x \in \Theta_X$

$$\begin{aligned} m(x) &= \sum_{y \subseteq \Theta_Y} m_X(x \mid y) m_Y(y) \\ bel(x) &= \sum_{y \subseteq \Theta_Y} bel_X(x \mid y) m_Y(y) \\ pl(x) &= \sum_{y \subseteq \Theta_Y} pl_X(x \mid y) m_Y(y) \end{aligned} \tag{16}$$

2.3. DSm model and DSm rules

Consider a finite frame $\Theta = \{\theta_1, \cdots, \theta_n\}$ of the fusion problem under consideration. We abandon Shafer's model 2^Θ by assuming here that the fuzzy/vague/relative nature of elements of Θ can be non-exclusive. Let $\Theta = \{\theta_1, \theta_2, \theta_3\}$, if all multiple focal element is not empty, D^Θ is a free-DSm model[11] as shown in Fig 2. In Fig. 2, "i" denotes the part of the diagram which belongs to θ_i only, "ij" denotes the part of the diagram which belongs to θ_i and θ_j only, "ijk" denotes the part of the diagram which belongs to θ_i and θ_j and θ_k only, etc. called Smarandache Codification [15].

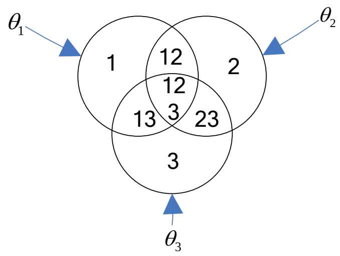

Fig. 2 Venn Diagram for free-DSm model

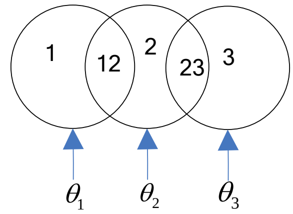

Fig. 3 Venn Diagram for hybrid DSm model

We have used in Fig. 2 and 3 the Smarandache Codifications 1, 2, 3, 12, 13, 23 and respectively 1, 2, 3, 12, 23 for each corresponding DSm model.

A hybrid DSm model[11] is defined from the free-DSm model by introducing some integrity constraints on some elements. Indeed, in some fusion problems, some elements θ_i and θ_j of Θ can be fully discernable because they are truly exclusive while other elements cannot be refined into finer exclusive elements. Hybrid DSm models are not only fit for this situation. The other situations is introduced in [11]. Suppose the frame of the fusion problem as show in Fig.2. Let $\theta_1 \cap \theta_3 = \varnothing$, the hybrid DSm model D^{Θ} is shown in Fig 3.

2.4. PCR5 based information fusion rule within DSmT framework

Instead of applying a direct transfer of partial conflicts onto partial uncertainties as with Dezert Smarandache Hybrid (DSmH) combination rule, the idea behind the Proportional Conflict Redistribution (PCR) rule[11] is to transfer (total or partial) conflicting masses to non-empty sets involved in the conflicts proportionally with respect to the masses assigned to them by sources as follows[11]:

1. calculation of the conjunctive rule of the belief masses of sources;

2. calculation of the total or partial conflicting masses;

3. redistribution of the (total or partial) conflicting masses to the non-empty sets involved in the conflicts proportionally with respect to their masses assigned by the sources.

The way the conflicting mass is redistributed yields actually several versions of PCR rules. PCR5 is the most mathematically exact redistribution method of conflicting mass. This rule redistributes the partial conflicting mass to the elements involved in the partial conflict, considering the conjunctive normal form of the partial conflict. It does a better redistribution of the conflicting mass than Dempster’s rule since PCR5 goes backwards on the tracks of the conjunctive rule and redistributes the conflicting mass only to the sets involved in the conflict and proportionally to their masses put in the conflict.

The PCR5 formula for the combination of two sources (s = 2) is given by[11]:

$$m_{\mathrm{PCR5}}[\varnothing] = 0, \forall X \in G^{\Theta} \setminus \varnothing$$

$$m_{\mathrm{PCR5}}[X] = m_{12}(X) + \sum_{\substack{Y \in G^{\Theta} \setminus X \\ X \cap Y = \phi}} \left[\frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X) + m_1(Y)}\right] \tag{17}$$

where all sets involved in formulas are in canonical form and where G^{Θ} corresponds to classical power set 2^{Θ} if Shafer’s model is used, or to a constrained hyper-power set D^{Θ} if any other hybrid DSm model is used instead, or to the super-power set S^{Θ} if the minimal refinement Θ^{ref} of Θ is used; $m_{12}(X) = m_{\cap}(X)$ corresponds to the conjunctive consensus on X between both sources and where all denominators are different from zero. If a denominator is zero, that fraction is discarded.

