Relativistic Cosmology and Einstein's 'Gravitational Waves'

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2nd August 2016

1 Introduction

The LIGO Scientific Collaboration and Virgo Collaboration have announced [1] that on the 14th of September 2015, at 09:50:45 UTC, they detected a transient Einstein gravitational wave, designated GW150914, produced by two merging black holes forming a single black hole. Not so long ago similar media excitement surrounded the announcement by the BICEP2 Team of detection of primordial gravitational waves imprinted in B-mode polarisations of a Cosmic Microwave Background, which proved to be naught. The two black holes that merged are reported to have been at a distance of some 1.3 billion light years from Earth, of ≈29 solar masses and ≈36 solar masses respectively, the newly formed black hole at ≈62 solar masses, radiating away ≈3 solar masses as Einstein gravitational waves. The insurmountable problem for the credibility of the LIGO-Virgo Collaboration claims is the falsity of the theoretical assumptions upon which they are based.

The reported detection was obtained, not during an observation run of LIGO, but during an engineering test run, prior to the first scheduled observation run on 18 September 2015.

> "*In the early morning hours of September 14, 2015 - during an engineering run just days before official data-taking started - a strong signal, consistent with merging black holes, appeared simultaneously in LIGO's two observatories, located in Hanford, Washington and Livingston, Louisiana.*" Conover [2]

> "*The eighth engineering run (ER8) began on 2015 August 16 at 15:00 and critical software was frozen by August 30. The rest of ER8 was to be used to calibrate the detectors, to carry out diagnostic studies, to practice maintaining a high coincident duty cycle, and to train and tune the data analysis pipelines. Calibration was complete by September 12 and O1 was scheduled to begin on September 18. On 2015 September 14, cWB reported a burst candidate to have occurred at 09:50:45 with a network signal-to-noise ratio (S/N) of 23.45 . . .*" Abbott *et al*. [3]

Magnitudes with error margins have been presented by the LIGO-Virgo Collaborations for the masses of the black holes, along with other related source quantities, in their TABLE I [1], reproduced herein as figure 1.

TABLE I. Source parameters for GW150914. We report median values with 90% credible intervals that include statistical errors, and systematic errors from averaging the results of different waveform models. Masses are given in the source frame; to convert to the detector frame multiply by $(1+z)$ [90]. The source redshift assumes standard cosmology [91].

Primary black hole mass	$36^{+5}_{-4} M_\odot$
Secondary black hole mass	$29^{+4}_{-4} M_\odot$
Final black hole mass	$62^{+4}_{-4} M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

Fig. 1: Reproduced from Abbott, B.P. et al., Observation of Gravitational Waves from a Binary Black Hole Merger, PRL 116, 061102 (2016), DOI: 10.1103/PhysRevLett.116.061102

There are two ways by which the LIGO report can be analysed: (a) examination of the LIGO interferometer operation and data acquisition, and (b) consideration of the theories of black holes and gravitational waves upon which all else relies. Only theoretical considerations are considered herein, as there is no need to analyse the LIGO apparatus and its signal data to understand that the claim for detection of Einstein gravitational waves and black holes is built upon theoretical fallacies and conformational bias.

2 Gravitational waves, black holes and big bangs combined

Presumably the gravitational waves reported by LIGO-Virgo are present inside some big bang expanding universe as there has been no report that big bang cosmology has been abandoned. Of the gravitational wave detection the LIGO-Virgo Collaborations have stated,

> "*It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole.*" Abbott *et al.* [1]

All purported black hole equations are solutions to corresponding specific forms of Einstein's nonlinear field equations. Gravitational waves on the other hand are obtained from a linearisation of Einstein's nonlinear field equations, combined with a deliberate selection of coordinates that produce the assumed aforehand propagation at the speed of light in vacuo. Because General Relativity is a nonlinear theory, the Principle of Superposition does not hold.

> "*In a gravitational field, the distribution and motion of the matter producing it cannot at all be assigned arbitrarily - on the contrary it must be determined (by solving the field equations for given initial conditions) simultaneously with the field produced by the same matter.*" Landau & Lifschitz [4]

> "*An important characteristic of gravity within the framework of general relativity is that the theory is non-linear. Mathematically, this means that if g_{ab} and γ_{ab} are two solutions of the field equations, then $ag_{ab} + b\gamma_{ab}$ (where a, b are scalars) may not be a solution. This fact manifests itself physically in two ways. First, since a linear combination may not be a solution, we cannot take the overall gravitational field of the two bodies to be the summation of the individual gravitational fields of each body.*" McMahon [5]

Let **X** be some black hole universe and **Y** be some big bang universe. Then the linear combination (i.e. superposition) **X + Y** is not a universe. Indeed, **X** and **Y** pertain to completely different sets of Einstein field equations and so they have absolutely nothing to do with one another. The same argument holds if **X** and **Y** are both black hole universes, be they the same or not, and if **X** and **Y** are big bang universes, be they the same or not. Consequently, a black hole universe cannot co-exist with any other black hole universe or with any big bang universe.

All black hole universes:

1. are spatially infinite
2. are eternal
3. contain only one mass
4. are not expanding (i.e. are not non-static)
5. are asymptotically flat (or, even more exotically, asymptotically curved).

All big bang universes:

1. are either spatially finite (1 case; $k = 1$) or spatially infinite (2 different cases; $k = -1, k = 0$)
2. are of finite age (≈13.8 billion years)
3. contain radiation and many masses
4. are expanding (i.e. are non-static)
5. are not asymptotically anything.

Note also that no black hole universe even possesses a big bang universe k-curvature. It is clearly evident that black holes and big bang universes cannot co-exist by their very definitions.

All the different black hole 'solutions' are also applicable to stars, planets and the like. Thus, these equations do not permit the presence of more than one star or planet in the universe. In the case of a body such as a star, the only significant difference is that its spacetime does not bend to infinite curvature at the star because there is no singularity and no event horizon in this case. Newton's theory on the other hand, places no restriction on the number of masses that can be present.

Since a black hole universe is a solution to a specific set of Einstein's nonlinear field equations it is not possible to extract from it any gravitational waves that are produced from linearised field equations. No gravitational waves can in fact be extracted from Einstein's nonlinear field equations [6]. Superposing solutions obtained from the nonlinear system with those from the linearised system violates the mathematical structure of General Relativity. Accordingly, contrary to the report of the LIGO-Virgo Collaborations, gravitational waves cannot exist in any black hole universe. Neither can they exist in any big bang universe because all big bang models are in fact single mass universes by mathematical construction [6,7]. Nonetheless the LIGO-Virgo Collaborations superpose everything [1], depicted in figure 2 herein.

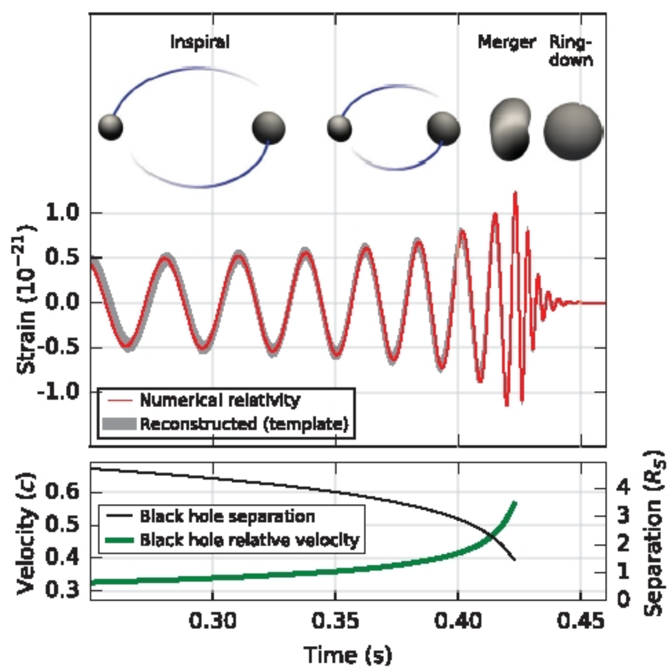

Fig. 2: "*Top: Estimated gravitational-wave strain amplitude from GW150914 projected onto H1. This shows the full bandwidth of the waveforms, without the filtering used for Fig. 1. The inset images show numerical relativity models of the black hole horizons as the black holes coalesce. Bottom: The Keplerian effective black hole separation in units of Schwarzschild radii ($R_S = 2GM/c^2$) and the effective relative velocity given by the post-Newtonian parameter $v/c = (GM\pi f/c^3)^{1/3}$, where f is the gravitational-wave frequency calculated with numerical relativity and M is the total mass (value from Table I).*" Reproduced from Abbott, B.P. et al., Observation of Gravitational Waves from a Binary Black Hole Merger, PRL 116, 061102 (2016), DOI: 10.1103/PhysRevLett.116.061102

Superposition where superposition does not hold is a standard method of cosmologists.

> "*From what I have said, collapse of the kind I have described must be of frequent occurrence in the Galaxy; and black-holes must be present in numbers comparable to, if not exceeding, those of the pulsars. While the black-holes will not be visible to external observers, they can nevertheless interact with one another and with the outside world through their external fields.*
>
> *"In considering the energy that could be released by interactions with black holes, a theorem of Hawking is useful. Hawking's theorem states that* ***in the interactions involving black holes, the total surface area of the boundaries of the black holes can never decrease****; it can at best remain unchanged (if the conditions are stationary).*
>
> *"Another example illustrating Hawking's theorem (and considered by him) is the following. Imagine two spherical (Schwarzschild) black holes, each of mass $\frac{1}{2}M$, coalescing to form a single black hole; and let the*

black hole that is eventually left be, again, spherical and have a mass M. Then Hawking's theorem requires that

$$16\pi\overline{M}^2 \geq 16\pi\left[2\left(\frac{1}{2}M\right)^2\right]^2$$

or

$$\overline{M} \geq \frac{M}{\sqrt{2}}$$

"Hence the maximum amount of energy that can be released in such a coalescence is

$$M\left(1-\frac{1}{\sqrt{2}}\right)c^2 = 0.293Mc^2\text{''}$$

Chandrasekhar [8]

"*Also, suppose two black holes collided and merged together to form a single black hole. Then the area of the event horizon of the final black hole would be greater than the sum of the areas of the event horizons of the original black holes.*" Hawking [9]

"*Hawking's area theorem: in any physical process involving a horizon, the area of the horizon cannot decrease in time. ... This fundamental theorem has the result that, while two black holes can collide and coalesce, a single black hole can never bifurcate spontaneously into two smaller ones.*

"Black holes produced by supernovae would be much harder to observe unless they were part of a binary system which survived the explosion and in which the other star was not so highly evolved." Schutz [10]

"*The extreme RN in isotropic coordinates is*

$$ds^2 = V^{-2}dt^2 + V^2\left(d\rho^2 + \rho^2 d\Omega^2\right)$$

where

$$V = 1 + \frac{M}{\rho}$$

This is a special case of the multi black hole solution

$$ds^2 = V^{-2}dt^2 + V^2 d\vec{x}\cdot d\vec{x}$$

where $d\vec{x}\cdot d\vec{x}$ *is the Euclidean 3-metric and* V *is any solution of* $\nabla^2 V = 0$. *In particular*

$$V = 1 + \Sigma_{i=1}^{N}\frac{M_i}{\left|\vec{x}-\vec{\overline{x}}\right|}$$

yields the metric for N extreme black holes of masses M_i *at positions* $\overline{x}_i$.*"* Townsend [11]

"*We not only accept the existence of black holes, we also understand how they can actually form under various circumstances. Theory allows us to calculate the behavior of material particles, fields or other substances near or inside a black hole. What is more, astronomers have now identified numerous objects in the heavens that completely match the detailed descriptions theoreticians have derived. These objects cannot be interpreted as anything else but black holes. The 'astronomical black holes' exhibit no clash whatsoever with other physical laws. Indeed, they have become rich sources of knowledge about physical phenomena under extreme conditions. General Relativity itself can also now be examined up to great accuracies.*" 't Hooft [12]

"*Black holes can be in the vicinity of other black holes.*" 't Hooft [6 §IX]

Much of the justification for the notion of irresistible gravitational collapse into an infinitely dense point-mass 'physical' singularity where spacetime is infinitely curved, and hence the formation of a black hole, is due to Oppenheimer and Snyder [13].

"*In an idealized but illustrative calculation, Oppenheimer and Snyder . . . showed in 1939 that a black hole in fact does form in the collapse of ordinary matter. They considered a 'star' constructed out of a pressureless 'dustball'. By Birkhof's Theorem, the entire exterior of this dustball is given by the Schwarzschild metric . . . . Due to the self-gravity of this 'star', it immediately begins to collapse. Each mass element of the pressureless star follows a geodesic trajectory toward the star's center; as the collapse proceeds, the star's density increases and more of the spacetime is described by the Schwarzschild metric. Eventually, the surface passes through $r = 2M$. At this point, the Schwarzschild exterior includes an event horizon: A black hole has formed. Meanwhile, the matter which formerly constituted the star continues collapsing to ever smaller radii. In short order, all of the original matter reaches $r = 0$ and is compressed (classically!) into a singularity*[4]." Hughes [14]

"[4] *Since all of the matter is squashed into a point of zero size, this classical singularity must be modified in a complete, quantum description. However, since all the singular nastiness is hidden behind an event horizon where it is causally disconnected from us, we need not worry about it (at least for astrophysical black holes).*"

Note that the 'Principle of Superposition' has again been incorrectly applied by Oppenheimer and Snyder, from the outset. They first assume a relativistic universe in which there are multiple mass elements present *a priori*, where the 'Principle of Superposition' however, does not apply. Then mass elements "*collapse*" into a central point (zero volume, finite mass, infinite density), due to 'self-gravity'. But the 'collapse' cannot be due to Newtonian gravitation, because gravity is not a force in General Relativity, and with the resulting black hole, which does not occur in Newton's theory of gravitation. A Newtonian universe cannot collapse into a non-Newtonian universe. Neither can a non-Newtonian universe collapse into a Newtonian universe. Furthermore, the black hole so formed is in an empty universe, since the 'Schwarzschild black hole' relates to $R_{\mu\nu} = 0$, a spacetime that by construction contains no matter. Nonetheless, Oppenheimer and Snyder permit, within the context of General Relativity, the presence of and the gravitational interaction of many mass elements, which coalesce and collapse into a point of zero volume to form an infinitely dense point-mass singularity, when there is no demonstrated general relativistic or Newtonian mechanism by which any of this can occur. Moreover, nobody has ever observed a body, celestial or otherwise, undergo irresistible gravitational collapse, and there is no laboratory evidence whatsoever for the existence of such a phenomenon.

In the 'self-gravity' of a star the cosmologists invoke Newtonian gravitational forces.

"*Assume that it obeys an equation of state. If, according to this equation of state, the pressure stays sufficiently low, one can calculate that this ball of matter will contract under its own weight.*" 't Hooft (see [6 §IV])

"*One must ask what happens when larger quantities of mass are concentrated in a small enough volume. If no stable soution (sic) exists, this must mean that the system collapses under its own weight.*" 't Hooft (see [6 §IV])

Weight is a force due to gravity, but in General Relativity gravity is not a force. Contrary to the practice of cosmologists, Newton's gravitational forces cannot be invoked in General Relativity, because there are none.

"*In Einstein's new theory, gravitation is of a much more fundamental nature: it becomes almost a property of space. . . . Gravitation is thus, properly speaking, not a 'force' in the new theory.*" de Sitter [15]

3 Gravitational wave propagation speed and the linearisation game

The LIGO-Virgo Collaborations opened the *Introduction* to their paper with the following:

"*In 1916, the year after the final formulation of the field equations of general relativity, Albert Einstein predicted the existence of gravitational waves. He found that the linearized weak-field equations had wave solutions: transverse waves of spatial strain that travel at the speed of light, generated by time variations of the mass quadrupole moment of the source.*" Abbott *et al.* [1]

The impression given here that the speed of propagation of Einstein's gravitational waves is uniquely that of light in vacuo is false. Propagation speed of Einstein's gravitational waves is arbitrary, because it is coordinate dependent. That Einstein's waves seem to travel uniquely at the speed of light *in vacuo* is simply because this speed was deliberately selected in order to conform to the presupposition. The method employed to determine the wave equation for Einstein's gravitational waves is the weak-field 'linearisation' of his field equations and concomitant selection of a specific set of coordinates.

Maxwell's electromagnetic theory predicts sinusoidal electromagnetic-wave propagation *in vacuo* at the unique speed v, given by,

$$v = \frac{1}{\sqrt{\epsilon_o \mu_o}} = c \tag{1}$$

The speed of light changes according to the permittivity and permeability of the dielectric medium in which it travels,

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{\sqrt{\kappa\kappa_m}} \tag{2}$$

wherein κ and κ_m are the dielectric constant and relative permeability respectively of the medium. Note that the speed of electromagnetic wave propagation *in vacuo* is not arbitrary. Since the speed of light 'in vacuo' plays a central role in Einstein's Relativity Theory, the motive for choosing coordinates in order to make gravitational waves, emerging from the linearisation game*, travel at the speed of light 'in vacuo', is abundantly clear.

Einstein's gravitational waves are extracted from his linearisation of his nonlinear field equations. Accordingly the metric tensor is written as,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{3}$$

where the $h_{\mu\nu} << 1$ and $\eta_{\mu\nu}$ represents the Galilean values $(1, -1, -1, -1)$. In the linearisation game the $h_{\mu\nu}$ slightly perturb the flat Minkowski spacetime $g_{\mu\nu} = \eta_{\mu\nu}$ from its flatness, and so suffixes are raised and lowered on the $h_{\mu\nu}$ by the $\eta_{\mu\nu}$. Here the $h_{\mu\nu}$ and their first derivatives $\partial h_{\mu\nu}/\partial x^\sigma \equiv h_{\mu\nu,\sigma}$, and higher derivatives, are small, and all products of them are neglected. Since the $\eta_{\mu\nu}$ are constants, the derivatives of Eq.(3) are $g_{\mu\nu,\sigma} = h_{\mu\nu,\sigma}$. The validity of the linearisation game is merely taken on trust because it leads to the desired result.

