On Scale Invariance and Particle Localization in Quantum Field Theory

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Abstract

It is known that *microcausality* is a cornerstone principle of relativistic Quantum Field Theory (QFT). It requires commutativity of local fields defined at space-like separations and prohibits physical effects to propagate at superluminal speeds. However, it is also known that the exact localization of quantum fields fails to apply to quantum particles, which behave as *non-local* entities (the Reeh-Schlieder and Malamert theorems). Over the years, challenges associated with the point-wise description of quantum particles have inspired many attempts to revisit the particle interpretation of QFT. All these proposals suffer from specific limitations and have not gained universal acceptance. Here we suggest that a field theory approaching scale invariance near the fixed points of the Renormalization Group flow blurs the distinction between locality and non-locality. In particular, *self-similarity* resolves the issue of particle localization in QFT, while reinforcing microcausality by default.

Introduction

The principle of microcausality in QFT implies that the *destruction* operators (positive field frequencies) act in a fully symmetrical way with the *creation* operators (negative field frequencies). This is equivalent to stating that particles and antiparticles share the same mass but opposite additive quantum numbers [1].

QFT is constructed from local fields that commute at arbitrarily small space-like separations. Let Ω_1 and Ω_2 denote two compact spacetime regions such that for all $x \in \Omega_1,\ y \in \Omega_2$ the separation $(x-y)$ is space-like and let $f_1(x),\ f_2(x)$ represent test functions with support in Ω_1, Ω_2. Then the "smeared" scalar fields $\varphi_{f_i} = \int d^4x \, f_i(x)\varphi(x)$ satisfy the commutation relation

$$\left[\varphi_{f_1}, \varphi_{f_2}\right] = 0 \tag{1}$$

In particular, (1) enable construction of mutually compatible quantum states in which the two fields simultaneously assume sharp values.

By contrast, the commutator of the destruction operator:

$$\left[\varphi^{(+)}(x), \varphi^{(-)}(y)\right] = \Delta_+(x-y;m) \tag{2}$$

assumes the form [1]

$$\Delta_+(x-y;m) = \frac{m^2}{4\pi^2}\frac{1}{m\sqrt{-(x-y)^2}}K_1(m\sqrt{-(x-y)^2}) \tag{3}$$

in which K_1 is a modified Bessel function. The commutator (2) does not vanish when the separation $(x-y)$ is space-like, $(x-y)^2 < 0$. Instead, (2) falls off exponentially as $\exp(-2m|\vec{x}|)$, with a scale defined by the Compton wavelength of the quantum particle (m^{-1}).

The goal of this brief note is to suggest that a field theory approaching scale invariance near the fixed points of the Renormalization Group (RG) flow blurs the distinction between locality and non-locality. In particular, *self-similarity* resolves the issue of particle localization in QFT and automatically reinforces microcausality.

Let us start from the observation that quantification of QFT observables is always limited by a non-vanishing cutoff Δ, which inherently sets the *measurement resolution*. In particular, if the numerical value of (1) or (2) falls below the commutator resolution ΔC, the two variables are deemed *commuting*. If the reverse is true, variables are *non-commuting*. The sharp distinction between the commuting and anti-commuting variables lies at the issue of particle localization in QFT [2]. We wish to show that, at least in principle, the approach to scale invariance solves this tension by blending locality and non-locality into a single concept.

Begin by assuming non-commutativity of (1) at an initial scale λ_0

$$\left[\varphi_{f_1}(x,\lambda_0),\, \varphi_{f_2}(y,\lambda_0)\right] = C_{f_1 f_2}(x,y,\lambda_0) > \Delta C_{f_1 f_2}(x,y,\lambda_0) \neq 0 \tag{4}$$

where $\Delta C_{f_1 f_2}(x,y,\lambda_0)$ stands for the scale-dependent resolution associated with measurement of (4). Demanding the approach to scale invariance near the fixed points of the RG flow implies a power-law behavior of $\varphi_{f_1}, \varphi_{f_2}$ and $\Delta C_{f_1 f_2}(x,y,\lambda_0)$ under a change of scale $\lambda = t\lambda_0$, namely

$$\varphi_{f_1}(tx, t\lambda_0) = t^{s_1}\varphi_{f_1}(x,\lambda_0) \tag{5a}$$

$$\varphi_{f_2}(ty, t\lambda_0) = t^{s_2}\varphi_{f_2}(y,\lambda_0) \tag{5b}$$

$$\Delta C_{f_1 f_2}(tx, ty, t\lambda_0) = t^{s_c}\Delta C_{f_1 f_2}(x,y,\lambda_0) \tag{5c}$$

Let

$$C_{f_1 f_2}(x,y,\lambda_0) = q\,\Delta C_{f_1 f_2}(x,y,\lambda_0),\ q > 1 \tag{6}$$

Imposing the condition that the commutator (4) measured at $\lambda = t\lambda_0$ drops below its resolution at this new scale yields

$$C_{f_1 f_2}(tx, ty, t\lambda_0) = t^{s_1+s_2} C_{f_1 f_2}(x, y, \lambda_0) < t^{s_c} \Delta C_{f_1 f_2}(x, y, \lambda_0) \tag{7}$$

Replacing (6) in (7) fixes the range of scales satisfying (7), namely

$$\boxed{t < q^{-1/(s_1+s_2-s_c)}} \tag{8}$$

An important observation is in order: the self-similarity argument *does not apply* when the commutator amounts to a physical constant, which is by default a scale-invariant entity. The immediate example is the canonical commutator of space and momentum,

$$[p, x] = -i\hbar \tag{9}$$

References

[1] A. Duncan, "Conceptual Framework of Quantum Field Theory", Oxford Univ. Press, 2012.

[2] E. Pessa, "*The Concept of Particle in Quantum Field Theory*", in "Vision of Oneness", Iganazio Licata and Ammar Sakaji (editors), Aracne Editrice, 2011.