3. A reasoning method in Conditional Evidential Network based on DSm-PCR5

3.1. A reasoning method in Conditional Evidential Network based on DSm model

3.1.1 The reasoning formulas in Conditional Evidential Network based on DSm model

The reasoning formulas in Conditional Evidential Network based on DSm model is given as follows

Theorem 1 Assume D^X and D^Y is the hyper-power set in DSm model and $x\in D^X$, $y_i\in D^Y$, then $\forall x\in D^X$

$$m_{D^X}(x)=\sum_{y_i\in D^Y} m_{D^Y}(x\mid y_i)m_{D^Y}(y_i) \tag{18}$$

$$bel_{D^X}(x)=\sum_{y_i\in G^Y} bel_{D^X}(x\mid y_i)m_{D^Y}(y_i) \tag{19}$$

$$pl_{D^X}(x)=\sum_{y_i\in D^Y} pl_{D^X}(x\mid y_i)m_{D^Y}(y_i) \tag{20}$$

Proof: As the focal elements of evidence in DSm model can be refined to the union of the focal elements which is exclusive in DS model, denoted by $x=x_i'\cup\cdots\cup x_j':x\in D^X, x_i'\cup\cdots\cup x_j'\in 2^X$, $y_i=y_i'\cup\cdots\cup y_j':y_i\in D^Y, y_i'\cup\cdots\cup y_j'\in 2^Y$. So the mass assignments functions $m_{D^X}(x\mid y_i)$ in DSm model can be transformed to $\{m_{2^X}(x_i'\cup\cdots\cup x_j'\mid y_i'\cup\cdots\cup y_j'):x_i'\cup\cdots\cup x_j'\in 2^X, y_i'\cup\cdots\cup y_j'\in 2^Y\}$ and the prior mass assignments functions of evidence $m_{D^Y}(y_i)$ can be transformed to $\forall y_i\in D^Y, m_{D^Y}(y_i)=m_{2^Y}(y_i'\cup\cdots\cup y_j')$.

As the a prior mass assignments functions of evidence $m_{D^Y}(y_i)$ in DSm model is complete, through the equivalent transformation mass assignments functions of evidence $m_{2^Y}(y_i'\cup\cdots\cup y_j')$ is also complete, denoted by $\sum_{y_i'\cup\cdots\cup y_j'=y_i} m_{2^Y}(y_i'\cup\cdots\cup y_j')=1$. So $\forall y_i'\cup\cdots\cup y_j'=y_i, y_i'\cup\cdots\cup y_j'$ can be the focal elements in power set in DS model of the evidence and $\forall y_i'\cup\cdots\cup y_j'=y_i, m_{2^Y}(y_i'\cup\cdots\cup y_j')$ can be the mass assignments functioning power set in DS model of the evidence.

Then, from the equation (16), we can obtain

$$m_{D^X}(x_i'\cup\cdots\cup x_j')=\sum_{y_i'\cup\cdots\cup y_j'\in D^Y} m_X(x_i'\cup\cdots\cup x_j'\mid y_i'\cup\cdots\cup y_j')m_Y(y_i'\cup\cdots\cup y_j')$$

So $m_{D^X}(x)=\sum_{y_i\in D^Y} m_{D^Y}(x\mid y_i)m_{D^Y}(y_i)$. QED.

As the proof of Equation (19) and (20) are similar with Equation (18), we omit it.

3.1.2 The reasoning algorithm in Conditional Evidential Network based on DSm model

Based on the reasoning formulas proved in the Section 3.1.2, the reasoning algorithm in Conditional Evidential Network based on DSm model is given.

Consider the evidential network based on DSm model has two nodes X and Y defined in hyper-power sets D^X and D^Y, respectively. Suppose that there exists some a priori information over D^X given by mass function m_0^X and some a prior information over D_Y given by m_0^Y. We assume that we have obtained conditional mass assignments functions of focal elements in hyper-power set of DSm model, denoted by $\{m_{D^Y}(y\,|\,x): x\in D^X\}$ and $\{m_{D^X}(x\,|\,y): y\in D^Y\}$.

