# The misuse of the No-communication Theorem

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**Abstract**

This short note seeks to draw together and clarify the author's early papers on the matter of a communication system. The enquiry occurred from the investigation of two interferometer based communication systems: one two photon entanglement, the other single photon path entanglement. The state vector treatment confirmed the communication protocol but the No-communication theorem (NCT), couched in the density matrix treatment forbade it. Since both state vector and density matrix formalisms contain the same treatment of quantum mechanics, the only conclusion is the NCT is being cited inappropriately. Here we clarify that error cogently and with brevity. The NCT only applies to non-factorisable systems and the communication system surprisingly entails Schmidt decomposition and hence factorisability, yet keeps entanglement information by the "which path information" of the interferometer.

## 1. Introduction

The author investigated two schemes using entangled communication[1, 2] (appendices 1 and 2) and is currently seeking partners to corroboration the latter path-entangled method. Mathematically the two entangled polarised photon system is identical to the one photon path-entangled system, as both are forms of the Bell states[3].

$$\begin{aligned}\left|\Phi^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}\left|A_1\right\rangle\left|A_2\right\rangle \pm \frac{1}{\sqrt{2}}\left|B_1\right\rangle\left|B_2\right\rangle \\ \left|\Psi^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}\left|A_1\right\rangle\left|B_2\right\rangle \pm \frac{1}{\sqrt{2}}\left|B_1\right\rangle\left|A_2\right\rangle\end{aligned} \quad \text{eqn. 1}$$

The author has directly interacted with two of the founders of the No-communications theorem (NCT), Michael Hall (Australian patent office, whom granted a patent) and Giancarlo Ghirardi, whom offered a repost[4]. In this note, Ghirardi used the density matrix treatment. We seek to counter those arguments coherently and concisely.

## 2. The state vector approach

We are interested in two endpoints of communication, so the joint evolution of a two particle system is used[=]. The state vector formalism gives this as:

$$\left|\psi_1'\right\rangle \otimes \left|\psi_1'\right\rangle = O_1\left|\psi_1\right\rangle \otimes O_2\left|\psi_1\right\rangle \quad \text{eqn. 2}$$

Where the operators $O_1$ and $O_2$ (which themselves may be several operators) act on their respective quantum states, be they unitary or non-unitary.
The ensuing bone of contention, as we shall see, arises when the states can't be factorised and we shall discuss the first apparatus[1] in this context. The evolution (not writing explicitly the tensor product symbol) is then:

$$\left|\psi_{12}'\right\rangle = O_1 O_2\left|\psi_{12}\right\rangle \quad \text{eqn. 3}$$

For the first apparatus[1], if the input is:

$$\psi_{12} = \frac{1}{\sqrt{2}}\left(\left|H_1\right\rangle \otimes \left|V_2\right\rangle + \left|V_1\right\rangle \otimes \left|H_2\right\rangle\right) \quad \text{eqn. 4}$$

Then the evolution is: $O_1 O_2\left|\psi_{12}\right\rangle \to \left|\psi_{12}'\right\rangle$

$$\left|\psi_{12}'\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \hat{U}_1\left|H_1\right\rangle & \otimes e^{-i\theta} e^{-i\frac{\pi}{4}} \hat{U}_{PBS}\left|V_2\right\rangle \\ + \;\; \hat{U}_1\left|V_1\right\rangle & \otimes e^{+i\frac{\pi}{4}} \hat{U}_{PBS}\left|H_2\right\rangle \end{pmatrix} \quad \text{eqn. 5}$$

Where,

- The first photon travels through free space: $\hat{U}_1$
- The polarising beam-splitter is the projection: $\hat{U}_{PBS} = \left|H_2\right\rangle\left\langle H_2\right| + \left|V_2\right\rangle\left\langle V_2\right|$
- The Faraday rotators (there can be just one with angle π/2) are shown: $e^{-i\frac{\pi}{4}}$ and $e^{+i\frac{\pi}{4}}$
- Then the phase plate to adjust the interference fringe is: $e^{-i\theta}$

**Before measurement**, the state is (D represents the diagonal basis) :

$$\left|\psi_{12}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|H_1\right\rangle \otimes e^{-i\theta}\left|D_2\right\rangle + \left|V_1\right\rangle \otimes \left|D_2\right\rangle\right) \quad \text{eqn. 6}$$

And one final operation gives the effect of the <u>detector</u> by the number operator $\hat{n}_2 = \hat{a}_2^{\dagger}\hat{a}_2$ projecting into the number basis:

$$\begin{aligned}&\left|\psi_{12}'\right\rangle = \hat{U}_1\hat{n}_2\left|\psi_{12}\right\rangle \\ &\Rightarrow \\ &\left|\psi_{12}'\right\rangle = \hat{U}_1\left|\psi_1\right\rangle \otimes \frac{1}{\sqrt{2}}\left(1 - e^{-i\theta}\right)\left|1_2\right\rangle\end{aligned} \quad \text{eqn. 7}$$