The selection of a specific coordinate system in order to ensure that gravitational waves propagate at the presupposed speed of light $c = 2.998 \times 10^8$m/s is exposed by the approximation of the Ricci tensor $R_{\mu\nu}$, in accordance with Eq.(3). The Ricci tensor can be first written in the following form by a contraction on the Riemann-Christoffel curvature tensor $R_{\rho\mu\nu\sigma}$, as follows,

$$R_{\mu\nu} = g^{\sigma\rho}\left[\frac{1}{2}\left(g_{\sigma\rho,\mu\nu} - g_{\nu\rho,\sigma\mu} - g_{\mu\sigma,\nu\rho} + g_{\mu\nu,\sigma\rho}\right) + \Gamma_{\beta\rho\sigma}\Gamma^{\beta}_{\mu\nu} - \Gamma_{\beta\rho\nu}\Gamma^{\beta}_{\mu\sigma}\right] \tag{4}$$

Since the last two terms of Eq.(4) are products of the components of the metric tensor $g_{\mu\nu}$ and their first derivatives, by the linearisation game they are neglected, so that the Ricci tensor reduces to,

$$R_{\mu\nu} = g^{\sigma\rho}\left[\frac{1}{2}\left(g_{\sigma\rho,\mu\nu} - g_{\nu\rho,\sigma\mu} - g_{\mu\sigma,\nu\rho} + g_{\mu\nu,\sigma\rho}\right)\right] \tag{5}$$

which can be broken into two parts,

$$R_{\mu\nu} = \frac{1}{2}g^{\sigma\rho}g_{\mu\nu,\sigma\rho} + \frac{1}{2}g^{\sigma\rho}\left[\left(g_{\sigma\rho,\mu\nu} - g_{\nu\rho,\sigma\mu} - g_{\mu\sigma,\nu\rho}\right)\right] \tag{6}$$

If the coordinates x^α are chosen so that the second part of Eq.(6) vanishes, the Ricci tensor reduces further as follows,

$$R_{\mu\nu} = \frac{1}{2}g^{\sigma\rho}g_{\mu\nu,\sigma\rho} = \frac{1}{2}g^{\sigma\rho}\frac{\partial^2 g_{\mu\nu}}{\partial x^\rho \partial x^\sigma} \tag{7}$$

$$g^{\sigma\rho}\left(g_{\sigma\rho,\mu\nu} - g_{\nu\rho,\sigma\mu} - g_{\mu\sigma,\nu\rho}\right) = 0 \tag{8}$$

According to Eq.(3), $g_{\mu\nu,\beta} = h_{\mu\nu,\beta}$ and so on for higher derivatives. Hence,

$$R_{\mu\nu} = \frac{1}{2}\eta^{\sigma\rho}h_{\mu\nu,\sigma\rho} = \frac{1}{2}\eta^{\sigma\rho}\frac{\partial^2 h_{\mu\nu}}{\partial x^\rho \partial x^\sigma} \tag{9}$$

$$\eta^{\sigma\rho}\left(h_{\sigma\rho,\mu\nu} - h_{\nu\rho,\sigma\mu} - h_{\mu\sigma,\nu\rho}\right) = 0 \tag{10}$$

(remembering that suffixes on the kernel h are raised and lowered by $\eta^{\mu\nu}$ according to tensor type). Contracting Eq.(9) yields the Ricci scalar,

$$R = \eta^{\nu\mu}R_{\mu\nu} = \frac{1}{2}\eta^{\nu\mu}\eta^{\sigma\rho}\frac{\partial^2 h_{\mu\nu}}{\partial x^\rho \partial x^\sigma} = \frac{1}{2}\eta^{\rho\sigma}\frac{\partial^2 h}{\partial x^\sigma \partial x^\rho} \tag{11}$$

*"*The rules of the 'linearization game' are as follows: (a) $h_{\mu\nu}$ together with its first derivatives $h_{\mu\nu,\rho}$ and higher derivatives are small, and all products of these are ignored; (b) suffixes are raised and lowered using $\eta^{\mu\nu}$ and $\eta_{\mu\nu}$, rather than $g^{\mu\nu}$ and $g_{\mu\nu}$.*" Foster & Nightingale [16]

Einstein's field equations (without cosmological constant) are,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\kappa T_{\mu\nu} \tag{12}$$

In terms of $h_{\mu\nu}$ these become, using Eq.(9) and Eq.(11),

$$\eta^{\sigma\rho}\frac{\partial^2 h_{\mu\nu}}{\partial x^\rho \partial x^\sigma} - \frac{1}{2}\eta^{\sigma\rho}\frac{\partial^2 h}{\partial x^\sigma \partial x^\rho}\eta_{\mu\nu} = -2\kappa T_{\mu\nu} \tag{13}$$

The d'Almbertian operator is defined by,

$$\Box \equiv -\eta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu} \tag{14}$$

Recalling that $\eta^{\mu\nu}$ represents the Galilean values and that hence $\eta^{\mu\nu} = 0$ when $\mu \neq \nu$, Eq.(14) gives,

$$\Box = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \tag{15}$$

where ∇ is the differential operator *del* (or *nabla*), defined as,

$$\nabla \equiv \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \tag{16}$$

Taking the dot product of *del* with itself gives the Laplacian operator ∇^2,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{17}$$

Setting $x_0 = ct, x_1 = x, x_2 = y, x_3 = z$, Eq.(13) can be written as,

$$\Box\left(h_{\mu\nu} - \frac{1}{2}\delta_{\mu\nu}h\right) = -2\kappa T_{\mu\nu} \tag{18}$$

These are the linearised field equations. They are subject to the condition Eq.(8), which can be condensed to the following condition [17],

$$\frac{\partial}{\partial x^\alpha}\left(h^\alpha_\mu - \frac{1}{2}\delta^\alpha_\mu h\right) = 0 \tag{19}$$

Using Eq.(14), Eq.(9) can be written as,

$$\Box h_{\mu\nu} = 2R_{\mu\nu} \tag{20}$$

For empty space this becomes,

$$\Box h_{\mu\nu} = 0 \tag{21}$$

which by Eq.(15) describes a wave propagating at the speed of light *in vacuo*.

However, the crucial point of the foregoing mathematical development is that Einstein's gravitational waves do not have a unique speed of propagation. The speed of the waves is coordinate dependent, as the condition at Eq.(8) attests. It is the constraint at Eq.(8) that selects a set of coordinates to produce the propagation speed c. A different set of coordinates yields a different speed of propagation, as Eq.(5) does not have to be constrained by Eq.(8). Einstein deliberately chose a set of coordinates that yields the desired speed of propagation at that of light in vacuum (i.e. $c = 2.998 \times 10^8$m/s) in order to satisfy the presupposition that propagation is at speed c. There is no *a priori* reason why this particular set of coordinates is better than any other. The sole purpose for the choice is to obtain the desired and presupposed result.

> "*All the coordinate-systems differ from Galilean coordinates by small quantities of the first order. The potentials $g_{\mu\nu}$ pertain not only to the gravitational influence which has objective reality, but also to the coordinate-system which we select arbitrarily. We can 'propagate' coordinate-changes with **the speed of thought**, and these may be mixed up at will with the more dilatory propagation discussed above. There does not seem to be any way of distinguishing a physical and a conventional part in the changes of $g_{\mu\nu}$.*
>
> "*The statement that in the relativity theory gravitational waves are propagated with the speed of light has,*

I believe, been based entirely upon the foregoing investigation; but it will be seen that it is only true in a very conventional sense. If coordinates are chosen so as to satisfy a certain condition which has no very clear geometrical importance, the speed is that of light; if the coordinates are slightly different the speed is altogether different from that of light. The result stands or falls by the choice of coordinates and, so far as can be judged, the coordinates here used were purposely introduced in order to obtain the simplification which results from representing the propagation as occurring with the speed of light. The argument thus follows a vicious circle." Eddington [17 §57]

4 A black hole is a universe

Each and every black hole is an independent universe by its very definition, no less than the big bang universes are independent universes, because the black hole universe is not contained within its event horizon. Its spacetime extends indefinitely far from its singularity. All types of black hole universes are spatially infinite and eternal, and are either asymptotically flat or, in more esoteric cases, asymptotically curved. There is no bound on asymptotic, for otherwise it would not be asymptotic. Thus every type of black hole constitutes an independent infinite and eternal universe; bearing in mind also that each different type of black hole universe pertains to a different set of Einstein field equations and therefore have nothing to do with one another. Without the asymptotic condition the black hole equations do not result, and one can then write as many non-asymptotic solutions to the corresponding Einstein field equations for the supposed different types of black holes as one pleases [6, 18], none of which produces a black hole.

"*Black holes were first discovered as purely mathematical solutions of Einstein's field equations. This solution, the Schwarzschild black hole, is a nonlinear solution of the Einstein equations of General Relativity. It contains no matter, and exists forever in an asymptotically flat space-time.*" Dictionary of Geophysics, Astrophysics and Astronomy [19]

"*The Kerr-Newman solutions ... are explicit asymptotically flat stationary solutions of the Einstein-Maxwell equation* ($\lambda = 0$) *involving just three free parameters* ***m***, ***a*** *and* ***e***. *... the mass, as measured asymptotically, is the parameter* ***m*** *(in gravitational units). The solution also possesses angular momentum, of magnitude* ***am***. *Finally, the total charge is given by* ***e***. *When* ***a***=***e***=0 *we get the Schwarzschild solution.*" Penrose [20]

"*The charged Kerr metrics are all stationary and axisymmetric ... They are asymptotically flat ...*" Wald [21]

All the different black hole equations are also applicable to stars and planets. Thus, these equations do not permit the presence of more than one star or planet in the universe. In the case of a body such as a star, the only significant difference is that the spacetime does not go to infinite curvature at the star, because there is no singularity and no event horizon in the case of a star (or planet).

5 Black hole gravity

Cosmology maintains that the finite mass of a black hole is concentrated at its 'singularity', where volume is zero, density is infinite, and spacetime curvature is infinite. There are two types of black hole singularity proposed by cosmologists, according to whether or not the black hole is rotating. In the case of no rotation the singularity is a point, adorned with mass: a 'point-mass'. In the case of rotation the singularity is the circumference of a circle, adorned with mass: a circumference-mass. Cosmologists call them 'physical singularities'. These and other mathematical singularities of black hole equations are reified so as to contain the masses of black holes and to locate their event horizons*. Singularities are actually only places in an equation where the equation is undefined, owing for example, to a division by zero. Although they have been construed as such by cosmology, singularities are not in fact physical entities. A singularity also occurs in Newton's theory; it is called the *centre of gravity* or the *centre of mass*. The centre of gravity of a body is not a physical object, rather a mathematical artifice.

"*Let me be more precise as to what one means by a black hole. One says that a black hole is formed when the gravitational forces on the surface become so strong that light cannot escape from it.*
"*...A trapped surface is one from which light cannot escape to infinity.*" Chandrasekhar [8]

*An event horizon is also called 'a trapped surface' or 'a Schwarzschild sphere'.

There are forces in General Relativity but gravity is not one of them, because it is spacetime curvature. It immediately follows that according to cosmologists, gravity is infinite at a black hole singularity. Infinities of density, spacetime curvature, and gravity are claimed to be physically real.

> "*The work that Roger Penrose and I did between 1965 and 1960 showed that, according to general relativity, there must be a singularity of infinite density and space-time curvature, within the black hole. This is rather like the big bang at the beginning of time ...*" Hawking [60]

> "*Once a body of matter, of any mass m, lies inside its Schwarzschild radius* 2*m it undergoes gravitational collapse ... and the singularity becomes physical, not a limiting fiction.*" Dodson and Poston [22]

> "*A nonrotating black hole has a particularly simple structure. At the center is the singularity, a point of zero volume and infinite density where all of the black hole's mass is located. Spacetime is infinitely curved at the singularity. ... The black hole's singularity is a real physical entity. It is not a mathematical artifact ...*" Carroll and Ostlie [23]

> "*As r decreases, the space-time curvature mounts (in proportion to r^{-3}), becoming theoretically infinite at $r = 0$.*" Penrose [20]

> "*One says that '$r = 0$ is a physical singularity of spacetime.'* " Misner, Thorne & Wheeler [24]

> "*Black holes, the most remarkable consequences of Einstein's theory, are not just theoretical constructs. There are huge numbers of them in our Galaxy and in every other galaxy, each being the remnant of a star and weighing several times as much as the Sun. There are much larger ones, too, in the centers of galaxies*." Rees [25]

> "*We not only accept the existence of black holes, we also understand how they can actually form under various circumstances. Theory allows us to calculate the behavior of material particles, fields or other substances near or inside a black hole. What is more, astronomers have now identified numerous objects in the heavens that completely match the detailed descriptions theoreticians have derived.*" 't Hooft [26]

> "*We've got the black holes cornered.*" Stern [27]

However, no finite mass possesses zero volume, infinite density, or infinite gravity, anywhere.

6 The mathematical theory of black holes

The LIGO-Virgo Collaborations have invoked a binary black hole system, merging to cause emission of their reported gravitational waves.

> "*The basic features of GW150914 point to it being produced by the coalescence of two black holes-i.e., their orbital inspiral and merger, and subsequent final black hole ringdown. Over 0.2s, the signal increases in frequency and amplitude in about 8 cycles from 35 to 150 Hz, where the amplitude reaches a maximum. The most plausible explanation for this evolution is the inspiral of two orbiting masses, m_1 and m_2, due to gravitational-wave emission.*" Abbott *et al*. [1]

In the *Introduction* of their paper the LIGO-Virgo Collaborations incorrectly attribute the black hole to K. Schwarzschild.

> "*Also in 1916, Schwarzschild published a solution for the field equations [4] that was later understood to describe a black hole [5,6], and in 1963 Kerr generalized the solution to rotating black holes [6].*" Abbott *et al*. [1]

The resultant black hole type is identified in [1].

> "*A pair of neutron stars, while compact, would not have the required mass, while a black hole neutron star binary with the deduced chirp mass would have a very large total mass, and would thus merge at much lower frequency. This leaves black holes as the only known objects compact enough to reach an orbital frequency of 65 Hz without contact. Furthermore, the decay of the waveform after it peaks is consistent with the damped oscillations of a black hole relaxing to a final stationary Kerr configuration.*" Abbott *et al*. [1]

All the black holes are identified in [28].

> "*Here we perform several studies of GW150914, aimed at detecting deviations from the predictions of GR. Within the limits set by LIGO's sensitivity and by the nature of GW150914, we find no statistically significant evidence against the hypothesis that, indeed, GW150914 was emitted by a binary system composed of two black holes (i.e., by the Schwarzschild [16] or Kerr [18] GR solutions), that the binary evolved dynamically toward merger, and that it formed a merged rotating black hole consistent with the GR solution.*" Abbott *et al.* [28]

Note the invalid superposition of the two 'Schwarzschild' or 'Kerr' black holes, due to violation of their asymptotic flatness (each encounters infinite spacetime curvature i.e. infinite gravity, at the singularity of the other). The Kerr configuration subsumes the Schwarzschild configuration and so depends upon the existence of the latter. The Schwarzschild solution has no physical meaning because it is the solution to a set of physically meaningless equations (see §6 and §7 below). Furthermore, all black hole equations are obtained by violations of the rules of pure mathematics, which will now be proven.

Satisfaction of the Einstein field equations is a necessary but insufficient condition for determination of Einstein's gravitational field. Einstein's field equations "*in the absence of matter*" [29] are,

$$R_{\mu\nu} = 0 \tag{22}$$

To determine his gravitational field *in the absence of matter*, Einstein prescribed the following conditions:

1. the solution must be static
2. it must be spherically symmetric
3. it must satisfy the field equations
4. it must be asymptotically flat.

Consider Schwarzschild's [30] actual solution to Eq.(22), which is not the solution that has been assigned to him by cosmologists:

$$\begin{gathered} ds^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \\ R = \left(r^3 + \alpha^3\right)^{\frac{1}{3}}, \quad 0 \leq r \end{gathered} \tag{23}$$

Here α is a positive real-valued constant and $r = \sqrt{x^2 + y^2 + z^2}$. The speed of light *in vacuo* is set to unity, i.e. $c = 1$. Eq.(23) is singular (i.e. undefined) only at $r = 0$ (i.e. when $x = y = z = 0$). Contrast this with the so-called 'Schwarzschild solution' attributed to Schwarzschild but actually due to the German mathematician D. Hilbert [31-33],

$$\begin{gathered} ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \\ 0 \leq r \end{gathered} \tag{24}$$

Here $c = 1$, Newton's gravitational constant $G = 1$, and M is claimed to be the mass that produces the gravitational field. Note that *prima facie* Eq.(24) is singular (i.e. undefined) at $r = 2M$ and at $r = 0$. Eq.(24) is not equivalent to Eq.(23) owing to $0 \leq r$ in Eq.(24). If they are equivalent then in Eq.(23) it must be that $-\alpha \leq r = \sqrt{x^2 + y^2 + z^2}$.

Eq.(24) is somewhat deceptive. Rewriting it explicitly with c and G is much more informative,

$$\begin{gathered} ds^2 = c^2\left(1 - \frac{2GM}{c^2 r}\right) dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \\ 0 \leq r \end{gathered} \tag{25}$$

In Eq.(24) the so-called 'Schwarzschild radius' is $r_s = 2M$. From Eq.(25) the Schwarzschild radius r_s is easily identified in full, "*This value is known as the Schwarzschild radius. In terms of the mass of the object that is the source of the gravitational field, it is given by*

$$r_s = \frac{2GM}{c^2}$$

"*For ordinary stars, the Schwarzschild radius lies buried deep in the stellar interior.*" McMahon [5]

"*Remarkably, as we shall see this is exactly the modern formula for the radius of a black hole in general relativity* ..." Schutz [10]

According to cosmologists, the Schwarzschild radius (or 'gravitational radius') is the radius of the event horizon of a black hole. That r is incorrectly treated as the radius by cosmologists is most clearly evident by the very 'Schwarzschild radius', which for stars, "*lies buried deep in the stellar interior*" [5].

"*The Schwarzschild radius for the Earth is about 1.0 cm and that of the Sun is 3.0 km.*" d'Inverno [34]

"*For example, a Schwarzschild black hole of mass equal to that of the Earth, $M_E = 6 \times 10^{26} g$, has $r_s = 2GM_E/c^2 \approx 1$ cm. ... A black hole of one solar mass has a Schwarzschild radius of only 3km.*" Wald [21]

"... *'ordinary' stars and planets contain matter ($T_{\mu\nu} = 0$) within a certain radius $r > 2M$, so that for them the validity of the Schwarzschild solution stops there.*" 't Hooft [12]

In relation to Hilbert's solution, the cosmologists Celotti, Miller and Sciama [35], make the following assertion:

"*The 'mean density' $\overline{\rho}$ of a black hole (its mass M divided by $\frac{4}{3}\pi r_s^3$) is proportional to $1/M^2$*" [35]

where r_s is the 'Schwarzschild radius'. However, the expression $\frac{4}{3}\pi r^3$ gives the volume of a Euclidean sphere where r is the radius of the sphere. It does not give the volume of the non-Euclidean sphere within Hilbert's solution, where the volume is in fact given by [36-48],

$$V = \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi \int_{2M}^{r} \frac{r^2 \, dr}{\sqrt{1 - \frac{2M}{r}}} = 4\pi \int_{2M}^{r} \frac{r^2 \, dr}{\sqrt{1 - \frac{2m}{r}}} \tag{26}$$

which is a particular case of the general expression [36, 37],

$$V = \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi \int_{r_o}^{r} \frac{R_c^2(r)}{\sqrt{1 - \frac{\alpha}{R_c(r)}}} \frac{dR_c}{dr} dr = 4\pi \int_{r_o}^{r} \frac{R_c^2(r)}{\sqrt{1 - \frac{\alpha}{R_c(r)}}} \frac{dR_c}{dr} dr \tag{27}$$

wherein the value of the real number r_o, although arbitrary, affects the form of $R_c(r)$.