If we want to compute m^X of the node X, which is the parent of Y, we combine its prior mass function with the message coming from Y,

$$m^X = m_0^X \underset{\text{DSmT}}{\oplus} m_{Y\to X} \tag{21}$$

Where $\underset{\text{DSmT}}{\oplus}$ represents DSm rules and $m_{Y\to X}$ is a mass function on X representing the message coming from Y, and is computed by

$$\forall x\in D^X, m_{Y\to X}(x) = \sum_{y\in D^Y} m_0^Y(y) m_{D^X}(x\,|\,y) \tag{22}$$

If we want to compute m^Y of the node Y, which is the child of X, we combine its prior mass function with the message coming from X,

$$m^Y = m_0^Y \underset{\text{DSmT}}{\oplus} m_{X\to Y} \tag{23}$$

where $\underset{\text{DSmT}}{\oplus}$ represents DSm rules and $m_{X\to Y}$ is a mass function on Y representing the message coming from X, and is computed by

$$\forall y\in D^Y, m_{X\to Y}(y) = \sum_{x\in D^X} m_0^X(x) m_{D^Y}(y\,|\,x) \tag{24}$$

3.2. The method of calculating the conditional mass assignments functions for evidence information based on DSm model

In this section, the Disjunctive Rule of Combination (DRC) based on DSm-PCR5 and the Generalized Bayesian Theorem(GBT) for multiple intersection sets of focal elements are proposed in the premise that the conditional mass functions of focal elements in refinement of hyper-power set is known.

3.2.1 DRC based on DSm-PCR5

Assume that there are 2 focal elements θ_i, θ_j based on DSm model which have the multiple intersection sets of focal elements $\theta_i \cap \theta_j$ as shown in Fig.4. Assume that the conditional mass functions $m_X(x\,|\,\theta_i')$ of focal elements in refinement $\{\theta_i'\}$ of hyper-power set $\{\theta_i\}$ is known

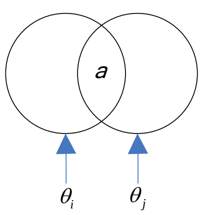

Fig. 4 two focal elements θ_i, θ_j based on DSm model

DRC based on DSm model is given

$$m_X(x\,|\,\theta_i) = \sum_{(\cup x_i)=x} m_X(x\,|\,\theta_i') \cdot m_X(x\,|\,\theta_i \cap \theta_j) \tag{25}$$

Then DRC based on DSm model for focal elements of $k(k\geqq 2)$ intersection sets based on DSm model is given

$$m_X(x\,|\,\theta)=\sum_{(\bigcup_{i:\theta_i\in\theta}x_i)=x}\ \prod_{i:\theta_i\in\theta}m_X(x\,|\,\theta_i) \tag{26}$$

The equations (25-26) are only fit for calculating the conditional mass assignments functions when the known conditional mass assignments functions $m_X(x\,|\,\theta_i\cap\cdots\cap\theta_j\in\theta_i)$ have the same focal element x. Obviously, if the known conditional mass assignments functions are highly conflict, the results of equation (25-26) are quietly not accurate. So the idea of PCR5 is applied to process the conflicts of conditional mass assignments functions. DRC for focal elements of 2 intersection sets based on DSm-PCR5 is given as follows

$$m_X(x\,|\,\theta_i)=\sum_{(\bigcup x_i)=x}\left\{m_X(x\,|\,\theta_i')\cdot m_X(x\,|\,\theta_i\cap\theta_j)+\sum_{\substack{y\in X\setminus x\\ x\cap y=\varnothing}}\left[\frac{m(x\,|\,\theta_i')^2\,m(y\,|\,\theta_i\cap\theta_j)}{m(x\,|\,\theta_i')+m(y\,|\,\theta_i\cap\theta_j)}+\frac{m(x\,|\,\theta_i\cap\theta_j)^2\,m(y\,|\,\theta_i')}{m(x\,|\,\theta_i\cap\theta_j)+m(y\,|\,\theta_i')}\right]\right\} \tag{27}$$

The conditional mass assignments functions for focal elements of k ($k\geqq 2$) intersection sets based on DSm-PCR5 can be obtained by the equation (27) in the sequence of 2 focal elements to 2 focal elements.