It is easy to then trace out system one by a **Schmidt decomposition**[3] to see system two:

$$\left|\psi_2\right\rangle = \frac{1}{\sqrt{2}}\left(1 - e^{-i\theta}\right)\left|1_2\right\rangle \quad \text{eqn. 8}$$

**Measurement** on either system and the spectral theory yield the mixed state:

$$\frac{e^{-iRand}}{\sqrt{2}}\left(\left|H_1\right\rangle \otimes e^{-i\theta}\left|D_2\right\rangle\ \right) \quad \text{or} \quad \frac{e^{-iRand}}{\sqrt{2}}\left(\left|V_1\right\rangle \otimes \left|D_2\right\rangle\ \right)$$

Where $e^{-iRand}$ is the random phase relation between them. The number operator and trace out yields (the phase of individual events is not important nor is there superposition) at the detector:

$$\left|\psi_2\right\rangle = \frac{1}{\sqrt{2}}\left|1_2\right\rangle \quad \text{eqn. 9}$$

The state vector approach permits discernment of distant measurement on an entangled system (not least a single photon path entangled[2]) and this should give identical results to density matrix analysis.

3. The density matrix approach and the NCT

The state vector and density matrix formulations of quantum mechanics are of course identical[5]. If we can write: $|\psi'\rangle = U|\psi\rangle$ and $\langle\psi'| = \langle\psi|U^\dagger$, then it is correct to write: $U|\psi\rangle\langle\psi|U$ or $\rho' = U\rho U^\dagger$.

Considering only two particles (though the results are extensible to any number of particles), if the states were separable, we'd write the evolution as, $\rho_1' \otimes \rho_2' = U_1\rho_1 U_1^\dagger \otimes U_2\rho_2 U_2^\dagger$. If the states cannot be factorised, the evolution is, $\rho_{12}' = U_{12}\rho_{12}U_{12}^\dagger$. These results, are no different than eqn. 2 or eqn. 3, respectively.

Let us now explore the NCT which is couched in terms of the density matrix treatment. The system before measurement and during measurement evolves jointly. The partial trace is then taken to isolate one system.

An archetypal Bell state (eqn. 1) on measurement or through the *procedure* of taking the partial trace, can only lead to a mixed state as the system is not factorisable. This is easy to see with eqn. 1:

$$\rho_2' = \sum_{i=A,B} \langle i_1| \;\; |\Phi^\pm\rangle\langle\Phi^\pm| \;\; |i_1\rangle \qquad \text{eqn. 10}$$

Orthogonal off-diagonal terms drop out, such as:

$$\underbrace{\cancel{\langle B_1|A_1\rangle}}_{=0}|A_2\rangle\langle A_2|\underbrace{\cancel{\langle A_1|B\rangle_1}}_{=0}$$

As opposed to orthonormal terms:

$$\underbrace{\cancel{\langle A_1|A_1\rangle}}_{=1}|A_2\rangle\langle A_2|\underbrace{\cancel{\langle A_1|A_1\rangle}}_{=1} = |A_2\rangle\langle A_2|$$

However the situation with the interferometer communication setup[1, 2] and the Schmidt decomposition is different, because the system is factorisable:

**Before measurement**,

$$\rho_{12} = \frac{1}{2}\left(1-e^{-i\theta}\right)|D_2\rangle\left(|H_1\rangle+|V_1\rangle\right)\left(\langle V_1|+\langle H_1|\right)\langle D_2|\left(1-e^{-i\theta}\right)$$

Whereupon the partial trace is:

$$\rho_2 = \frac{1}{2}\left(1-e^{-i\theta}\right)^2|D_2\rangle\langle D_2| \qquad \text{eqn. 11}$$

**After measurement**,

$$\rho_{12} = \frac{1}{2}|D_2\rangle|H_1\rangle\langle H_1|\langle D_2| + \frac{1}{2}e^{-2i\theta}|D_2\rangle|V_1\rangle\langle V_1|\langle D_2|$$

And the partial trace is:

$$\rho_2 = \frac{1}{2}|D_2\rangle\langle D_2| + \frac{1}{2}e^{-2i\theta}|D_2\rangle\langle D_2| \qquad \text{eqn. 12}$$

The two cases eqn. 11 and eqn. 12 are different.

4. Conclusion

The No-communication theorem since its inception has been extensively cited. The simple and obviously correct proof herein shows a glaring flaw in its application regarding a system that maintains entanglement information, by performing a Schmidt decomposition, which renders the system factorisable. The NCT only applies to non-factorisable systems and the slavish, unthinking citing of it must cease.

Single particle path-entanglement experiments alone[2], without all the machinery of multi-particle quantum systems, show obvious known experimental fact; yet if one transforms it into a two particle system by considering the vacuum state as a quasi-particle and uses the incorrectly applied density matrix rationale of the NCT, we arrive at a result not just in abeyance of experimental fact *and* not just in abeyance of the state vector treatment for a multi-particle system *but also* in abeyance of the state vector treatment for a single particle system too. This is a ridiculous situation.