Cosmology confounds the quantity r as radial distance, which ultimately gives rise to the 'Schwarzschild radius' r_s. It is variously and vaguely called the '*areal radius*', the '*Schwarzschild r-coordinate*', the '*distance*', '*the radius*', the '*radius of a 2-sphere*', the '*radial coordinate*', the '*reduced circumference*', the '*radial space coordinate*', the '*coordinate radius*', and even "*a gauge choice: it determines the coordinate r*" 't Hooft [6 §7]. None of these mere labels correctly identifies the geometric significance of the quantity r in Hilbert's solution.

Cosmologists maintain that the Schwarzschild radius $r = r_s$ is a 'coordinate' or 'apparent' or 'removable' singularity, and that $r = 0$ is a 'physical singularity' (because it is endowed with the fantastic physical properties in §5 above).

The quantity R in Schwarzschild's solution and the quantity r in Hilbert's solution can be replaced by any analytic real-valued function $R_c(r)$ of the real variable r without violating $R_{\mu\nu} = 0$ or spherical symmetry. However, not simply any analytic function of r is permissible. Satisfaction of the field equations is a necessary but insufficient condition for determination of Einstein's 'gravitational field'. For example, replace Hilbert's r with $R_c(r) = e^r$. The resulting metric is singular only at $r = \ln(2M)$. At $r = 0$ nothing special happens; on the unproven assumption that $0 \leq r \leq \ln(2M)$ is permissible. But $R_c(r) = e^r$ is forbidden because the resulting metric is not asymptotically flat. The infinite equivalence class of permissible analytic functions $R_c(r)$ must be ascertained.

Let r' be the radius of a Euclidean sphere. It is routinely claimed for Eq.(24) and Eq.(25) that $r = r' = \sqrt{x^2 + y^2 + z^2}$ (Einstein [48]), from which the black hole was constructed. This is incorrect [6,7,18,36-48] because here,

$$r = \sqrt{x_o^2 + y_o^2 + z_o^2} + \sqrt{(x - x_o)^2 + (y - y_o)^2 + (z - z_o)^2} = r_o + r' \tag{28}$$

where

$$r_o = \sqrt{x_o^2 + y_o^2 + z_o^2} = \frac{2GM}{c^2} \tag{29}$$

Notwithstanding, r is neither the radius nor even a distance in any black hole equation [6,7,18,36-48]; a mathematical fact which subverts the entire theory of black holes. The reader is referred to [6,7,18,36-48] for all the mathematical details, which I only summarise herein.

Geometrically speaking Eq.(25) means that the black hole is the result of unwittingly moving a sphere, initially centred at the origin of coordinates, to some other place, but leaving its centre behind. Analytically this is revealed by,

$$\begin{gathered} ds^2 = \left(1 - \frac{\alpha}{R_c}\right) dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right) \\ R_c = \left(|r - r_o|^n + \alpha^n\right)^{\frac{1}{n}}, \quad r, r_o \in \Re, \quad n \in \Re^+ \end{gathered} \tag{30}$$

Eqs.(30) satisfy Einstein's prescription, and constitute an infinite equivalence class because every element of the class describes the very same metric space.

The radius R_p for Eq.(30) is,

$$R_p = \int \frac{dR_c}{\sqrt{1 - \frac{\alpha}{R_c}}} = \sqrt{R_c \left(R_c - \alpha\right)} + \alpha \ln\left(\frac{\sqrt{R_c} + \sqrt{R_c - \alpha}}{\sqrt{\alpha}}\right) \tag{31}$$

Note that $R_c\,(r_o) = \alpha \;\forall r_o \;\forall n$ and $R_p\,(r_o) = 0 \;\forall r_o \;\forall n$. The constants r_o and n are arbitrary. Setting $r_o = 0, n = 3, r_o \leq r$ yields Schwarzschild's actual solution [16]. Setting $r_o = 0, n = 1, r_o \leq r$ yields Brillouin's solution [36]. Setting $r_o = \alpha, n = 1, r_o \leq r$ yields Droste's solution [36]. Hilbert's solution is not an element of the infinite equivalence class. Note that Hilbert's solution is an alleged 'extension' of Droste's solution to $0 \leq r$, for an 'event horizon' at 'the radius' $r = \alpha$ and a 'physical singularity' at 'the origin' $r = 0$. Although $r = 0$ denotes the origin of a coordinate system, it does not denote the centre of spherical symmetry of Eq.(24) and Eq.(25), as Eq.(30) reveals. The centre of spherical symmetry is at $r = r_o$. When a sphere initially centred at the origin of coordinates is moved, it takes its centre with it, and the position of the sphere is specified by the coordinates of its centre (x_0, y_0, z_0) so that whereupon the radius r' of the sphere is no longer given by $r = r' = \sqrt{x^2 + y^2 + z^2}$, but by $r' = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$. The intrinsic geometry of a sphere is not altered by changing its position and so its radius does not change with a change of position. When Hilbert set r^2 as the coefficient of $\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$ in the derivation of his solution, he unwittingly shifted the centre of Schwarzschild's Euclidean sphere from $r = r_o = 0$ to the coordinates (x_0, y_0, z_0) at the distance $r = 2m$ from the origin of coordinates, mistakenly thinking the centre of that sphere still at $r = 0$. Hilbert shifted Schwarzschild's Euclidean sphere but left its centre behind. The result was fantastic. David Hilbert had separated the Euclidean sphere from its centre and even placed its centre outside the sphere!

Owing to equivalence, if any element of the infinite equivalence class determined by Eq.(30) cannot be extended then none can be extended, owing to equivalence. It is immediately apparent that none can be extended because $|r - r_o|^n \geq 0$. This is amplified by the case $r_o = 0, n = 2$, in which case Eq.(30) is defined for all real values of r except $r = r_o = 0$. In this case the black hole requires that,

$$-\alpha^2 \leq r^2 = \left(x^2 + y^2 + z^2\right) \tag{32}$$

Thus, the 'Schwarzschild' black hole is invalid because it violates the rules of pure mathematics - it requires the square of a real number to be less than zero. In general, the mathematical theory of black holes requires that the positive power of the absolute value of a real number must take on values less that zero. The same violation of the rules of pure mathematics produces all the black hole universes [6,7,18,36-47]. All purported means of extending Droste's solution to Hilbert's are consequently invalid [6,46,47].

Schwarzschild spacetime can be written in the 'isotropic coordinates'. The infinite equivalence class in this case is given by [6,7,49] [4,6,34]*,

$$\begin{gathered} ds^2 = \left(\frac{4R_c - \alpha}{4R_c + \alpha}\right)^2 dt^2 - \left(1 + \frac{\alpha}{4R_c}\right)^4 \left[dR_c^2 + R_c^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)\right] \\ R_c = \left[|r - r_o|^n + \left(\frac{\alpha}{4}\right)^n\right]^{\frac{1}{n}}, \quad r, r_o \in \Re, \quad n \in \Re^+ \end{gathered} \tag{33}$$

*Here $c = 1$.

and the radius is,

$$R_p = \int \left(1 + \frac{\alpha}{4R_c}\right)^2 dR_c = R_c + \frac{\alpha}{2}\ln\left(\frac{4R_c}{\alpha}\right) - \frac{\alpha^2}{8R_c} + \frac{\alpha}{4} \tag{34}$$

Note that $R_c(r_o) = \alpha/4 \ \forall r_o \ \forall n$ and $R_p(r_o) = 0 \ \forall r_o \ \forall n$. Once again it is evident that no black hole is possible without a violation of the rules of pure mathematics, as the case $r_o = 0, n = 2$ again amplifies.

The Kerr-Newman solution adds charge q and angular momentum a to the 'Schwarzschild solution'*. The infinite equivalence class for Kerr-Newman spacetime is given by [6,18,40,],

$$\begin{gathered} ds^2 = -\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta\, d\varphi\right)^2 + \frac{\sin^2\theta}{\rho^2}\left[\left(R_c^2 + a^2\right)d\varphi - a dt\right]^2 + \frac{\rho^2}{\Delta}dR_c^2 + \rho^2 d\theta^2 \\ \Delta = R_c^2 - \alpha R_c + a^2 + q^2, \ \ \rho^2 = R_c^2 + a^2\cos^2\theta \\ R_c = \left(|r - r_o|^n + \zeta^n\right)^{\frac{1}{n}}, \ \ r, r_o \in \Re, \ \ n \in \Re^+ \\ \zeta = \frac{\alpha + \sqrt{\alpha^2 - 4q^2 - 4a^2\cos^2\theta}}{2}, \ \ a^2 + q^2 < \frac{\alpha^2}{4} \end{gathered} \tag{35}$$

The infinite equivalence class for Kerr spacetime is obtained from Eqs.(35) by setting $q = 0$. Similarly, the infinite equivalence class for Reissner-Nordström spacetime is obtained from Eqs.(35) by setting $a = 0$. Setting $a = 0$ and $q = 0$ in Eqs.(35) yields the infinite equivalence class for Schwarzschild spacetime. No black hole is possible without a violation of the rules of pure mathematics, as the case $r_o = 0, n = 2$ yet again amplifies.

Black hole 'escape velocity'

On the one hand, cosmologists assign an escape speed to the black hole. At the event horizon it is the speed of light. Rearranging the 'Schwarzschild radius' for c gives,

$$c = \sqrt{\frac{2GM}{r_s}} \tag{36}$$

which is immediately recognised as the Newtonian expression for escape speed. Although only one term for mass appears in this expression (i.e. M), it is an implicit two-body relation: one body 'escapes' from another body. Consequently, the Newtonian expression for escape speed cannot rightly appear in a solution for a one-body problem. The Schwarzschild solution is supposedly for a one-body problem. It is by this incorrect insinuation of the Newtonian expression for escape speed that cosmologists assign the black hole an escape speed, especially at its 'event horizon'.

> "***black hole*** *A region of spacetime from which the escape velocity exceeds the velocity of light. In Newtonian gravity the escape velocity from the gravitational pull of a spherical star of mass M and radius R is*
>
> $$v_{esc} = \sqrt{\frac{2GM}{R}},$$
>
> *where G is Newton's constant. Adding mass to the star (increasing M), or compressing the star (reducing R) increases v_{esc}. When the escape velocity exceeds the speed of light c, even light cannot escape, and the star becomes a black hole. The required radius R_{BH} follows from setting v_{esc} equal to c:*
>
> $$R_{BH} = \frac{2GM}{c^2}.$$
>
> "*. . . In General Relativity for spherical black holes (Schwarzschild black holes), exactly the same expression R_{BH} holds for the surface of a black hole. The surface of a black hole at R_{BH} is a null surface, consisting of those photon trajectories (null rays) which just do not escape to infinity. This surface is also called the black hole horizon.*" Dictionary of Geophysics, Astrophysics and Astronomy [19]

*The pronumeral a is called 'the angular momentum parameter': $a = J/M$ where J is angular momentum and M is the mass of the source of a 'gravitational field' (i.e. the mass of a star or a black hole).

"***black hole*** *A massive object so dense that no light or any other radiation can escape from it; its escape velocity exceeds the speed of light.*" Collins Encyclopaedia of the Universe [50]

"*A black hole is characterized by the presence of a region in space-time from which no trajectories can be found that escape to infinity while keeping a velocity smaller than that of light.*" 't Hooft [26]

On the other hand, nothing can even leave the event horizon of a black hole, not even light.

"*The problem we now consider is that of the gravitational collapse of a body to a volume so small that a trapped surface forms around it; as we have stated, from such a surface no light can emerge.*" Chandrasekhar [8]

"*It is clear from this picture that the surface $r = 2m$ is a one-way membrane, letting future-directed timelike and null curves cross only from the outside (region I) to the inside (region II).*" d'Inverno [34]

"*Things can go into the horizon (from $r > 2M$ to $r < 2M$), but they cannot get out; once inside, all causal trajectories (timelike or null) take us inexorably into the classical singularity at $r = 0$. . . . The defining property of black holes is their event horizon. Rather than a true surface, black holes have a 'one-way membrane' through which stuff can go in but cannot come out.*" Hughes [14]

"*Einstein predicts that nothing, not even light, can be successfully launched outward from the horizon . . . and that light launched outward EXACTLY at the horizon will never increase its radial position by so much as a millimeter.*" Taylor and Wheeler [51]

"*In the exceptional case of a ∂_v photon parametrizing the positive v axis, $r = 2M$, though it is racing 'outward' at the speed of light the pull of the black hole holds it hovering at rest. . . . No particle, whether material or lightlike, can escape from the black hole.*" O'Neill [52]

"*Thus we cannot have direct observational knowledge of the region $r < 2m$. Such a region is called a black hole, because things can fall into it (taking an infinite time, by our clocks, to do so) but nothing can come out.*" Dirac [53]

"*The most obvious asymmetry is that the surface $r = 2m$ acts as a one-way membrane, letting future-directed timelike and null curves cross only from the outside ($r > 2m$) to the inside ($r < 2m$).*" Hawking and Ellis [54]

"*It turned out that, at least in principle, a space traveller could go all the way in such a 'thing' but never return. Not even light could emerge out of the central region of these solutions. It was John Archibald Wheeler who dubbed these strange objects 'black holes'.*" 't Hooft [26]

Escape speed however means that things can either leave or escape from some other body, depending upon initial speed at the place of departure. It does not mean that nothing can leave. To escape from some body, the escapee must achieve the escape speed. If it fails to do so it can leave, but not escape, unless its initial speed is precisely 0 m/s, in which case it neither leaves nor escapes, because its does not move. If it achieves the escape speed it can leave and escape. Escape speed does not mean that nothing can leave. The black hole event horizon has an escape speed, the speed of light c, yet nothing, not even light, can leave (light hovers forever at the event horizon as it tries to 'escape'). As the foregoing citations attest, cosmologists assert that the black hole event horizon has the unique property of having and not having an escape speed simultaneously at the same place. However, no material body can have and not have an escape speed simultaneously, anywhere.

"*A black hole is, ah, a massive object, and it's something which is so massive that light can't even escape. . . . some objects are so massive that the escape speed is basically the speed of light and therefore not even light escapes. . . . so black holes themselves are, are basically inert, massive and nothing escapes.*" Bland-Hawthorn [55]

If the escape speed at the event horizon of a black hole is the speed of light, and light travels at the speed of light, then, by the very definition of escape speed, light must escape. Cosmologists however assert the opposite; that the escape speed at the event horizon is the speed of light, so light cannot escape! In fact, light cannot even leave the event horizon, hovering there instead, forever. In other words, the speed of light at the event horizon along a radially outward direction is $c = 0$ m/s and thereby light cannot either leave or escape, because light is not moving. On the other hand, the speed of light at the event horizon, the 'escape' speed, according to the cosmologists, is $c = 2.998 \times 10^8$m/s $= \sqrt{2GM/r_s}$, Einstein's 'speed of light *in vacuo*'. Thus, the speed of light at the black hole event horizon has a split personality; two different values at the same place, *in vacuo*. Furthermore, if the escape speed is zero, any speed greater than zero must ensure leaving and escape. Presumably, no physical object can even achieve the escape speed $c = 0$, because, according to the cosmologists, nothing at all can even leave the event horizon, let alone escape from it. In Relativity Theory the speed of any material body is always restricted to values less than that of $c = 2.998 \times 10^8$m/s, not to $c = 0$ m/s. If the escape speed at the event horizon is 0 m/s, this contradicts the escape speed obtained from the 'Schwarzschild radius': $v_{esc} = \sqrt{\frac{2GM}{r_s}} = c = 2.998 \times 10^8$m/s which is > 0. In fact, the 'Schwarzschild radius' is itself obtained by setting $v_{esc} = c = 2.998 \times 10^8$m/s in the Newtonian expression for escape speed. Thus, on the one hand, according to the cosmologists, the escape speed at the event horizon of a black hole is the speed of light $c = 2.998 \times 10^8$m/s.

By various mathematical approaches which amount to the same thing, the cosmologists on the other hand claim that the escape speed at the event horizon (the speed of light) is 0m/s. One of their means is to set $\theta = const.$ and $\varphi = const.$ in Hilbert's solution to yield for 'radial motion'. For light $ds = c\,d\tau = 0$, because the so-called 'proper time' $\tau = 0$*. Hence,

$$0 = c^2\left(1 - \frac{2GM}{c^2 r}\right)dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 \tag{37}$$

Rearrangement for what cosmologists call 'the radial velocity' [56,57] gives,

$$v = \frac{dr}{dt} = \pm c\left(1 - \frac{2GM}{c^2 r}\right) \tag{38}$$

> "*The + sign is for a light ray heading outwards i.e. r increasing with time, and the − is for a light ray heading inwards, i.e. r decreasing with time.*" Rennie [57]

At the event horizon $r = r_s = 2GM/c^2$ (the 'Schwarzschild radius'). Putting this value into Eq.(38) yields,

$$v = v_{esc} = \frac{dr}{dt} = \pm c\left(1 - \frac{2GM}{c^2}\frac{c^2}{2GM}\right) = 0 \tag{39}$$

Thus, according to the cosmologists, the speed of light at the event horizon is zero for light travelling either outward or inward.

> "*We find that the velocity of light at the event horizon is zero.*" Rennie [57]

This is the other cosmologist 'escape speed' (here the outward radial speed for the + sign) at the black hole event horizon. Consequently, light cannot leave or escape because it is unable to even move. Contrast this with the 'escape speed' at the black hole event horizon obtained from the 'Schwarzschild radius': $v_{esc} = \sqrt{\frac{2GM}{r_s}} = c = 2.998 \times 10^8$m/s. A body freely falling from rest 'at infinity' along a radial line acquires a speed equal to that of the escape speed, according to the 'Schwarzschild radius' r_s, because r_s is obtained from the Newtonian relation for escape speed. Note that in Eq.(39) the cosmologists give the speed of light two different values: the escape speed $c = 2.998 \times 10^8$m/s by $r_s = 2GM/c^2$ and the escape speed 0 m/s by means of $dr/dt = 0$. The speed of light (the 'escape speed') cannot have two different values in the one equation. This logical absurdity however does not stop cosmologists.