3.2.2 GBT for multiple intersection sets of focal elements

Assume that the conditional plausibility functions $pl_X(x\,|\,\theta_i')$ of focal elements in refinement $\{\theta_i'\}$ of hyper-power set $\{\theta_i\}$ is known, the backward conditional assignments functions of multiple intersection sets of focal elements in hyper-power sets can be calculated as follows:

1) Calculate the backward conditional plausibility functions of refinement focal elements $\{\theta_i'\}$

$$pl_\Theta(\theta'\,|\,x)=K\cdot(1-\prod_{\theta_i'\in\theta'}(1-pl_X(x\,|\,\theta_i'))) \tag{28}$$

$$K^{-1}=1-\prod_{\theta_i'\in D^\Theta}(1-pl_X(x\,|\,\theta_i')) \tag{29}$$

2) Calculate the backward conditional plausibility functions of focal elements in hyper-power sets based on DSm model by the refinement and the relation between plausibility functions and assignments functions.

$$\begin{aligned}
&m(\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k}\,|\,x)=bel(\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k}\,|\,x)=1-\underset{\theta'=(D^\Theta-\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k})}{pl_\Theta}(\theta'\,|\,x)\\
&m(\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k-1}\,|\,x)=bel(\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k-1}\,|\,x)-\sum_{\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k}\subset\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k-1}}m(\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k}\,|\,x)\\
&=1-\underset{\theta'=(D^\Theta-\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k-1})}{pl_\Theta}(\theta'\,|\,x)-\sum_{\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k}\subset\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k-1}}m(\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k}\,|\,x)\\
&\vdots\\
&m(\theta_i\cap\theta_j\,|\,x)=bel(\theta_i\cap\theta_j\,|\,x)-\sum_{\underbrace{\theta_i\cap\cdots\cap\theta_j}_{3}\subset\theta_i\cap\theta_j}m(\underbrace{\theta_i\cap\cdots\cap\theta_j}_{3}\,|\,x)-\cdots-\sum_{\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k}\subset\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k-1}}m(\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k}\,|\,x)\\
&=1-\underset{\theta'=(D^\Theta-\theta_i\cap\theta_j)}{pl_\Theta}(\theta'\,|\,x)-\sum_{\underbrace{\theta_i\cap\cdots\cap\theta_j}_{3}\subset\theta_i\cap\theta_j}m(\underbrace{\theta_i\cap\cdots\cap\theta_j}_{3}\,|\,x)-\cdots-\sum_{\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k}\subset\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k-1}}m(\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k}\,|\,x)\\
&m(\theta_i\,|\,x)=bel(\theta_i\,|\,x)-\sum_{\theta_i\subset\theta_i\cap\theta_j}m(\theta_i\cap\theta_j\,|\,x)-\sum_{\underbrace{\theta_i\cap\cdots\cap\theta_j}_{3}\subset\theta_i\cap\theta_j}m(\underbrace{\theta_i\cap\cdots\cap\theta_j}_{3}\,|\,x)-\cdots-\sum_{\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k}\subset\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k-1}}m(\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k}\,|\,x)\\
&=1-\underset{\theta'=(D^\Theta-\theta_i)}{pl_\Theta}(\theta'\,|\,x)-\sum_{\theta_i\subset\theta_i\cap\theta_j}m(\theta_i\cap\theta_j\,|\,x)-\sum_{\underbrace{\theta_i\cap\cdots\cap\theta_j}_{3}\subset\theta_i\cap\theta_j}m(\underbrace{\theta_i\cap\cdots\cap\theta_j}_{3}\,|\,x)-\cdots-\sum_{\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k}\subset\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k-1}}m(\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k}\,|\,x)
\end{aligned}$$

(30)

4. Monte Carlo Simulation experiments of situation assessment

Through the research on the air target situation assessment, we find it's difficult to require the evidence of the types of air target to be exclusive as some types of air target have multi type's characters and the sources cannot distinguish the type of air target which has good stealthy performance. So the source's evidence of types of air target is possible based on DSm model. In this section, we apply our method to the air target situation assessment.

An evidential network model of air target situation assessment is shown in Fig.5. Suppose the exclusive discernment frame of 'air target situation level' is {1, 2, 3}, the exclusive discernment frame of 'distance of air target' is {far, middle, near} and non-exclusive discernment frame of 'types of air target' is {t1, t2, t3} in evidence, where $t1 \cap t3 = t2$.

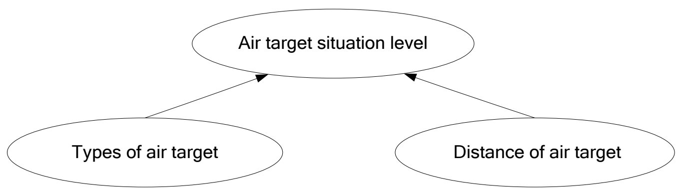

Fig.5 An evidential network model of air target situation assessment

Assume that types of air target t1, t2 and t3 are random variables of normal distribution. The interval distribution of t1 is [1000, 2000], the interval distribution of t3 is [2000, 3000]. As t3 has good stealthy performance and the sources cannot distinguish t3 and t2 or t1 and t2. So the interval distribution of t2 is [1000, 2000] or [2000, 3000]. Assume a prior conditional mass assignments functions of 'types of air target' as show in Table 1. Assume 'distance of air target' can be defined as near, middle and far, which are also random variables of normal distribution. The interval distribution of near is [500, 1000], the interval distribution of middle is [1000, 1500] and the interval distribution of far is [1500, 2000]. A prior conditional assignments functions of 'distance of air target' are given as show in Table 2, which is the exclusive discernment frame.

Table 1 A prior conditional assignments functions of 'types of air target'

Types of air target / Situation level	t1	t2	t3
1	0.1	0.1	0.8
2	0.1	0.8	0.1
3	0.8	0.1	0.1

Table 2 A prior conditional assignments functions of 'distance of air target'

Types of air target / Situation level	far	middle	near
1	0	0	1
2	0	1	0

3	1	0	0

The measurement signal parameters are obtained by addition of the random Gaussian noise to real signal parameters. Monte Carlo Simulation experiments are carried out 1000 times in each noise condition. The experiment procedures of each time is given as follows

1) Select 1000 data randomly for t1 which obeys normal distribution in [1000, 2000], select 1000 data randomly for t2 which obeys normal distribution in [1000, 2000], select 1000 data randomly for t3 which obeys normal distribution in [2000, 3000], and also select 1000 data randomly for t2 which also obeys normal distribution in [2000, 3000].

2) Select the data in each types of air target to each situation level 1, 2 or 3 randomly as Table 1. Calculate the number of data belonging to level 1, level 2 or level 3, denoted by numberl1, numberl2 and numberl3.

3) Select numberl1 data randomly for near which obeys normal distribution in [500, 1000], select numberl2 data randomly for middle which obeys normal distribution in [1000, 1500], select numberl3 data randomly for far which obeys normal distribution in [1500, 2000].

4) Situation database is established by matching the data of ‘types’ and ‘distance’ with the same situation level.

5) Calculate the conditional mass assignments functions for focal elements of t1’ and t3’ based on DSm model by the method in the Section 3.2.1.

6) Select one data randomly in Situation database, then add white Gauss noise whose variance is x times to the variance of the selected type to generate uncertain measurement evidences, denoted by a. x is from 0.1 to 5 as the step 0.2. The mass assignments of the uncertain measurement evidences of ‘types’ based on DSm model is obtained by the bell shaped membership function as follows:

$$\begin{aligned} &u(a \in A) = \mathrm{e}^{-\frac{(x-\mathrm{Ex})^2}{500\times \mathrm{En}^2}}, A \in \{\mathrm{t1'}, \mathrm{t3'}\} \\ &u(a \in \Theta) = 1 - \max(u(a \in A)) \\ &m(a \in A) = \frac{u(a \in A)}{sum(u(a \in A)) + u(a \in \Theta)} \\ &m(a \in \Theta) = \frac{u(a \in \Theta)}{sum(u(a \in A)) + u(a \in \Theta)} \end{aligned} \tag{31}$$

where a represents the uncertain measurement evidences, A represents the focal elements of the types in DSm model, and Ex and En represent the mathematical expectation and variance of A.

Similarly, the mass assignments of the uncertain measurement evidences of ‘distance’ is obtained by the bell shaped membership function as follows:

$$\begin{aligned} &u(a \in B) = \mathrm{e}^{-\frac{(x-\mathrm{Ex})^2}{500\times \mathrm{En}^2}}, B \in \{\mathrm{t1}, \mathrm{t2}, \mathrm{t3}\} \\ &u(a \in \Theta) = 1 - \max(u(a \in B)) \\ &m(a \in B) = \frac{u(a \in B)}{sum(u(a \in B)) + u(a \in \Theta)} \\ &m(a \in \Theta) = \frac{u(a \in \Theta)}{sum(u(a \in B)) + u(a \in \Theta)} \end{aligned} \tag{32}$$

where a represents the uncertain measurement evidences, B represents the focal elements of the types in DS model, and Ex and En represent the mathematical expectation and variance of *B*.