The proof that the Newtonian relation for escape speed is a two-body relation is elementary. According to Newton's theory,

$$F_g = -\frac{GMm}{r^2} = ma = m\frac{dv}{dt} = mv\frac{dv}{dr} \tag{40}$$

where G is the gravitational constant and r is the distance between the centre of mass of m and the centre of mass of M. A centre of mass is not a physical object; merely a mathematical artifice. Although Newton's F_g is singular at $r = 0$, this does not produce a 'physical singularity'. Separating variables and integrating gives,

$$\int_v^0 mv\,dv = \lim_{r_f \to \infty} \int_R^{r_f} -GMm\frac{dr}{r^2} \tag{41}$$

*The motion of light is 'light-like', or 'null': hence $ds = 0$.

whence,

$$v = \sqrt{\frac{2GM}{R}} \tag{42}$$

where R is the radius of the mass M. Thus, although Eq.(42) contains only one mass term (M), escape speed necessarily involves two bodies: m and M.

In any event, contrary to cosmology, the Newtonian implicit two-body escape speed relation cannot be involved because the black hole pertains to a universe that contains only one mass (that of the black hole itself) by hypothesis. The impossible duality of cosmology's black hole 'escape velocity' is now clear. The black hole event horizon has an escape speed and no escape speed simultaneously at the same place. But, contrary to cosmology, nothing can have and not have an escape speed simultaneously at the same place. Furthermore, according to Einstein, no material body can move with a speed that is equal to or greater than the speed of light *in vacuo*, i.e. $c = 2.998 \times 10^8$m/s, but can certainly move with a speed v such that $0 < v < c = 2.998 \times 10^8$m/s. Cosmologists, with an escape speed $v_{esc} = c = 0$ do not permit any material object to have a speed greater than zero at their event horizon, contrary to Einstein's fundamental tenet, because, they say, no material body can move at or greater than the speed of light. In other circles this is called 'an each-way bet'.

Since Hilbert's solution is utilised by cosmologists, $0 \leq r$. Therefore, if $0 < r < 2M$ the escape speed from Eq.(38) becomes negative and hence is no longer an escape speed. Beneath the event horizon, say cosmologists, the 'escape velocity' is greater than that of light. Cosmologists have an additional and equally bizarre interpretation for this: time-convergence "*inexorably into the classical singularity at* $r = 0$" [14], into the black hole's 'physical singularity', because Hilbert's metric changes its signature and becomes time-dependent (i.e. non-static). Eqs.(30) maintain a fixed signature, $(+, -, -, -)$. It is not possible for the signature to change to $(-, +, -, -)$, for instance. Cosmologists admit that when $0 < r < 2m$ in Hilbert's Eq.(24), the roles of t and r are interchanged. This produces a non-static solution to a static problem, i.e. a solution that is time-dependent for a problem that is time-independent. To further illustrate this violation, when $2m < r$ the signature of Eq.(24) is $(+, -, -, -)$; but if $0 < r < 2m$ in Eq.(24), then

$$g_{oo} = \left(1 - \frac{2M}{r}\right) \text{ is negative, and } g_{11} = -\left(1 - \frac{2M}{r}\right)^{-1} \text{ is positive.} \tag{43}$$

So the signature of Eq.(24) changes from $(+, -, -, -)$ to $(-, +, -, -)$. Thus the roles of t and r are exchanged. According to Misner, Thorne and Wheeler, who use the spacetime signature $(-, +, +, +)$ instead of $(+, -, -, -)$,

> "*The most obvious pathology at* $r = 2M$ *is the reversal there of the roles of t and r as timelike and spacelike coordinates. In the region* $r > 2M$*, the t direction,* $\partial/\partial t$*, is timelike* ($g_{tt} < 0$) *and the r direction,* $\partial/\partial r$*, is spacelike* ($g_{rr} > 0$)*; but in the region* $r < 2M$*,* $\partial/\partial t$*, is spacelike* ($g_{tt} > 0$) *and* $\partial/\partial r$*, is timelike* ($g_{rr} < 0$).
>
> "*What does it mean for r to 'change in character from a spacelike coordinate to a timelike one'? The explorer in his jet-powered spaceship prior to arrival at* $r = 2M$ *always has the option to turn on his jets and change his motion from decreasing r (infall) to increasing r (escape). Quite the contrary in the situation when he has once allowed himself to fall inside* $r = 2M$*. Then the further decrease of r represents the passage of time. No command that the traveler can give to his jet engine will turn back time. That unseen power of the world which drags everyone forward willy-nilly from age twenty to forty and from forty to eighty also drags the rocket in from time coordinate* $r = 2M$ *to the later time coordinate* $r = 0$*. No human act of will, no engine, no rocket, no force (see exercise 31.3) can make time stand still. As surely as cells die, as surely as the traveler's watch ticks away 'the unforgiving minutes', with equal certainty, and with never one halt along the way, r drops from* $2M$ *to* 0.
>
> "*At* $r = 2M$*, where r and t exchange roles as space and time coordinates,* g_{tt} *vanishes while* g_{rr} *is infinite.*"
> Misner, Thorne and Wheeler [24]

Note that at $r = 2M$, $g_{rr} = (1 - 2M/r)^{-1}$ is not in fact infinite. At $r = 2M$, $g_{rr} = 1/0$, which is undefined. Similarly, if $r = 0$, $2M/r = 2M/0$ which is undefined. Contrary to the cosmologists, division by zero does not produce 'infinity', it is actually undefined, and infinity is not even a number*.

> "*There is no alternative to the matter collapsing to an infinite density at a singularity once a point of no-return is passed. The reason is that once the event horizon is passed, all time-like trajectories must necessarily get to the singularity: 'all the King's horses and all the King's men' cannot prevent it.*" Chandrasekhar [8]

*Cantor's theory of 'transfinite numbers' has no relevence here either.

"*This is worth stressing; not only can you not escape back to region I, you cannot even stop yourself from moving in the direction of decreasing r, since this is simply the timelike direction. (This could have been seen in our original coordinate system; for r < 2GM, t becomes spacelike and r becomes timelike.) Thus you can no more stop moving toward the singularity than you can stop getting older.*" Carroll [23]

"*For $r < 2GM/c^2$, however, the component g_{oo} becomes negative, and g_{rr}, positive, so that in this domain, the role of time-like coordinate is played by r, whereas that of space-like coordinate by t. Thus in this domain, the gravitational field depends significantly on time (r) and does not depend on the coordinate t.*" Vladmimirov, Mitskiévich and Horský [58]

To amplify this, set $t = r^*$ and $r = t^*$. Then for $0 < r < 2M$, Eq.(24) becomes,

$$ds^2 = \left(1 - \frac{2M}{t^*}\right) dr^{*2} - \left(1 - \frac{2M}{t^*}\right)^{-1} dt^{*2} - t^{*2}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$
$$0 \le t^* < 2M \tag{44}$$

It now becomes quite clear that this is a time-dependent metric since all the components of the metric tensor are functions of the timelike t^*. Therefore this metric bears no relationship to the original time-independent problem to be solved. In other words, this metric is a non-static solution to a static problem (see also Brillouin [59]).

Infinite densities

The 'infinite density' of the black hole's 'physical singularity' produced by irresistible 'gravitational collapse' violates Special Relativity. The singularity of big bang cosmology is also infinitely dense. Yet according to Special Relativity, infinite densities are forbidden because their existence implies that a material object can acquire the speed of light c in vacuo i.e. 2.998×10^8m/s (or equivalently, the existence of infinite kinetic energy), thereby violating the very basis of Special Relativity.

"*Eventually when a star has shrunk to a certain critical radius, the gravitational field at the surface becomes so strong that the light cones are bent inward so much that the light can no longer escape. According to the theory of relativity, nothing can travel faster than light. Thus, if light cannot escape, neither can anything else. Everything is dragged back by the gravitational field. So one has a set of events, a region of space-time from which it is not possible to escape to reach a distant observer. This region is what we now call a black hole. Its boundary is called the event horizon. It coincides with the paths of the light rays that just fail to escape from the black hole.*" Hawking [60]

Since General Relativity cannot violate Special Relativity, General Relativity must therefore also forbid infinite densities. Therefore, point-mass singularities are forbidden by the Theory of Relativity. Let a cuboid rest-mass m_o have sides of length L_o. Let m_o have a relative speed $v < c$ in the direction of one of three mutually orthogonal Cartesian axes attached to an observer of rest-mass M_o. According to Einstein [61] the observer M_o reckons the moving mass m is,

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{45}$$

and the volume is,

$$V = L_o^2\sqrt{1 - \frac{v^2}{c^2}}. \tag{46}$$

The density of m according to M_o is therefore,

$$D = \frac{m}{V} = \frac{m_o}{L_o^2\left(1 - \frac{v^2}{c^2}\right)}. \tag{47}$$

Hence, $v \to c \Rightarrow D \to \infty$. Since, according to Special Relativity, no material object can acquire the speed c, infinite densities are forbidden by Special Relativity, and so point-mass singularities and circumference-mass singularities are forbidden. Since General Relativity cannot repudiate Special Relativity, it too must thereby forbid infinite densities and hence forbid point-mass singularities and circumference-mass singularities. It does not matter how it is alleged that a 'physical singularity' is generated by General Relativity because the infinitely dense physical singularity cannot be reconciled with Special Relativity.

Point-charges and circumference-charges too are therefore forbidden by the Theory of Relativity since there can be no charge without mass.

Curvature invariants

The squared differential element of arc of a curve in a surface is given by the First Fundamental Quadratic Form for a surface,

$$ds^2 = E\,du^2 + 2F\,du\,dv + G\,dv^2 \tag{48}$$

wherein u and v are curvilinear coordinates. If either u or v is constant the resulting line-elements are called parametric curves in the surface. The differential element of surface area is given by,

$$dA = \sqrt{EG - F^2}\,du\,dv \tag{49}$$

An expression which depends only on E, F, G, and their first and second derivatives, is called *a bending invariant*. It is an intrinsic (or absolute) property of a surface. The Gaussian (or Total) curvature of a surface is an important intrinsic property of a surface.

The 'Theorema Egregium' of Gauss: *The Gaussian curvature K at any point P of a surface depends only on the values at P of the coefficients in the First Fundamental Quadratic Form and their first and second derivatives.*

Hence, the Gaussian curvature of a surface is a bending invariant.

The Euclidean plane has a constant Gaussian curvature of $K = 0$. A surface of positive constant Gaussian curvature is called *a spherical surface*. A surface of constant negative curvature is called *a pseudo-spherical surface*.

Being an intrinsic geometric property of a surface, Gaussian curvature is independent of any embedding space.

> "*And in any case, if the metric form of a surface is known for a certain system of intrinsic coordinates, then all the results concerning the intrinsic geometry of this surface can be obtained without appealing to the embedding space.*" Efimov [62]

All black hole spacetime metrics contain a surface from which various invariants and geometric identities can be deduced by purely mathematical means. Such identities are independent of the area of the surface and of the length of any curve in the surface. The Kerr-Newman form subsumes the Kerr, Reissner-Nordström, and Schwarzschild forms. The Gaussian curvature of the surface in the Kerr-Newman metric therefore subsumes the Gaussian curvatures of the surfaces in the subordinate forms to which it can be reduced. Gaussian curvature reveals the type of surface and uniquely identifies the terms that appear in its general form. Gaussian curvature reveals that no purported black hole metric can in fact be extended to produce the black hole it is said to contain.

The Gaussian curvature K of a surface can be calculated by means of the following relation,

$$K = \frac{R_{1212}}{g} \tag{50}$$

where R_{1212} is a component of the Riemann-Christoffel curvature tensor of the first kind and g is the determinant of the metric tensor. Note that neither the area of the surface nor the length of any curve in the surface is involved.

If $r = const. \neq 0$ and $t = const.$, Eq.(35) reduces to the surface [6],

$$ds^2 = \rho^2 d\theta^2 + \frac{\left(R_c^2 + a^2\right) - a^2\Delta\sin^2\theta}{\rho^2}\sin^2\theta\, d\varphi^2 \tag{51}$$

where

$$\begin{gathered}\rho^2 = R_c^2 + a^2\cos^2\theta, \quad \Delta = R_c^2 - \alpha R_c + a^2 + q^2, \quad R_c = \left(|r - r_o|^n + \zeta^n\right)^{\frac{1}{n}}, \quad r, r_o \in \Re, \quad n \in \Re^+ \\ \zeta = \frac{\alpha + \sqrt{\alpha^2 - 4q^2 - 4a^2\cos^2\theta}}{2}, \quad a^2 + q^2 < \frac{\alpha^2}{4}\end{gathered} \tag{52}$$

The Gaussian curvature K of this surface is given by [4],

$$K = \frac{1}{2hf}\frac{\partial\beta}{\partial\theta}\frac{\partial h}{\partial\theta} - \frac{a^2\cos^2\theta}{h^2} - \frac{1}{2f}\frac{\partial^2\beta}{\partial\theta^2} + \frac{1}{h} + \frac{a^2\sin\theta\cos\theta}{2hf}\frac{\partial\beta}{\partial\theta} + \frac{h}{4f^2}\left(\frac{\partial\beta}{\partial\theta}\right)^2 + \frac{2a^2(f - \Delta h)\cos^2\theta}{h^2 f} \tag{53}$$

where

$$f = \left(R_c^2 + a^2\right)^2 - a^2 \Delta \sin^2\theta, \quad h = R_c^2 + a^2 \cos^2\theta, \quad \beta = \frac{f}{h}$$

$$\Delta = R_c^2 - \alpha R_c + a^2 + q^2, \quad R_c = \left(|r - r_o|^n + \zeta^n\right)^{\frac{1}{n}}, \quad r, r_o \in \Re, \quad n \in \Re^+ \tag{54}$$

$$\zeta = \frac{\alpha + \sqrt{\alpha^2 - 4q^2 - 4a^2\cos^2\theta}}{2}, \quad a^2 + q^2 < \frac{\alpha^2}{4}$$

It is clearly evident from this that the Gaussian curvature is not a positive constant and so the surface is not a spherical surface. Thus, the Kerr-Newman metric is not spherically symmetric. Furthermore, by virtue of this result, the quantity R_c is neither the radius nor even a distance because it is defined by the intrinsic geometry of the surface. Since the intrinsic geometry of a surface is independent of any embedding space the quantity R_c retains its identity when the surface is embedded in Kerr-Newman spacetime. If $r_o = \zeta$ and $n = 1$, then $R_c = r$. Hence, r is not the radius of anything nor even a distance in Kerr-Newman spacetime. This result is independent of the area of the surface or the length of any curve in the surface.

The Gaussian curvature for the Kerr-Newman surface is dependent on θ, because it is axially-symmetric. When $\theta = 0$ and $\theta = \pi$, it becomes [6],

$$K = \frac{R_c^2}{(R_c^2 + a^2)^2} - \frac{a^2\left(\alpha R_c - q^2\right)}{(R_c^2 + a^2)^3} \tag{55}$$

Since $R_c\,(r_o) = \zeta\ \forall r_o\ \forall n$, for $\theta = 0$ and $\theta = \pi$ the Guassian curvature becomes [4]

$$K_{r_o} = \frac{\zeta^2}{(\zeta^2 + a^2)^2} - \frac{a^2\left(\alpha\zeta - q^2\right)}{(\zeta^2 + a^2)^3} \tag{56}$$

Similarly, when $\theta = \pi/2$, the Gaussian curvature becomes [4],

$$K = \frac{1}{R_c^2} + \frac{a^2\left(R_c^2 + a^2\right)\left(\alpha R_c - q^2\right)}{R_c^4\left[R_c^2\left(R_c^2 + a^2\right) + a^2\left(\alpha R_c - q^2\right)\right]} \tag{57}$$

Since $R_c\,(r_o) = \zeta\ \forall r_o\ \forall n$, for $\theta = \pi/2$ the Guassian curvature becomes [4],

$$K_{r_o} = \frac{1}{\zeta^2} + \frac{a^2\left(\zeta^2 + a^2\right)\left(\alpha\zeta - q^2\right)}{\zeta^4\left[\zeta^2\left(\zeta^2 + a^2\right) + a^2\left(\alpha\zeta - q^2\right)\right]} \tag{58}$$

If $a = 0$ then the Gaussian curvature is independent of θ and reduces to the spherically-symmeytric Reissner-Nordström form [4],

$$K = \frac{1}{R_c^2} \tag{59}$$

and hence, when $r = r_o$ [4],

$$K_{r_o} = \frac{1}{\zeta^2} = \frac{1}{\left[\frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - q^2}\right]^2} \tag{60}$$

where ζ is reduced accordingly. This is an invariant for the Reissner-Nordström form.

If both $a = 0$ and $q = 0$ then the Gaussian curvature reduces to the spherically-symmetric Schwarzschild form [6],

$$K = \frac{1}{R_c^2} \tag{61}$$

so that when $r = r_o$,

$$K_{r_o} = \frac{1}{\alpha^2} \tag{62}$$

which is an invariant for the Schwarzschild form.

The minimum value for Δ is,

$$\Delta_{min} = a^2 \sin^2\theta \tag{63}$$

which occurs when $r = r_o$, irrespective of the values of r_o and n. $\Delta_{min} = 0$ only when $\theta = 0$ and when $\theta = \pi$, in which cases the metric is undefined.

Similarly, the minimum value of R_c^2 is,

$$R_c^2 = \zeta^2 + a^2 \cos^2\theta \tag{64}$$

which occurs when $r = r_o$, irrespective of the values of r_o and n. Since ζ^2 is always greater than zero, R_c^2 can never be less than or equal to zero.

Note that if $a = 0$ and $q = 0$, the Gaussian curvature for the surface embedded in Kerr-Newman spacetime reduces to that for the surface in the Schwarzschild metric ground-form [6],

$$K = \frac{1}{R_c^2} \tag{65}$$

Because $R_c\,(r_o) = \alpha \; \forall r_o \; \forall n$,

$$K_{r_o} = \frac{1}{R_c^2\,(r_o)} = \frac{1}{\alpha^2} \tag{66}$$

If, further, $r_o = \alpha$ and $n = 1$, then $R_c = r$ and,

$$K = \frac{1}{r^2} \tag{67}$$

Hence, r in Hilbert's solution is not the radius of anything, or even a distance therein. Once again, this result is independent of the area of the surface or the length of any curve in the surface. Indeed, the length of a curve in the surface and the area of the surface are determined by the metric and r. The length L of a closed geodesic (a closed parametric curve where $r = const. \neq 0$, $\theta = \pi/2$) in the surface embedded in Hilbert's metric space is given by,

$$L = \int_0^{2\pi} r\, d\varphi = 2\pi r \tag{68}$$

Applying the relation for the area A of a surface, the area of the surface embedded in Hilbert's spacetime is,

$$A = r^2 \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi = 4\pi r^2 \tag{69}$$

Since this is a surface, r is not the radius of anything, nor is it even a distance in the surface. The geometric identity of r is not lost when the surface is embedded in any other space because the Gaussian curvature of a surface is intrinsic. It is now clear why the cosmologist notions of 'areal radius' ($r = \sqrt{A/4\pi}$) and 'reduced circumference' ($r = L/2\pi$) are vacuous. Neither the length of any curve in a surface nor the area of the surface or part thereof determines the geometric identity of r in Hilbert's metric.

The impossibility of a black hole is reaffirmed by Riemannian curvature. Riemannian curvature is a generalisation of Gaussian curvature to dimensions greater than two. The Riemannian curvature K_S at any point in a metric space of dimensions $n > 2$ depends upon the Riemann-Christoffel curvature tensor of the first kind, R_{ijkl}, the components of the metric tensor g_{ik}, and two arbitrary n-dimensional linearly independent contravariant direction vectors U^i and V^i, as follows:

$$K_S = \frac{R_{ijkl} U^i V^j U^k V^l}{G_{pqrs} U^p V^q U^r V^s}, \quad G_{pqrs} = g_{pr} g_{qs} - g_{ps} g_{qr} \tag{70}$$

Definition 1: *If the Riemannian curvature at any point is independent of direction vectors at that point then the point is called an isotropic point.*

The Riemannian curvature K_S for Schwarzschild spacetime is given by [4],

$$K_S = \frac{2\alpha\,(R_c - \alpha)\, W_{0101} - \alpha R_c\,(R_c - \alpha)^2\, W_{0202} - \alpha R_c\,(R_c - \alpha)^2\, W_{0303} \sin^2\theta + \alpha R_c^3 W_{1212} + \alpha R_c^3 W_{1313} \sin^2\theta - 2\alpha R_c^4\,(R_c - \alpha)\, W_{2323} \sin^2\theta}{-2R_c^3\,(R_c - \alpha)\, W_{0101} - 2R_c^4\,(R_c - \alpha)^2\, W_{0202} - 2R_c^4\,(R_c - \alpha)^2\, W_{0303} \sin^2\theta + 2R_c^6 W_{1212} + 2R_c^6 W_{1313} \sin^2\theta + 2R_c^6\,(R_c - \alpha)\, W_{2323} \sin^2\theta}$$

$$\begin{gathered} W_{ijki} = \begin{vmatrix} U^i & U^j \\ V^i & V^j \end{vmatrix} \begin{vmatrix} U^k & U^l \\ V^k & V^l \end{vmatrix}, \qquad R_c = \left(|r - r_o|^n + \alpha^n\right)^{\frac{1}{n}}, \quad r, r_o \in \Re, \quad n \in \Re^+, \\ r = \sqrt{x_o^2 + y_o^2 + z_o^2} + \sqrt{(x - x_o)^2 + (y - y_o)^2 + (z - z_o)^2} \end{gathered} \tag{71}$$

Since $R_c(r_o) = \alpha$ irrespective of the values of r_o and n, at $r = r_o$ the Riemannian curvature is,

$$K_S(r_o) = \frac{1}{2\alpha^2} = \frac{K_{r_o}}{2}$$

which is entirely independent of the direction vectors U^i and V^j, and of θ. Thus, $r = r_o$ produces an isotropic point (the only isotropic point), which reaffirms that Schwarzschild spacetime cannot be extended to produce a black hole. Note that $K_S(r_o) = K_{r_o}/2$, i.e. at $r = r_o$ the Riemannian curvature invariant of Schwarzschild 4-dimensional spacetime is half the Gaussian curvature invariant of the embedded spherical surface.

Similarly, the Riemannian curvature of Schwarzschild spacetime in isotropic coordinates is [6],

$$K_S = \frac{A+B}{C+D}$$

$$\begin{aligned}
A &= \frac{16\alpha(4R_c-\alpha)^2}{R_c(4R_c+\alpha)^4}W_{0101} - \frac{8\alpha R_c(4R_c-\alpha)^2}{(4R_c+\alpha)^4}W_{0202} - \frac{8\alpha R_c(4R_c-\alpha)^2\sin^2\theta}{(4R_c+\alpha)^4}W_{0303} \\
B &= \frac{\alpha(8R_c-\alpha)(4R_c+\alpha)^2}{2\cdot 4^3R_c^4}W_{1212} + \frac{\alpha(4R_c+\alpha)^2\sin^2\theta}{2\cdot 4^2R_c^3}W_{1313} - \frac{\alpha(4R_c+\alpha)^2\sin^2\theta}{4^2R_c}W_{2323} \\
C &= -\frac{\alpha(4R_c-\alpha)^2(4R_c+\alpha)^2}{2\cdot 4^4R_c^4}W_{0101} - \frac{\alpha(4R_c-\alpha)^2(4R_c+\alpha)^2}{2\cdot 4^4R_c^2}W_{0202} - \frac{\alpha(4R_c-\alpha)^2(4R_c+\alpha)^2\sin^2\theta}{2\cdot 4^4R_c^2}W_{0303} \\
D &= \frac{(4R_c+\alpha)^8}{4^8R_c^6}W_{1212} + \frac{(4R_c+\alpha)^8\sin^2\theta}{4^8R_c^6}W_{1313} + \frac{(4R_c+\alpha)^8\sin^2\theta}{4^8R_c^4}W_{2323} \\
W_{ijkl} &= \begin{vmatrix} U^i & U^j \\ V^i & V^j \end{vmatrix}\begin{vmatrix} U^k & U^l \\ V^k & V^l \end{vmatrix}, \qquad R_c = \left[|r-r_o|^n + \left(\frac{\alpha}{4}\right)^n\right]^{\frac{1}{n}}, \qquad r, r_o \in \Re,\ \ n \in \Re^+, \\
r &= \sqrt{x_o^2+y_o^2+z_o^2} + \sqrt{(x-x_o)^2+(y-y_o)^2+(z-z_o)^2}
\end{aligned} \tag{72}$$

This isotropic Riemannian curvature depends upon θ.

When $R_c(r_o) = \alpha/4\ \forall r_o\ \forall n$, so the Riemannian curvature becomes,

$$K_S(r_o) = \frac{8\left(W_{1212} + W_{1313}\sin^2\theta\right) - \alpha^2 W_{2323}\sin^2\theta}{16\alpha^2\left(W_{1212} + W_{13123}\sin^2\theta\right) + \alpha^2 W_{2323}\sin^2\theta} \tag{73}$$

Note that this differs from that for the ordinary Schwarzschild equivalence class only by the terms in W_{2323} (i.e. if not for the W_{2323} terms the Riemannian curvature $K_S(r_o)$ would be $1/2\alpha^2$ as for the ordinary Schwarzschild form). For $\theta = 0$ and $\theta = \pi$ it reduces to the Riemannian curvature invariant for the Schwarzschild form:

$$K_S(r_o) = \frac{1}{2\alpha^2} \tag{74}$$

Hence, for $\theta = 0$ and $\theta = \pi$, r_o is an isotropic point (the only isotropic point). The W_{2323} terms appear due to the conformal mapping of ordinary Schwarzschild equivalence class into isotropic Schwarzschild equivalence class [6].

When $\theta = \pi/2$ and $r = r_o$ the Riemannian curvature is,

$$K_S(r_o) = \frac{8(W_{1212} + W_{1313}) - \alpha^2 W_{2323}}{16\alpha^2(W_{1212} + W_{13123}) + \alpha^2 W_{2323}} \tag{75}$$

The Riemannian curvature for the Reissner-Nordström equivalence class is [4],

$$K_S = \frac{A+B+C}{D+E+F}$$

$$
\begin{aligned}
A &= 2\left(R_c^2-\alpha R_c+q^2\right)\left(\alpha R_c-3q^2\right)W_{0101}-\left(R_c^2-\alpha R_c+q^2\right)^2\left(\alpha R_c-2q^2\right)W_{0202}\\
B &= -\left(R_c^2-\alpha R_c+q^2\right)^2\left(\alpha R_c-2q^2\right)\sin^2\theta\, W_{0303}+R_c^4\left(\alpha R_c-2q^2\right)W_{1212}\\
C &= R_c^4\left(\alpha R_c-2q^2\right)\sin^2\theta\, W_{1313}-2R_c^4\left(\alpha R_c-2q^2\right)\left(R_c^2-\alpha R_c+q^2\right)\sin^2\theta\, W_{2323}\\
D &= -2R_c^4\left(R_c^2-\alpha R_c+q^2\right)W_{0101}-2R_c^4\left(R_c^2-\alpha R_c+q^2\right)^2 W_{0202}\\
E &= -2R_c^4\left(R_c^2-\alpha R_c+q^2\right)^2\sin^2\theta\, W_{0303}+2R_c^8 W_{1212}\\
F &= 2R_c^8\sin^2\theta W_{1313}+2R_c^8\left(R_c^2-\alpha R_c+q^2\right)\sin^2\theta\, W_{2323}\\
W_{ijkl} &= \begin{vmatrix} U^i & U^j\\ V^i & V^j\end{vmatrix}\begin{vmatrix} U^k & U^l\\ V^k & V^l\end{vmatrix},\qquad R_c=\left(|r-r_o|^n+\zeta^n\right)^{\frac{1}{n}},\qquad \zeta=\frac{\alpha+\sqrt{\alpha^2-4q^2}}{2},\\
q^2 &< \frac{\alpha^2}{4},\qquad r,r_o\in\Re,\qquad n\in\Re^+,\qquad r=\sqrt{x_o^2+y_o^2+z_o^2}+\sqrt{(x-x_o)^2+(y-y_o)^2+(z-z_o)^2}
\end{aligned}
\tag{76}
$$

At $r=r_o$ this becomes,

$$
K_S\left(r_o\right)=\frac{\alpha\zeta-2q^2}{2\zeta^4}=\frac{4\left(\alpha^2+\alpha\sqrt{\alpha^2-4q^2}-4q^2\right)}{\left(\alpha+\sqrt{\alpha^2-4q^2}\right)^4}
\tag{77}
$$

which is independent of the direction vectors U^i and V^j. Therefore r_o is an isotropic point (the only isotropic point). This reaffirms that the Reissner-Nordström equivalence class cannot be extended to produce a black hole.

The Riemannian curvature for Reissner-Nordström equivalence class in isotropic coordinates is,

$$
K_S=\frac{R_{0101}W_{0101}+R_{0202}\left(W_{0202}+W_{0303}\sin^2\theta\right)+R_{1212}\left(W_{1212}+W_{1313}\sin^2\theta\right)+R_{2323}W_{2323}}{G_{0101}W_{0101}+G_{0202}\left(W_{0202}+W_{0303}\sin^2\theta\right)+G_{1212}\left(W_{1212}+W_{1313}\sin^2\theta\right)+G_{2323}W_{2323}}
$$

$$
\begin{aligned}
R_{0101} =& \frac{8\left(16R_c^2-\alpha^2+4q^2\right)\left(32q^2-32\alpha R_c-8\alpha^2\right)+4^4R_cY}{\left(4R_c+\alpha+2q\right)^3\left(4R_c-\alpha+2q\right)^3}+\frac{32\left(16R_c^2-\alpha^2+4q^2\right)Y}{\left(4R_c+\alpha+2q\right)^3\left(4R_c-\alpha+2q\right)^4}+\\
&+\frac{32\left(16R_c^2-\alpha^2+4q^2\right)Y}{\left(4R_c+\alpha+2q\right)^4\left(4R_c-\alpha+2q\right)^3}+\frac{16\left(16R_c^2-\alpha^2+4q^2\right)\left(4q^2-4\alpha R_c-\alpha^2\right)Y}{R_c\left(4R_c+\alpha+2q\right)^4\left(4R_c-\alpha+2q\right)^4}-\\
&-\frac{64Y^2}{\left(4R_c+\alpha+2q\right)^4\left(4R_c-\alpha+2q\right)^4}
\end{aligned}
$$

$$
\begin{aligned}
&Y = \left\{4R_c\left[(4R_c+\alpha)^2 - 4q^2\right] - (4R_c+\alpha)\left(16R_c^2 - \alpha^2 + 4q^2\right)\right\} \\
&R_{0202} = -\frac{8R_c\left(16R_c^2-\alpha^2+4q^2\right)^2 Y}{(4R_c+\alpha+2q)^4\,(4R_c-\alpha+2q)^4} \qquad R_{0303} = R_{0202}\sin^2\theta \\
&R_{1212} = \frac{Y}{32R_c^3} \qquad R_{1313} = R_{1212}\sin^2\theta \qquad R_{2323} = -\frac{\left(Y+16q^2R_c\right)}{4^2R_c}\sin^2\theta \\
&G_{0101} = -\frac{\left(16R_c^2-\alpha^2+4q^2\right)^2}{4^4R_c^4} \qquad G_{0202} = -\frac{\left(16R_c^2-\alpha^2+4q^2\right)^2}{4^4R_c^2} \qquad G_{0303} = G_{0202}\sin^2\theta \\
&G_{1212} = \frac{(4R_c+\alpha+2q)^4\,(4R_c-\alpha+2q)^4}{4^8R_c^6} \qquad G_{1313} = G_{1212}\sin^2\theta \\
&G_{2323} = \frac{(4R_c+\alpha+2q)^4\,(4R_c-\alpha+2q)^4\sin^2\theta}{4^8R_c^4} \\
&W_{ijkl} = \begin{vmatrix} U^i & U^j \\ V^i & V^j \end{vmatrix}\begin{vmatrix} U^k & U^l \\ V^k & V^l \end{vmatrix}, \qquad R_c = \left(|r-r_o|^n + \zeta^n\right)^{\frac{1}{n}}, \qquad \zeta = \frac{\sqrt{\alpha^2-4q^2}}{4}, \\
&q^2 < \frac{\alpha^2}{4}, \qquad r, r_o \in \Re, \qquad n \in \Re^+, \qquad r = \sqrt{x_o^2+y_o^2+z_o^2} + \sqrt{(x-x_o)^2+(y-y_o)^2+(z-z_o)^2}
\end{aligned}
\tag{78}
$$

which depends upon θ.

Since $R_c\,(r_o) = \zeta\;\forall r_o\;\forall n$ it then reduces to,

$$
K_S\,(r_o) = \frac{\frac{4\left(\alpha^2-4q^2+\alpha\sqrt{\alpha^2-4q^2}\right)}{\left(\alpha^2-4q^2\right)}\left(W_{1212}+W_{1313}\sin^2\theta\right) - \frac{\left(\sqrt{\alpha^2-4q^2}+\alpha\right)^2}{4}W_{2323}\sin^2\theta}{\frac{\left[\left(\sqrt{\alpha^2-4q^2}+\alpha\right)^2-4q^2\right]^4}{4^2\left(\alpha^2-4q^2\right)^3}\left(W_{1212}+W_{1313}\sin^2\theta\right) + \frac{\left[\left(\sqrt{\alpha^2-4q^2}+\alpha\right)^2-4q^2\right]^4}{4^4\left(\alpha^2-4q^2\right)^2}W_{2323}\sin^2\theta}
\tag{79}
$$

If $q = 0$ this reduces to the Riemannian curvature invariant for the isotropic Schwarzschild equivalence class.

When $\theta = 0$ and $\theta = \pi$, the Riemannian curvature is,

$$
K_S = \frac{R_{0101}W_{0101} + R_{0202}W_{0202} + R_{1212}W_{1212}}{G_{0101}W_{0101} + G_{0202}W_{0202} + G_{1212}W_{1212}}
\tag{80}
$$

and hence if also $r = r_o$ this reduces further to the Riemannian curvature invariant,

$$
K_S = \frac{R_{1212}}{G_{1212}} = \frac{4\left(\alpha^2-4q^2\right)^2}{\left(\alpha^2-4q^2+\alpha\sqrt{\alpha^2-4q^2}\right)^3} = \frac{4\left(\alpha^2+\alpha\sqrt{\alpha^2-4q^2}-4q^2\right)}{\left(\alpha+\sqrt{\alpha^2-4q^2}\right)^4}
\tag{81}
$$

which is the same as the isotropic Riemannian curvature invariant for the ordinary Reissner-Nordström equivalence class; and r_o is an isotropic point (the only isotropic point). This reaffirms that Reissner-Nordström spacetime cannot be extended to produce a black hole.