7) Calculate the conditional reasoning results from ‘types’ to ‘level’ by the our method and DRC in DS model separately. Calculate the conditional reasoning results from ‘distance’ to ‘level’. The PCR5 fusion results in ‘level’ can be obtained by fusing the conditional reasoning results from ‘types’ to ‘level’ and ‘distance’ to ‘level’, separately.

6) Fo the PCR5 fusion results of our method, Select out the Situation level number which has the maximum value of mass assignments. If the Situation level number is the same as the prior Situation level number which is known in Situation database, the correct times of situation assessment by our method increase one time. Similarly, the correct times of situation assessment by the method in DS model can also be obtained.

The correct rate comparison of situation assessment by the method in this paper and the method in DS model with the increase of the variance of white Gauss noise are given as the Table 3 and Fig.6. It is shown by the Table 3 and Fig.6. that the method in this paper have more high correct rates of situation assessment than the method in DS model in the same noise circumstance.

Table 3 correct rates comparison of situation assessment

Different methods	Correct rates of situation assessment in different methods (%)
The method in this paper	99.8, 99.4, 99.9, 99.5, 98.5, 99, 98.6, 98.2, 97.9, 97.3, 96.9, 96.7, 94.8, 93.9, 93, 93.1, 91.4, 88.4, 87.4, 85.3, 85.5, 83.6, 83.8, 81.4, 79.0, 79.4, 77.7, 76.1, 74.7, 74.4
The method in DS model	91.9, 92.6, 91.6, 92.3, 89.1, 89.7, 90.7, 89.5, 88.1, 88.0, 88.2, 86.4, 87.6, 86.6, 82.3, 83.4, 82.4, 81.5, 77.9, 77.2, 79.7, 77.1, 74.3, 74.7, 71.1, 72.8, 70, 70.3, 67.6, 67.6

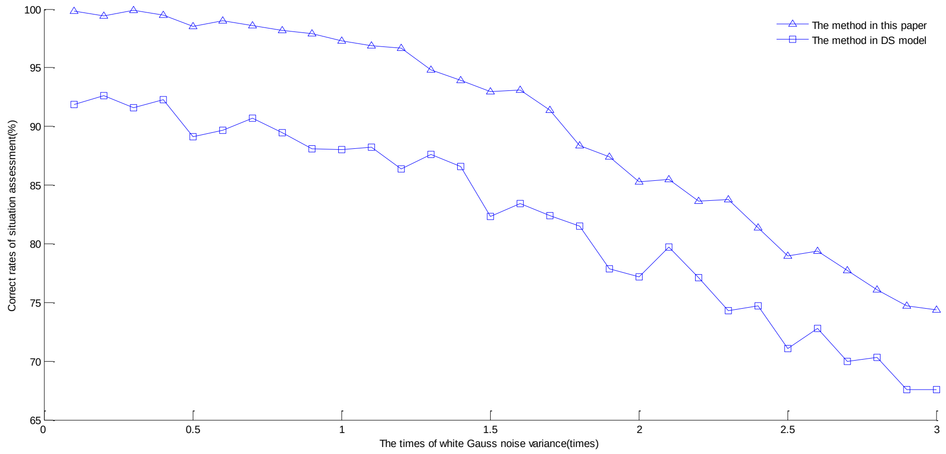

Fig.6. Correct rates comparison of situation assessment

6. Conclusions

A situation assessment method in Conditional Evidential Network based on DSm-PCR5 is proposed in this paper. The method improves ENC to dealing with heterogeneous evidence in DSm model. According to the dynamic changes of information model of the evidence in evidential networks, the method calculate different conditional belief functions based on different models, which makes effective and efficient fusion and reasoning of the heterogeneous evidence in different discernment frames based on DSm model or DS model. Simulation results show that the method in this paper can be successfully applied to the field of situation assessment and has a profound theoretical significance and engineering practical value.

However, we think that it is important to look at fast approximate evidence fusion methods because if there is multi-source evidence in the network, the computation complexity of mass assignments functions fusion grows exponentially with the increasing focal numbers of the evidence.

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