Then if $q = 0$ Eq.(81) reduces finally to the Riemannian curvature invariant for the isotropic Schwarzschild equivalence class,

$$
K_S = \frac{1}{2\alpha^2}
\tag{82}
$$

When $\theta = \pi/2$, the Riemannian curvature for the Reissner-Nordström equivalence class accordingly becomes,

$$
K_S = \frac{R_{0101}W_{0101} + R_{0202}\left(W_{0202}+W_{0303}\right) + R_{1212}\left(W_{1212}+W_{1313}\right) + R_{2323}W_{2323}}{G_{0101}W_{0101} + G_{0202}\left(W_{0202}+W_{0303}\right) + G_{1212}\left(W_{1212}+W_{1313}\right) + G_{2323}W_{2323}}
\tag{83}
$$

The Kretschmann scalar f is also called the Riemann tensor scalar curvature invariant. It is defined by $f = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$. Cosmologists incorrectly assert that their 'physical singularity' must occur where the Kretschmann scalar is 'infinite'. For the Schwarzschild equivalence class it is actually given by [6],

$$\begin{gathered} f = \frac{12\alpha^2}{R_c^6} = \frac{12\alpha^2}{\left(|r - r_o|^n + \alpha^n\right)^{\frac{6}{n}}} \\ R_c = \left(|r - r_o|^n + \alpha^n\right)^{\frac{1}{n}} \end{gathered} \tag{84}$$

Hence, at $r = r_o$,

$$f(r_o) = \frac{12}{\alpha^4} \tag{85}$$

In the case of the Reissner-Nordström equivalence class it is given by [6],

$$\begin{gathered} f = \frac{8\left[6\left(\frac{\alpha R_c}{2} - q^2\right)^2 + q^4\right]}{R_c^8} \\ R_c = \left(|r - r_o|^n + \zeta^n\right)^{\frac{1}{n}}, \qquad \zeta = \frac{\alpha + \sqrt{\alpha^2 - 4q^2}}{2} \end{gathered} \tag{86}$$

Hence, at $r = r_o$,

$$f(r_o) = \frac{8\left[6\left(\frac{\alpha\zeta}{2} - q^2\right)^2 + q^4\right]}{\zeta^8} = \frac{8\left\{6\left[\frac{\alpha\left(\frac{\alpha + \sqrt{\alpha^2 - 4q^2}}{2}\right)}{2} - q^2\right]^2 + q^4\right\}}{\left(\frac{\alpha + \sqrt{\alpha^2 - 4q^2}}{2}\right)^8} \tag{87}$$

For the Kerr-Newman equivalence class the Kretschmann scalar is given by [6],

$$\begin{gathered} f = \frac{8}{\left(R_c^2 + a^2\cos^2\theta\right)^6}\left[\begin{array}{c} \frac{3\alpha^2}{2}\left(R_c^6 - 15a^2R_c^4\cos^2\theta + 15a^4R_c^2\cos^4\theta - a^6\cos^6\theta\right) - \\ -6\alpha q^2 R_c\left(R_c^4 - 10a^2R_c^2\cos^2\theta + 5a^4\cos^4\theta\right) + \\ +q^4\left(6R_c^4 - 34a^2R_c^2\cos^2\theta + 6a^4\cos^4\theta\right) \end{array}\right] \\ R_c = \left(|r - r_o|^n + \zeta^n\right)^{\frac{1}{n}}, \qquad r, r_o \in \Re, \quad n \in \Re^+ \\ \zeta = \frac{\alpha + \sqrt{\alpha^2 - 4\left(q^2 + a^2\cos^2\theta\right)}}{2}, \qquad a^2 + q^2 < \frac{\alpha^2}{4}, \\ r = \sqrt{x_o^2 + y_o^2 + z_o^2} + \sqrt{(x - x_o)^2 + (y - y_o)^2 + (z - z_o)^2} \end{gathered} \tag{88}$$

Note that here f depends upon θ.

Since $R_c(r_o) = \zeta\ \forall r_o\ \forall n$, when $r = r_o$ the Kretschmann scalar for the Kerr-Newman equivalence class becomes,

$$f(r_o) = \frac{8}{\left(\zeta^2 + a^2\cos^2\theta\right)^6}\left[\begin{array}{c} \frac{3\alpha^2}{2}\left(\zeta^6 - 15a^2\zeta^4\cos^2\theta + 15a^4\zeta^2\cos^4\theta - a^6\cos^6\theta\right) - \\ -6\alpha q^2\zeta\left(\zeta^4 - 10a^2\zeta^2\cos^2\theta + 5a^4\cos^4\theta\right) + \\ +q^4\left(6\zeta^4 - 34a^2\zeta^2\cos^2\theta + 6a^4\cos^4\theta\right) \end{array}\right] \tag{89}$$

The Kretschmann scalar is finite when $r = r_o$, irrespective of the values of r_o and n. When $\theta = 0$ and when $\theta = \pi$,

$$f(r_o) = \frac{8}{\left(\zeta^2 + a^2\right)^6}\left[\begin{array}{c} \frac{3\alpha^2}{2}\left(\zeta^6 - 15a^2\zeta^4 + 15a^4\zeta^2 - a^6\right) - \\ -6\alpha q^2\zeta\left(\zeta^4 - 10a^2\zeta^2 + 5a^4\right) + \\ +q^4\left(6\zeta^4 - 34a^2\zeta^2 + 6a^4\right) \end{array}\right] \tag{90}$$

When $\theta = \frac{\pi}{2}$, $f(r_o)$ reduces to,

$$f(r_o) = \frac{8}{\zeta^8}\left[\frac{3\alpha^2\zeta^2}{2} - 6\alpha q^2\zeta + 6q^4\right] \tag{91}$$

Note that this does not contain the 'angular momentum' term a and that the result is precisely that for the Reissner-Nordström equivalence class (where $a = 0$).

When $q = 0$ the Kretschmann scalar for the Kerr-Newman equivalence class reduces to that for the Kerr equivalence class,

$$f(r_o) = \frac{12\alpha^2}{\left(\zeta^2 + a^2\cos^2\theta\right)^6}\left(\zeta^6 - 15a^2\zeta^4\cos^2\theta + 15a^4\zeta^2\cos^4\theta - a^6\cos^6\theta\right) \tag{92}$$

This too depends upon the value of θ. When $\theta = 0$ and when $\theta = \pi$, $f(r_o)$ becomes,

$$f(r_o) = \frac{12\alpha^2}{\left(\zeta^2 + a^2\right)^6}\left(\zeta^6 - 15a^2\zeta^4 + 15a^4\zeta^2 - a^6\right) \tag{93}$$

When $\theta = \frac{\pi}{2}$, $f(r_o)$ reduces to,

$$f(r_o) = \frac{12}{\alpha^4} \tag{94}$$

which is precisely the Kretschmann scalar for the Schwarzschild form, where both $q = 0$ and $a = 0$ in the Kretschmann scalar for Kerr-Newman spacetime.

In the case of the Schwarzschild equivalence class in isotropic coordinates, the Kretschmann scalar is given by [6],

$$\begin{gathered} f = 3\cdot 4^{13}\frac{\alpha^2 R_c^6}{\left(4R_c + \alpha\right)^{12}} \\ R_c = \left[\left|r - r_o\right|^n + \left(\frac{\alpha}{4}\right)^n\right]^{\frac{1}{n}}, \qquad r, r_o \in \Re, \qquad n \in \Re^+ \end{gathered} \tag{95}$$

Since $R_c(r_o) = \frac{\alpha}{4}\ \forall r_o\ \forall n$,

$$f(r_o) = \frac{12}{\alpha^4} \tag{96}$$

which is the very same finite value as that for the ordinary Schwarzschild equivalence class.

The Kretschmann scalar for the isotropic Reissner-Nordström equivalence class is [6],

$$\begin{gathered} f = \frac{4^{13}R_c^6\left\{\left[\alpha\left(4R_c + \alpha\right)^2 - 4q^2\left(8R_c + \alpha\right)\right]^2 + \left[\alpha\left(4R_c + \alpha\right)^2 - 4q^2\left(12R_c + \alpha\right)\right]^2 + \left[\alpha\left(4R_c + \alpha\right)^2 - 4q^2\left(4R_c + \alpha\right)\right]^2\right\}}{\left(4R_c + \alpha + 2q\right)^8\left(4R_c + \alpha - 2q\right)^8} \\ R_c = \left(\left|r - r_o\right|^n + \zeta^n\right)^{\frac{1}{n}}, \qquad \zeta = \frac{\sqrt{\alpha^2 - 4q^2}}{4}, \qquad 4q^2 < \alpha^2 \qquad r, r_o \in \Re, \qquad n \in \Re^+ \end{gathered} \tag{97}$$

Since $R_c(r_o) = \zeta\ \forall r_o\ \forall n$, the Kretschmann scalar reduces to the invariant,

$$f = \frac{4^{13}\zeta^6\left\{\left[\alpha\left(4\zeta + \alpha\right)^2 - 4q^2\left(8\zeta + \alpha\right)\right]^2 + \left[\alpha\left(4\zeta + \alpha\right)^2 - 4q^2\left(12\zeta + \alpha\right)\right]^2 + \left[\alpha\left(4\zeta + \alpha\right)^2 - 4q^2\left(4\zeta + \alpha\right)\right]^2\right\}}{\left(4\zeta + \alpha + 2q\right)^8\left(4\zeta + \alpha - 2q\right)^8} \tag{98}$$

The Kretschmann scalar is always finite. For it to be undefined by a division by zero, as contended by cosmologists, it requires that the positive real power of the absolute value of a real number must take on values less than zero, which is a violation of the rules of pure mathematics, as the case $r_o = 0, n = 2$ amplifies.

Geodesic incompleteness

A geodesic is a line in some space. In Euclidean space the geodesics are simply straight lines. This is because the Riemannian curvature of Euclidean space is zero. If the Riemannian curvature is not zero throughout the entire space, the space is not Euclidean and the geodesics are curved lines rather than straight lines. If a geodesic terminates at some point in the space it is said to be incomplete, and the manifold or space in which it lies is also said to be geodesically incomplete. If no geodesic in some manifold is incomplete then the manifold is said to be geodesically complete. More specifically, "*A semi-Riemannian manifold M for which every maximal geodesic is defined on the entire real line is said to be geodesically complete - or merely complete. Note that if even a single point p is removed from a complete manifold M then M − p is no longer complete, since geodesics that formerly went through p are now obliged to stop.*" O'Neill [52]

Consider now Hilbert's solution. In 1931, Hagihara [63] proved that all geodesics therein that do not run into the boundary at $r = 2Gm/c^2$ are complete. Hence this is also the case at $r = r_o$ for all the solutions generated by the Schwarzschild infinite equivalence class. This is also the case at $r = r_o$ for the isotropic forms. The geodesics terminate at the origin; the point from which the radius emanates; $R_p = 0$. In other words, Hagihara effectively proved that all geodesics that do not run into the origin $R_p = 0$ are complete. This once again attests that Droste's solution cannot be 'extended' to produce a black hole.

The acceleration invariant

Doughty [64] obtained the following expression for the acceleration β of a point along a radial geodesic for the static spherically symmetric line-elements,

$$\beta = \frac{\sqrt{-g_{11}}\left(-g^{11}\right)\left|\frac{\partial g_{oo}}{\partial r}\right|}{2g_{oo}} \tag{99}$$

Since the Hilbert and Nordström metrics utilised by cosmologists are particular cases of their respective infinite equivalence classes, the foregoing expression becomes, in general,

$$\beta = \frac{\sqrt{-g_{11}}\left(-g^{11}\right)\left|\frac{\partial g_{oo}}{\partial R_c}\right|}{2g_{oo}}$$

$$R_c = \left(|r - r_o|^n + \zeta^n\right)^{\frac{1}{n}}, \qquad r, r_o \in \Re, \qquad n \in \Re^+ \tag{100}$$

$$\zeta = \frac{\alpha + \sqrt{\alpha^2 - 4q^2}}{2}, \qquad q^2 < \frac{\alpha^2}{4}$$

The acceleration is therefore given by,

$$\beta = \frac{\alpha R_c - 2q^2}{2R_c^2\sqrt{R_c^2 - \alpha R_c + q^2}} \tag{101}$$

In all cases, whether or not $q = 0$, $r \to r_o \Rightarrow \beta \to \infty$, which constitutes an invariant condition, and therefore reaffirms that the Schwarzschild and Reissner-Nordström forms cannot be extended to produce black holes.

The expression for acceleration appears at first glance to be a first-order intrinsic differential invariant since it is superficially composed of only the components of the metric tensor and their first derivatives. This is however, not so, because the expression applies only to the radial direction, i.e. to the motion of a point along a radial geodesic. In other words, it involves a direction vector. Consequently, although the acceleration expression is a first-order differential invariant, it is not intrinsic. First-order differential invariants exist, but first-order intrinsic differential invariants do not exist [73,74]. That the acceleration expression involves a direction vector is amplified by the Killing vector. Let X_a be a first-order tensor (i.e. a covariant vector). Then for it to be a Killing vector it must satisfy Killing's equations,

$$X_{a;b} + X_{b;a} = 0 \tag{102}$$

where $X_{a;b}$ denotes the covariant derivative of X_a. The condition for hypersurface orthogonality is [32,34],

$$X_{[a}X_{b;c]} = 0 \tag{103}$$

The two foregoing conditions determine a unique time-like Killing vector that fixes the direction of time [32]. By means of this Killing vector the four-velocity v_i is,

$$v^a = \frac{X^a}{\sqrt{X_a X^a}} \tag{104}$$

The absolute derivative of the four-velocity along its own direction gives the four-acceleration β^a,

$$\beta^a = \frac{Dv^a}{du} \tag{105}$$

The norm of the four-acceleration is,

$$\beta = \sqrt{-\beta_a \beta^a} \tag{106}$$

Applying this to the Reissner-Nordström equivalence class yields,

$$\begin{gathered}\beta = \frac{\alpha R_c - 2q^2}{2R_c^2 \sqrt{R_c^2 - \alpha R_c + q^2}} \\ R_c = \left(|r - r_o|^n + \zeta^n\right)^{\frac{1}{n}}, \qquad r, r_o \in \Re, \qquad n \in \Re^+ \qquad \zeta = \frac{\alpha + \sqrt{\alpha^2 - 4q^2}}{2}, \qquad q^2 < \frac{\alpha^2}{4}\end{gathered} \tag{107}$$

which is Eq.(101). Consequently, the acceleration expression is not intrinsic; it is a first-order differential invariant which is constructed with the metric tensor and an associated direction vector, as the motion of a point *along a radial geodesic* implies.

When $q = 0$ the acceleration expression reduces to,

$$\begin{gathered}\beta = \frac{\alpha}{2R_c^2 \sqrt{1 - \frac{\alpha}{R_c}}} \\ R_c = \left(|r - r_o|^n + \alpha^n\right)^{\frac{1}{n}}, \qquad r, r_o \in \Re, \qquad n \in \Re^+\end{gathered} \tag{108}$$

which can of course be calculated directly from the equations for the Schwarzschild equivalence class.

In the case of the isotropic Reissner-Nordström equivalence class the acceleration is given by [6],

$$\begin{gathered}\beta = \frac{8R_c^2\left[64R_c\left(4R_c + \alpha + 2q\right)\left(4R_c + \alpha - 2q\right) - 16\left(16R_c^2 - \alpha^2 + 4q^2\right)\left(4R_c + \alpha\right)\right]}{\left(4R_c + \alpha + 2q\right)^2\left(4R_c + \alpha - 2q\right)^2\left(16R_c^2 - \alpha^2 + 4q^2\right)} \\ R_c = \left(|r - r_o|^n + \zeta^n\right)^{\frac{1}{n}}, \qquad \zeta = \frac{\sqrt{\alpha^2 - 4q^2}}{4}, \qquad 4q^2 < \alpha^2 \qquad r, r_o \in \Re, \qquad n \in \Re^+\end{gathered} \tag{109}$$

If $q = 0$ this reduces to the acceleration for the isotropic Schwarzschild equivalence class [6].

In all cases $r \to r_o \Rightarrow \beta \to \infty$, which constitutes an invariant condition, and therefore reaffirms once again that the Schwarzschild and Reissner-Nordström forms cannot be extended to produce black holes.

The Newtonian 'black hole'

Cosmologists routinely assert, incorrectly, that the theoretical Michell-Laplace dark body, extracted from Newton's theory of gravity, is a black hole.

> "*Laplace essentially predicted the black hole ...*" Hawking and Ellis [54]

> "*On this assumption a Cambridge don, John Michell, wrote a paper in 1683 in the Philosophical Transactions of the Royal Society of London. In it, he pointed out that a star that was sufficiently massive and compact would have such a strong gravitational field that light could not escape. Any light emitted from the surface of the star would be dragged back by the star's gravitational attraction before it could get very far. Michell suggested that there might be a large number of stars like this. Although we would not be able to see them because light from them would not reach us, we could still feel their gravitational attraction. Such objects are what we now call black holes, because that is what they are - black voids in space.*" Hawking [60]

> "*Eighteenth-century speculators had discussed the characteristics of stars so dense that light would be prevented from leaving them by the strength of their gravitational attraction; and according to Einstein's General Relativity, such bizarre objects (today's 'black holes') were theoretically possible as end-products of stellar evolution, provided the stars were massive enough for their inward gravitational attraction to overwhelm the repulsive forces at work.*" Cambridge Illustrated History of Astronomy [65]

> "*Two important arrivals on the scene: the neutron star (1933) and the black hole (1695, 1939). No proper account of either can forego general relativity.*" Minser, Thorne, and Wheeler [24]

"*That such a contingency can arise was surmised already by Laplace in 1798. Laplace argued as follows. For a particle to escape from the surface of a spherical body of mass M and radius R, it must be projected with a velocity v such that $v^2/2 > GM/R$; and it cannot escape if $v^2 < 2GM/R$. On the basis of this last inequality, Laplace concluded that if $R < 2GM/c^2 = R_s$ (say) where c denotes the velocity of light, then light will not be able to escape from such a body and we will not be able to see it!*

"By a curious coincidence, the limit R_s discovered by Laplace is exactly the same that general relativity gives for the occurrence of the trapped surface around a spherical mass" Chandrasekhar [8]

But it is not "*a curious coincidence*" that General Relativity gives the same R_s "*discovered by Laplace*" because the Newtonian expression for escape speed is deliberately inserted, *post hoc*, into Hilbert's solution by Einstein and his followers in order to make a mass appear in equations that contain no material source by mathematical construction.

The theoretical Michell-Laplace dark body is not a black hole. It possesses an escape speed at its surface, but the black hole has both an escape speed and no escape speed simultaneously at its 'surface' (i.e. event horizon); masses and light can leave the Michell-Laplace dark body, but nothing can leave the black hole; it does not require irresistible gravitational collapse to form, whereas the black hole does; it has no infinitely dense 'physical singularity', but the black hole does; it has no event horizon, but the black hole does; it has 'infinite gravity' nowhere, but the black hole has infinite gravity at its 'physical singularity'; there is always a class of observers that can see the Michell-Laplace dark body [116], but there is no class of observers that can see the black hole; the Michell-Laplace dark body persists in a space which by consistent theory contains other Michell-Laplace dark bodies and other matter and they can interact with themselves and other matter, but the spacetime of all types of black holes pertains to a universe that contains, supposedly, only one mass (but actually contains no mass by mathematical construction) and so cannot interact with any other masses (in other words, the Principle of Superposition holds for the theoretical Michell-Laplace dark body but does not hold for the black hole); the space of the Michell-Laplace dark body is 3-dimensional and Euclidean, but that of the black hole is a 4-dimensional non-Euclidean (pseudo-Riemannian) spacetime; the space of the Michell-Laplace dark body is not asymptotically anything whereas the spacetime of the black hole is asymptotically flat or asymptotically curved; the Michell-Laplace dark body does not 'curve' a spacetime, but the black hole does; the gravity of the theoretical Michell-Laplace dark body is a force whereas the 'gravity' of a black hole is not a force. Hence, the theoretical Michell-Laplace dark body does not possess any of the characteristics of the black hole, other than a finite mass, and so it is not a black hole.

7 The paradox of black hole mass

Although one violation of the rules of pure mathematics is sufficient to invalidate it, the black hole violates other rules of logic. Einstein maintains that although $R_{\mu\nu} = 0$ contains no terms for material sources (since $T_{\mu\nu} = 0$), a material source is nonetheless present, in order to cause a gravitational field. The material source is rendered present linguistically by the assertion that $R_{\mu\nu} = 0$ describes the gravitational field outside a body such as a star. Indeed, concerning Hilbert's solution, Einstein writes,

$$\text{“}ds^2 = \left(1 - \frac{A}{r}\right)dt^2 - \left[\frac{dr^2}{1-\frac{A}{r}} + r^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right)\right] \tag{109a}$$

$$A = \frac{\kappa M}{4\pi}$$

M denotes the sun's mass centrally symmetrically placed about the origin of co-ordinates; the solution (109a) is valid only outside this mass, where all the $T_{\mu\nu}$ vanish." Einstein [48]

Note that Einstein has incorrectly asserted, in the standard fashion of cosmologists, that his mass M in his Eq.(109a) is "*centrally symmetrically placed about the origin of co-ordinates*".

"*In general relativity, the stress-energy or energy-momentum tensor T_{ab} acts as the source of the gravitational field. It is related to the Einstein tensor and hence to the curvature of spacetime via the Einstein equation.*" McMahon [5]

"*Again, just as the electric field, for its part, depends upon the charges and is instrumental in producing mechanical interaction between the charges, so we must assume here that the metrical field (or, in mathematical language, the tensor with components g_{ik}) is related to the material filling the world.*" Weyl [66]

On the one hand, Einstein removes all material sources by setting $T_{\mu\nu} = 0$ and on the other hand immediately reinstates the presence of a massive source with words, by alluding to a mass "*outside*" of which equations $R_{\mu\nu} = 0$ apply; since his gravitational field must be caused by matter. This contradiction is reiterated by cosmologists.

"*Einstein's equation, (6.26), should be exactly valid. Therefore it is interesting to search for exact solutions. The simplest and most important one is empty space surrounding a static star or planet. There, one has $T_{\mu\nu} = 0$.*" 't Hooft [26]

According to Einstein his equation (109a) contains a massive source, at "*the origin*", yet also according to Einstein, the universe modelled by $R_{\mu\nu} = 0$, from which (109a) is obtained, contains no material sources. The contradiction is readily amplified. That $R_{\mu\nu} = 0$ contains no material sources whatsoever is easily reaffirmed by the field equations,

$$R_{\mu\nu} = \lambda g_{\mu\nu} \tag{110}$$

The constant λ is the so-called 'cosmological constant'. The solution for Eq.(110) is de Sitter's empty universe, which is empty precisely because the energy-momentum tensor for material sources is zero, i.e. $T_{\mu\nu} = 0$. De Sitter's universe contains no matter:

"*This is not a model of relativistic cosmology because it is devoid of matter.*" d'Inverno [34]

"*the de Sitter line element corresponds to a model which must strictly be taken as completely empty.*" Tolman [67]

"*the solution for an entirely empty world.*" Eddington [17]

"*there is no matter at all!*" Weinberg [68]

Note that in $R_{\mu\nu} = 0$ and $R_{\mu\nu} = \lambda g_{\mu\nu}$, $T_{\mu\nu} = 0$. Thus, according to Einstein and the cosmologists, material sources are both present and absent by the very same mathematical constraint, which is a violation of the rules of logic. Since de Sitter's universe is devoid of material sources by virtue of $T_{\mu\nu} = 0$, the 'Schwarzschild' universe must also be devoid of material sources by the very same constraint. Thus, the universe modelled by $R_{\mu\nu} = 0$ contains no matter, whereby its solution is physically meaningless. But it is upon $R_{\mu\nu} = 0$ and its solution that the mathematical theory of black hole rests.

Not only does $R_{\mu\nu} = 0$ contain no matter, it also violates other 'physical principles' of General Relativity. According to Einstein his Principle of Equivalence and his Special Theory of Relativity must hold in his gravitational field.

"*Let now K be an inertial system. Masses which are sufficiently far from each other and from other bodies are then, with respect to K, free from acceleration. We shall also refer these masses to a system of co-ordinates K′, uniformly accelerated with respect to K. Relatively to K′ all the masses have equal and parallel accelerations; with respect to K′ they behave just as if a gravitational field were present and K′ were unaccelerated. Overlooking for the present the question as to the 'cause' of such a gravitational field, which will occupy us later, there is nothing to prevent our conceiving this gravitational field as real, that is, the conception that K′ is 'at rest' and a gravitational field is present we may consider as equivalent to the conception that only K is an 'allowable' system of co-ordinates and no gravitational field is present. The assumption of the complete physical equivalence of the systems of coordinates, K and K′, we call the 'principle of equivalence'; this principle is evidently intimately connected with the law of the equality between the inert and the gravitational mass, and signifies an extension of the principle of relativity to co-ordinate systems which are in non-uniform motion relatively to each other. In fact, through this conception we arrive at the unity of the nature of inertia and gravitation.*

"Stated more exactly, there are finite regions, where, with respect to a suitably chosen space of reference, material particles move freely without acceleration, and in which the laws of the special theory of relativity, which have been developed above, hold with remarkable accuracy." Einstein [48]

"*We may incorporate these ideas into the principle of equivalence, which is this: In a freely falling (nonrotating) laboratory occupying a small region of spacetime, the laws of physics are the laws of special relativity.*" Foster and Nightingale [16]

"*We can think of the physical realization of the local coordinate system K_o in terms of a freely floating, sufficiently small, box which is not subjected to any external forces apart from gravity, and which is falling under the influence of the latter. . . . It is evidently natural to assume that the special theory of relativity should remain valid in K_o*" Pauli [69]

"*General Relativity requires more than one free-float frame.*" Taylor and Wheeler [51]

"*Near every event in spacetime, in a sufficiently small neighborhood, in every freely falling reference frame all phenomena (including gravitational ones) are exactly as they are in the absence of external gravitational sources.*" Dictionary of Geophysics, Astrophysics and Astronomy [19]

Note that both the Principle of Equivalence and Special Relativity are defined in terms of the *a priori* presence of multiple arbitrarily large finite masses and photons. There can be no multiple arbitrarily large finite masses and photons in a spacetime that contains no matter by mathematical construction, and so neither the Principle of Equivalence nor Special Relativity can manifest therein. But $R_{\mu\nu} = 0$ is a spacetime that contains no matter by mathematical construction.

8 Localisation of gravitational energy and conservation laws

Without a theoretical framework by which the usual conservation laws for the energy and momentum of a closed system hold, as determined by a vast array of experiments, there is no means to produce gravitational waves by General Relativity. Einstein was aware of this and so devised a means for his theory to satisfy the usual conservation of energy and momentum for a closed system. However, Einstein's method of solving this problem is invalid because he violated the rules of pure mathematics. There is in fact no means by which the usual conservation laws for a closed system can be satisfied by General Relativity. Consequently the concept of gravitational waves has no valid theoretical basis in Einstein's theory.

It must first be noted that when Einstein talks of the conservation of energy and momentum he means that the sum of the energy and momentum of his gravitational field and its material sources is conserved *in toto*, in the usual way for a closed system, as experiment attests, for otherwise his theory is in conflict with experiments and therefore invalid.

"*It must be remembered that besides the energy density of the matter there must also be given an energy density of the gravitational field, so that there can be no talk of principles of conservation of energy and momentum of matter alone.*" Einstein [48]

The meaning of Einstein's 'matter' needs to be clarified.

"*We make a distinction hereafter between 'gravitational field' and 'matter' in this way, that we denote everything but the gravitational field as 'matter'. Our use of the word therefore includes not only matter in the ordinary sense, but the electromagnetic field as well.*" Einstein [29 §14]

"*In the general theory of relativity the doctrine of space and time, or kinematics, no longer figures as a fundamental independent of the rest of physics. The geometrical behaviour of bodies and the motion of clocks rather depend on gravitational fields, which in their turn are produced by matter.*" Einstein [70]

Einstein himself is not free from contamination by his followers. He states clearly that in his theory his gravitational field is not matter and that only matter as he conceives of it can produce his gravitational field. Nevertheless, cosmologists alter Einstein's theory *ad arbitrium* in order to attempt justification of their own claims about his theory. For instance, according to 't Hooft Einstein's gravitational field can "*have a mass of its own*" [6,71], in direct contradiction of Einstein's own account of his theory. Alteration of Einstein's theory to suit their purpose, pretending that their alterations are part of Einstein's theory and thereby have his seal of absolute authority, is another common method employed by his followers. Einstein's theory has the character of a chameleon, able to take on any colour required for any desired purpose. Einstein's enthusiastic followers have become "*more Einsteinich than he*" Heaviside [72]

The energy-momentum of Einstein's matter alone is contained in his energy-momentum tensor $T_{\mu\nu}$. To account for the energy-momentum of his gravitational field alone Einstein introduced his pseudotensor t^{α}_{σ}, defined by (Einstein [40 §15]),

$$\kappa t^{\alpha}_{\sigma} = \frac{1}{2}\delta^{\alpha}_{\sigma} g^{\mu\nu}\Gamma^{\lambda}_{\mu\beta}\Gamma^{\beta}_{\mu\lambda} - g^{\mu\nu}\Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\sigma} \tag{111}$$

where κ is a constant and δ^{α}_{σ} is the Kronecker-delta.

"*The quantities t^{α}_{σ} we call the 'energy components' of the gravitational field.*" Einstein [29 §15]

But t^{α}_{σ} is not a tensor. As such it is a coordinate dependent quantity, contrary to the basic coordinate independent tenet of General Relativity.

"*It is to be noted that t as is not a tensor*" Einstein [29 §15]

The justification is that t^{α}_{σ} acts 'like a tensor' under linear transformations of coordinates when subjected to certain strict conditions. Einstein then takes an ordinary divergence,

$$\frac{\partial t^{\alpha}_{\sigma}}{\partial x_{\alpha}} = 0 \tag{112}$$

and claims a conservation law for the energy and momentum of his gravitational field alone.

> "*This equation expresses the law of conservation of momentum and of energy for the gravitational field.*" Einstein [29 §15]

Einstein added his pseudotensor for his gravitational field alone to his energy-momentum tensor for matter alone to obtain the total energy-momentum equation for his gravitational field and its material sources.

$$E = \left(t^{\alpha}_{\sigma} + T^{\alpha}_{\sigma}\right) \tag{113}$$

Not being a tensor equation, Einstein cannot take a tensor divergence. He therefore takes an ordinary divergence, [29 §16],

$$\frac{\partial \left(t^{\alpha}_{\sigma} + T^{\alpha}_{\sigma}\right)}{\partial x_{\alpha}} = 0 \tag{114}$$

and claims the usual conservation laws of energy and momentum for a closed system:

> "*Thus it results from our field equations of gravitation that the laws of conservation of momentum and energy are satisfied. . . . here, instead of the energy components t^{σ}_{μ} of the gravitational field, we have to introduce the totality of the energy components of matter and gravitational field.*" Einstein [29 §16]

The mathematical error is profound, but completely unknown to cosmologists. Contract Einstein's pseudotensor by setting $\sigma = \alpha$ to yield the invariant $t = t^{\alpha}_{\alpha}$, thus,

$$\kappa t^{\alpha}_{\alpha} = \kappa t = \frac{1}{2}\delta^{\alpha}_{\alpha} g^{\mu\nu}\Gamma^{\lambda}_{\mu\beta}\Gamma^{\beta}_{\mu\lambda} - g^{\mu\nu}\Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha} \tag{115}$$

Since the $\Gamma^{\alpha}_{\beta\sigma}$ are functions only of the components of the metric tensor and their first derivatives, t is seen to be a first-order intrinsic differential invariant [4,23,24,41], i.e. it is an invariant that depends solely upon the components of the metric tensor and their first derivatives. However, the pure mathematicians proved in 1900 that first-order intrinsic differential invariants do not exist [73]. Thus, by *reductio ad absurdum*, Einstein's pseudotensor is a meaningless collection of mathematical symbols. Contrary to Einstein and the cosmologists, it cannot therefore be used to represent anything in physics or to make any calculations, including those for the energy of Einstein's gravitational waves. Nevertheless, cosmology calculates:

> "*It is not possible to obtain an expression for the energy of the gravitational field satisfying both the conditions: (i) when added to other forms of energy the total energy is conserved, and (ii) the energy within a definite (three dimensional) region at a certain time is independent of the coordinate system. Thus, in general, gravitational energy cannot be localized. The best we can do is to use the pseudotensor, which satisfies condition (i) but not condition (ii). It gives us approximate information about gravitational energy, which in some special cases can be accurate.*" Dirac [53]
>
> "*Let us consider the energy of these waves. Owing to the pseudo-tensor not being a real tensor, we do not get, in general, a clear result independent of the coordinate system. But there is one special case in which we do get a clear result; namely, when the waves are all moving in the same direction.*" Dirac [53]

Consider the following two equivalent forms of Einstein's field equations,

$$R^{\mu}_{\nu} = -\kappa\left(T^{\mu}_{\nu} - \frac{1}{2}T g^{\mu}_{\nu}\right) \tag{116}$$

$$T^{\mu}_{\nu} = -\frac{1}{\kappa}\left(R^{\mu}_{\nu} - \frac{1}{2}R g^{\mu}_{\nu}\right) \tag{117}$$

By Eq.(116), according to Einstein, if $T^{\mu}_{\nu} = 0$ then $R^{\mu}_{\nu} = 0$. But by Eq.(117), if $R^{\mu}_{\nu} = 0$ then $T^{\mu}_{\nu} = 0$. In other words, $R^{\mu}_{\nu} = 0$ and $T^{\mu}_{\nu} = 0$ must vanish identically - if either is zero then so is the other, and the field equations reduce to the identity $0 = 0$ [6,36,38,74]. Hence, if there are no material sources (i.e. $T^{\mu}_{\nu} = 0$) then there is no gravitational field, and no universe. Bearing this in mind, with Eq.(113) and Eq.(114), consideration of the conservation of energy and momentum, and tensor relations, Einstein's field equations must take the following form [6,36,38,74],

$$\frac{G^{\mu}_{\nu}}{\kappa} + T^{\mu}_{\nu} = 0 \tag{118}$$

where

$$G^{\mu}_{\nu} = R^{\mu}_{\nu} - \frac{1}{2}Rg^{\mu}_{\nu} \tag{119}$$

Comparing Eq.(118) to Eq.(113) it is clear that the G^{μ}_{ν}/κ actually constitute the energy-momentum components of Einstein's gravitational field [6,36,38,74], which is rather natural since the Einstein tensor G^{μ}_{ν} describes the geometry of Einstein's spacetime (i.e. his gravitational field). Eq.(118) also constitutes the total energy-momentum equation for Einstein's gravitational field and its material sources combined.

Spacetime and matter have no separate existence. Einstein's field equations,

> "*. . . couple the gravitational field (contained in the curvature of spacetime) with its sources.*" Foster and Nightingale [16]

> "*Since gravitation is determined by the matter present, the same must then be postulated for geometry, too. The geometry of space is not given a priori, but is only determined by matter.*" Pauli [69]

> "*Mass acts on spacetime, telling it how to curve. Spacetime in turn acts on mass, telling it how to move.*" Carroll and Ostlie [23]

> "*space as opposed to 'what fills space', which is dependent on the coordinates, has no separate existence*" Einstein [75]

> "*I wish to show that space-time is not necessarily something to which one can ascribe a separate existence, independently of the actual objects of physical reality.*" Einstein [76]

Unlike Eq.(113), Eq.(118) is a tensor equation. The tensor (covariant derivative) divergence of the left side of Eq.(118) is zero and therefore constitutes the conservation law for Einstein's gravitational field and its material sources. However, the total energy-momentum by Eq.(118) is always zero, the G^{μ}_{ν}/κ and the T^{μ}_{ν} must vanish identically because spacetime and matter have no separate existence in General Relativity, and hence gravitational energy cannot be localised, i.e. there is no possibility of gravitational waves [6,36,38,74]. Moreover, since the total energy-momentum is always zero the usual conservation laws for energy and momentum of a closed system cannot be satisfied [6,36,38,74]. General Relativity is therefore in conflict with a vast array of experiments on a fundamental level.

The so-called 'cosmological constant' can be easily included as follows,

$$\frac{\left(G^{\mu}_{\nu} + \lambda g^{\mu}_{\nu}\right)}{\kappa} + T^{\mu}_{\nu} = 0 \tag{120}$$

In this case the energy-momentum components of Einstein's gravitational field are the $\left(G^{\mu}_{\nu} + \lambda g^{\mu}_{\nu}\right)/\kappa$. When G^{μ}_{ν} or T^{μ}_{ν} is zero, all must vanish identically, and all the same consequences ensue just as for Eq.(118). Thus, once again, if there is no material source, not only is there no gravitational field, there is no universe, and Einstein's field equations violate the usual conservation of energy and momentum for a closed system.

The so-called 'dark energy' is attributed to λ by cosmologists. Dark energy is a mysterious aether *ad arbitrium*, because, according to Einstein [48,77], λ is not a material source for a gravitational field, but is vaguely implicated by him in his gravitational field,

> "*. . . by introducing into Hamilton's principle, instead of the scalar of Riemann's tensor, this scalar increased by a universal constant*" Einstein [77]

The ‘cosmological constant’ however falls afoul of de Sitter’s empty universe, which possesses spacetime curvature but contains no matter, and is therefore physically meaningless. By Eq.(120), if λg^{μ}_{ν} is to be permitted, for the sake of argument, it must be part of the energy-momentum of the gravitational field, which necessarily vanishes when $T^{\mu}_{\nu} = 0$. Recall that according to Einstein, everything except his gravitational field is matter and that matter is the cause of his gravitational field. The insinuation of λ can be more readily seen by writing Eq.(120) as,

$$\frac{\left[R^{\mu}_{\nu} - \frac{1}{2}\left(R - 2\lambda\right) g^{\mu}_{\nu}\right]}{\kappa} + T^{\mu}_{\nu} = 0 \tag{121}$$

Einstein’s “*scalar increased by a universal constant*” is clearly evident; it is the term $-(R - 2\lambda)/2$. Hence Einstein’s field equations “*in the absence of matter*” [29 §14], i.e. $R^{\mu}_{\nu} = 0$, once again, have no physical meaning, and so the Schwarzschild solution too has no physical meaning, despite putative observational verifications. Consequently, the theories of black holes and gravitational waves are invalid.

9 Numerical relativity and perturbations on black holes

Numerical analysis of merging black holes and perturbation of black holes are ill-posed procedures for the simple fact that such mathematical means cannot validate a demonstrable fallacy. Numerical analysis and ‘systematic perturbation expansions’ on a fallacy produce fallacies still. Similarly, no amount of observation or experiment can legitimise entities that are the products of violations of the rules of pure mathematics and logic. However, cosmologists systematically perturb:

> “*In a systematic perturbation expansion one can compute the interactions, due to nonlinearity, between black holes.*” ’t Hooft [78]

Since the premises are false and the conclusions drawn from them inconsistent with them, such numerical and perturbation procedures are consequently of no scientific merit. Nonetheless the LIGO-Virgo Collaboration has stated,

> “*The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of* 1.0×10^{-21}. *It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole.*” Abbott *et al*. [1]
>
> “*Using the fits to numerical simulations of binary black hole mergers in [92,93], we provide estimates of the mass and spin of the final black hole, the total energy radiated in gravitational waves, and the peak gravitational-wave luminosity [39].*” Abbott *et al*. [1]
>
> “*Several analyses have been performed to determine whether or not GW150914 is consistent with a binary black hole system in general relativity [94]. A first consistency check involves the mass and spin of the final black hole. In general relativity, the end product of a black hole binary coalescence is a Kerr black hole, which is fully described by its mass and spin. For quasicircular inspirals, these are predicted uniquely by Einstein’s equations as a function of the masses and spins of the two progenitor black holes. Using fitting formulas calibrated to numerical relativity simulations [92], we verified that the remnant mass and spin deduced from the early stage of the coalescence and those inferred independently from the late stage are consistent with each other, with no evidence for disagreement from general relativity.*” Abbott *et al*. [1]

Signal GW150914 was extracted from a database containing 250,000 numerically determined waveforms generated on the false assumptions of the existence of black holes and gravitational waves. A ‘generic’ signal cGW was initially reported by LIGO, after which powerful computers extracted GW150914 from the waveform database for a best fit element.

> “*The initial detection was made by low-latency searches for generic gravitational-wave transients [41] and was reported within three minutes of data acquisition [43]. Subsequently, matched-filter analyses that use relativistic models of compact binary waveforms [44] recovered GW150914 as the most significant event from each detector for the observations reported here.*” Abbott *et al*. [1]

With such powerful computing resources and so many degrees of freedom it is possible to best fit just about any LIGO instability with an element of its numerically determined waveform database. This is indeed the outcome for the LIGO-Virgo Collaborations, as they have managed to best fit a numerically determined waveform for and to entities that not only do not exist, but are not even consistent with General Relativity itself. This amplifies the futility of applying numerical and perturbation methods to ill-posed problems.

There are no known Einstein field equations for two or more masses and hence no known solutions thereto. There is no existence theorem by which it can even be asserted that Einstein's field equations contain latent capability for describing configurations of two or more masses [6,36,38,79]. General Relativity cannot account for the simple experimental fact that two fixed suspended masses approach one another upon release. It is for precisely these reasons that all the big bang models treat the universe as a single mass, an ideal indivisible fluid of macroscopic density and pressure that permeates the entire universe. Upon this model the cosmologists simply superpose, where superposition does not hold.

> "*We may, however, introduce a more specific hypothesis by assuming that the material filling the model can be treated as a perfect fluid.*" Tolman [67]
>
> "*We can then treat the universe as filled with a continuous distribution of fluid of proper macroscopic density ρ_{oo} and pressure p_o, and shall feel justified in making this simplification since our interest lies in obtaining a general framework for the behaviour of the universe as a whole, on which the details of local occurrences could be later superposed.*" Tolman [67]
>
> "*. . . it must be remembered that these quantities apply to the idealized fluid in the model, which we have substituted in place of the matter and radiation actually present in the real universe.*" Tolman [67]

10 Big bang cosmology

The central dogma of big bang cosmology is that the Universe created itself out of nothing [80]. Often this nothingness is vaguely called a big bang 'singularity'. Space, time, and matter, all came into existence with the big bang creation *ex nihilo*.

> "*General relativity plays an important role in cosmology. The simplest theory is that at a certain moment '$t = 0$', the universe started off from a singularity, after which it began to expand. . . . All solutions start with a 'big bang' at $t = 0$.*" 't Hooft [12]

> "*At the big bang itself, the universe is thought to have had zero size, and to have been infinitely hot.*" Hawking [60]

That which has zero size has no volume and hence cannot possess mass or have a temperature. What is temperature? According to the physicists and the chemists it is the motion of atoms and molecules. Atoms and molecules have mass. The more energy imparted to the atoms and molecules the faster they move about and so the higher the temperature. In the case of a solid the atoms or molecules vibrate about their equilibrium positions in a lattice structure and this vibration increases with increased temperature.

> "*As the temperature rises, the molecules become more and more agitated; each one bounds back and forth more and more vigorously in the little space left for it by its neighbours, and each one strikes its neighbours more and more strongly as it rebounds from them.*" Pauling [81]

Increased energy causes atoms or molecules of a solid to break down the long range order of its lattice structure to form a liquid or gas. Liquids have short range order, or long range disorder. Gases have a great molecular or atomic disorder. In the case of an ideal gas its temperature is proportional to the mean kinetic energy of its molecules [82-84],

$$\frac{3}{2}kT = \frac{1}{2}m\left\langle v^2\right\rangle \tag{122}$$

wherein $\left\langle v^2\right\rangle$ is the mean squared molecular speed, m the molecular mass, and k is Boltzman's constant.

Now that which has zero size has no space for atoms and molecules to exist in or for them to move about in. And just how fast must atoms and molecules be moving about to be infinitely hot? An entity of zero size and infinite hotness has no scientific meaning whatsoever. Nonetheless, according to Misner, Thorne and Wheeler [24],

> "*One crucial assumption underlies the standard hot big-bang model: that the universe 'began' in a state of rapid expansion from a very nearly homogeneous, isotropic condition of infinite (or near infinite) density and pressure.*"

Just how close to infinite must one get to be "*near infinite*"? No object can possesses infinite or near 'infinite density' and pressure either, just as no object can possess infinite gravity or infinite temperature. Even Special Relativity forbids infinite density.

Near infinities of various sorts are routinely and widely invoked by cosmologists and astronomers. Here is yet another example; this time from Professor Lawrence Krauss [85] of Arizona State University, on Australian national television:

"*But is that, in fact, because of discovering that empty space has energy, it seems quite plausible that our universe may be just one universe in what could be almost an infinite number of universes and in every universe the laws of physics are different and they come into existence when the universe comes into existence.*"

Just how close to infinite is "*almost an infinite number*"? There is no such thing as "*almost an infinite number*".

"*There's no real particles but it actually has properties but the point is that you can go much further and say there's no space, no time, no universe and not even any fundamental laws and it could all spontaneously arise and it seems to me if you have no laws, no space, no time, no particles, no radiation, it is a pretty good approximation of nothing.*" Krauss [85]

"*There was nothing there. There was absolutely no space, no time, no matter, no radiation. Space and time themselves popped into existence which is one of the reasons why it is hard . . .* " Krauss [89]

Thus, the Universe sprang into existence from absolutely nothing, by big bang creationism, "*at a certain moment* $'t = 0'$" [12] and nothing, apparently, is "*a good approximation of nothing*" [85]. And not only is nothing a good approximation of nothing, this rigmarole is pushed even further:

"*But I would argue that nothing is a physical quantity. It's the absence of something.*" Krauss [85]

Professor Brian Schmidt is a Nobel Prize winning cosmologist [86]. The following question was put to him on Australian national television:

"*How can something as infinitely large as the universe actually get bigger?*" Irvin [86]

Schmidt began his reply with the following:

"*Ah, yes, this is always a problem: infinity getting bigger. So, if you think of the universe and when we measure the universe it, as near as we can tell, is very close to being infinite in size, that is we can only see 13.8 billion light years of it because that's how old the universe is, but we're pretty sure there's a lot more universe beyond the part we can see, which light just simply can't get to us. And our measurements are such that we actually think that very nearly that may go out, well, well, thousands of times beyond what we can see and perhaps an infinite distance.*" Schmidt [86,87]

However, an infinite universe cannot get bigger*, bearing in mind that infinite simply means endless, and so is not even a real number. Professor Schmidt committed the very common cosmologist elementary error that "*very close to being infinite in size*" is a scientific quantity [88]. With this in mind, how likely is it that cosmologists actually measured the nearness to infinity that Professor Schmidt has claimed? Schmidt continued,

"*So, ultimately, we're expanding into the future but think of it this way: in school you would have done this little experiment in math where you will put a ray starting at zero and it will go out one, two, three and off to infinity. You put a little arrow, it goes off forever. So I can multiply that by two. So zero stays at zero, one goes to two, two goes to four, four goes to eight and you can do that for any number you want all the way up to infinity. And that's sort of what the universe is doing. Infinity is just getting bigger and we're allowed to do that in mathematics. That's what's so cool about math.*" [86,87]

Consider the two infinite sequences of integers that Professor Schmidt utilised (where the dots mean, 'goes on in like manner without end'),

$$0, 1, 2, 3, 4, \ldots$$

$$0, 2, 4, 6, 8, \ldots$$

First, all Schmidt has done here is to place the non-negative even integers (the lower sequence) into a *one-to-one correspondence* with the non-negative integers (the upper sequence). This does not make infinity get bigger. Both sequences are infinite. For every number in the upper sequence there is one and only one corresponding number in the lower sequence, according to position. Second, since infinity is not a real number, contrary to Professor Schmidt's claim, it cannot even be multiplied by 2 because, ultimately, numbers on the real number line can only be multiplied by numbers. Infinity is often denoted by the

*I shall not consider the esoteric purely mathematical issues of Cantor's 'transfinite numbers', as they have no relevance here.

symbol ∞. This is not a real number and so it cannot be used for the usual arithmetic or algebra. Substituting the symbol ∞ for the word 'endless' or the word 'infinity' or the word 'limitless' does not make ∞ a real number. Consequently, $2\times\infty$ does not mean that infinity is doubled; it is a meaningless concatenation of symbols. In like fashion, multiply Professor Schmidt's first sequence by $\frac{1}{2}$. The resulting sequence is,

$$0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$$

Does this mean that infinity has been halved? Is not this sequence also infinite? Professor Schmidt's doubling of infinity by means of the real number line is just as nonsensical as halving infinity with the real number line.

Yet despite the zero size, the infinities and near infinities possessed by nothing, the absence of something, and big bang creation *ex nihilo*, Hawking admits that,

> "*energy cannot be created out of nothing.*" Hawking [60]

Thus stands yet another cosmological contradiction.

The so-called 'Cosmic Microwave Background' (CMB) is inextricably intertwined with big bang cosmology. Without the 'CMB', big bang creationism and General Relativity are defunct. The reasons why the 'CMB' does not exist are simply stated:

1. Kirchhoff's Law of Thermal Emission is false [90].
2. Due to (1), Planck's equation for thermal spectra is not universal.

Consequently, when Penzias and Wilson [91] assigned a temperature to their residual signal and the theoreticians assigned that signal to the Cosmos, they violated the laws of thermal emission. It is a scientific fact that no monopole signal has ever been detected beyond $\approx$ 900 km of Earth. The signal is proximal (i.e. from the oceans on Earth [92-95]).

Nuclear Magnetic Resonance (NMR) and Magnetic Resonance Imaging (MRI) are thermal processes. That they exist is physical proof of the invalidity of Kirchhoff's Law of Thermal Emission and the non-universality of Planck's equation. If Kirchhoff's Law of Thermal Emission is true and Planck's equation is universal, then NMR and MRI would be impossible, because NMR and MRI utilise spin-lattice relaxation [96]. Hence, there is energy in the walls of an arbitrary cavity that is not available to thermal emission. Kirchhoff and Planck however, incorrectly permitted all energy in the walls of an arbitrary cavity to be available to the emission field. The very existence of clinical MRI, used in medicine everyday, proves that Kirchhoff's Law of Thermal Emission is false and that Planck's equation is not universal. This means that the 'CMB' does not exist because it requires the validity of Kirchhoff's Law of Thermal Emission and universality of Planck's equation. These facts alone invalidate big bang cosmology completely.

Put a glass of water inside a microwave oven then turn on the oven. The water gets hot because it absorbs microwaves [95]. That water absorbs microwaves is also well known for submarines, which is precisely why microwave radio communications cannot be used under water. It is well known from the laboratory that a good absorber is also a good emitter, and at the same frequencies. Approximately 70% of Earth's surface is covered by water. This water is not microwave silent. The reported 'CMB' is characterised by the monopole signal for the mean temperature of the microwave residue of the big bang. Its spectrum is a blackbody distribution at $\approx$ 2.725 K. Cosmologists claim the error bars of their CMB spectrum plot are some 400 times narrower than the thickness of the curve they have drawn for it. Yet it is a scientific fact that no monopole signal has ever been detected beyond $\approx$ 900 km of Earth. Without a monopole signal far from Earth, at say L2 (i.e. the second Lagrange point), all talk of a CMB and its alleged anisotropies is wishful thinking. All detections of the monople signal are of microwaves emitted by the oceans, scattered by the atmosphere.

The water molecule is bound by two bonds: (a) the hydroxyl bond, and (b) the hydrogen bond. The hydrogen bond weakly binds one water molecule to another. The hydroxyl bond strongly binds an oxygen atom to a hydrogen atom within the water molecule. Robitaille [97] has shown that the hydroxyl bond is $\approx$ 100 times stronger than the hydrogen bond. It is the hydrogen bond that is responsible for microwave emissions from water. If the oceans are at 300 K, then their microwave emission reports a temperature of $\approx$ 3 K. From this it is clear that a blackbody spectrum does not report the true temperature of the emission source, unless that source is a black material, such as soot; otherwise the temperature extracted from a blackbody spectrum is only apparent. Moreover, the Planckian (blackbody) distribution is continuous. Only condensed matter can emit a continuous spectrum. Gases can only emit in narrow bands, never a continuous spectrum, irrespective of pressure, and pressure requires the presence of a surface. Solar scientists and cosmologists believe the Sun and stars to be balls of hot gas, mostly hydrogen. Stars, they say, produce black holes by irresistible gravitational collapse. Liquids however are essentially incompressible. The photosphere of the Sun emits a Planckian spectrum. This alone is certain evidence that the Sun is condensed matter, not a ball of hot gas [98,99]. The most likely candidate for the constitution of the Sun and stars is liquid metallic hydrogen [98,99]. Furthermore, when a solar flare errupts it produces a radiating circular transverse wave emanating from its point of eruption in the Sun's surface, like that when a stone is flung into a pond. Gases cannot form or carry a transverse wave. The transverse wave produced by a solar flare too is certain evidence that the Sun is condensed matter.

11 Conclusions

LIGO did not detect Einstein gravitational waves or black holes. Black holes and Einstein's gravitational waves do not exist. The LIGO instability has been interpreted as gravitational waves produced by two merging black holes by a combination of theoretical fallacies, wishful thinking, and conformational bias. Black holes are products entire of violations of the rules of pure mathematics. Einstein's General Theory of Relativity is riddled with logical inconsistencies, invalid mathematics, and impossible 'physics'. The General Theory of Relativity violates the usual conservation of energy and momentum for a closed system and is thereby in conflict with a vast array of experiments, rendering it untenable at a fundamental level.

Arguments such as,

> "*What is more, astronomers have now identified numerous objects in the heavens that completely match the detailed descriptions theoreticians have derived. These objects cannot be interpreted as anything else but black holes.*" 't Hooft [26]

have no scientific merit [100].

LIGO is reported to have so far cost taxpayers $1,100,000,000.00 [101]. Just as with the Large Hadron collider at CERN, such large sums of public money demand justification by eventually finding what they said they would, despite the actual facts.

In the same fashion that people who believe in ghosts attribute the action of ghosts to that which they do not understand, cosmologists attribute the action of black holes and big bangs to that which they do not understand. No amount of experiment and observation can validate entities that have been extracted from theory by means of violations of the rules of pure mathematics, violations of basic logic, conflict with well established experimental findings, and just plain wishful thinking. With their litany of violations of scientific method it is perhaps not surprising that cosmologists, led by Professor Stephen Hawking, are now spending $\$100,000,000.00$ of Milner's money, looking for aliens [102]; the very same aliens that fly saucers, man UFO's, and abduct human beings for experiments and vivisection, because they all come from outer space. Radio telescopes around the world will assist the cosmologists in their quest for alien contact [102]. One such telescope is the Parkes Radio Telescope in New South Wales, Australia. For 17 years, cosmologists at the Parkes facility mistook microwave signals from the microwave oven in their lunchroom for cosmic signals, and even called them 'perytons' [103]. Is there any doubt that the cosmologists will soon report alien contact? The aliens must be out there because the cosmologists even have a journal for them [104].

Kirchhoff's Law of Thermal Emission is false and Planck's equation for thermal spectra is not universal; physically proven by the clinical existence of MRI. It follows immediately from this that the Cosmic Microwave Background does not exist because it requires Kirchhoff's Law and universality of Planck's equation. The 'CMB' is due to microwave emission from the oceans on Earth [92-95]. For this reason no monopole signal has ever been detected beyond $\approx$ 900 km of Earth.

Planck's theoretical proof of Kirchhiff's Law of Thermal Emission is false, owing to violations of the physics of optics and thermal emission [90]. Einstein's derivation of Planck's equation is valid only for black materials because he invoked a Wein's field; a characteristic of black materials such as soot [90]. Only black materials such as soot emit a blackbody spectrum. Other materials emit an approximate black spectrum whilst yet others do not. The radiation within arbitrary cavities is not black and their radiation fields, at thermal equilibrium, depend upon the nature of the cavity walls.

The Sun and stars are not balls of hot gas; they are condensed matter [98,99].

Modern physics is steeped in magic, mysticism and superstition. The proclivity of the Human Condition to magic and mysticism is well known to anthropologists:

> "*The reader may well be tempted to ask. How was it that intelligent men did not sooner detect the fallacy of magic? How could they continue to cherish expectations that were invariably doomed to disappointment? With what heart persist in playing venerable antics that led to nothing, and mumbling solemn balderdash that remained without effect? Why cling to beliefs which were so flatly contradicted by experience? How dare to repeat experiments that had failed so often? The answer seems to be that the fallacy was far from easy to detect, the failure by no means obvious, since in many, perhaps in most cases, the desired event did actually follow at a*

longer or shorter interval, the performance of the rite which was designed to bring it about; and a mind of more than common acuteness was needed to perceive that, even in these cases, the rite was not necessarily the cause of the event." Frazer [105]

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