References

1. Cornwall, R.O., *Secure Quantum Communication and Superluminal Signalling on the Bell Channel.* Infinite Energy, 2006, **69.** http://arxiv.org/abs/1106.2257

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3. Nielsen, M.C., Isaac, *Quantum Computation and Quantum Information*, 2000: Cambridge.

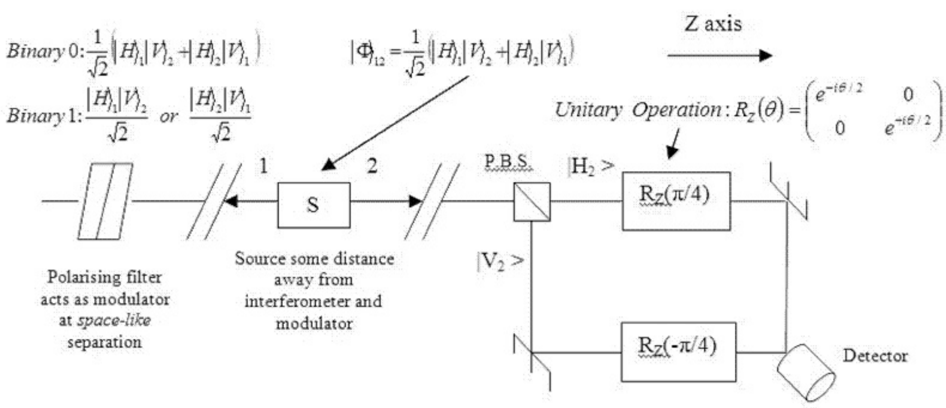
4. Ghirardi, G., *Comments on a recent proposal of superluminal communicatio.* 2014. http://arxiv.org/pdf/1411.0893.pdf

5. Landau, L., *A Course in Theoretical Physics: Quantum Mechanics*. Vol. 3. 1982: Butterworth-Heinemann.

6. Ghirardi, G.C.R., A.; Weber, T., *A General Argument against Superluminal Transmission through the Quantum Mechanical Measurement Process.* Lettere al Nuovo Cimento, 1980, 8th March. **27**(10): p. 293-298.

7. Hall, M.J.W., *Imprecise Measurements and Non-Locality in Quantum Mechanics.* Physics Letters A, 1987. **125**(2,3): p. 89,91.

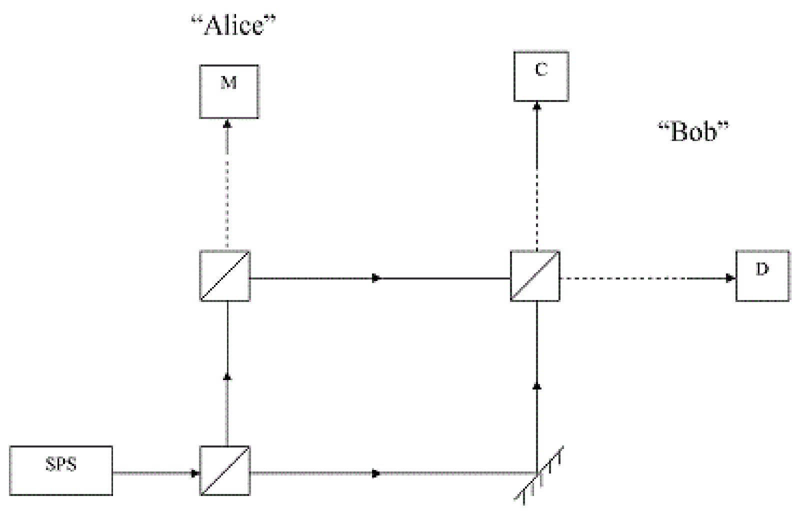
## Appendix 1 – Two H-V entangled photon communication scheme




| Measurement/Modulation at distant system and state of two photon system | State of distant system | State of local system | Local measurement by interferometer after modulation of distant system |
|---|---|---|---|
| No modulation: 'Binary 0' $\frac{1}{\sqrt{2}}\left(\|H\rangle_1\|V\rangle_2 + \|H\rangle_2\|V\rangle_1\right)$ | Entangled => Pure state $\frac{1}{\sqrt{2}}\left(\|H\rangle_1 + \|V\rangle_1\right)$ **Behaves as**, see Appendix1: arxiv.org/abs/1106.2258 | Entangled => Pure state $\frac{1}{\sqrt{2}}\left(\|V\rangle_2 + \|H\rangle_2\right)$ **Behaves as**, see Appendix1: arxiv.org/abs/1106.2258 | Pure state results in interference (Or at least some interference since source is not ideally pure) |
| Modulation: 'Binary 1' $\frac{\|H\rangle_1\|V\rangle_2}{\sqrt{2}}$ or $\frac{\|H\rangle_2\|V\rangle_1}{\sqrt{2}}$ | Not entangled <=> Mixed state $\frac{\|H\rangle_1}{\sqrt{2}}$ or $\frac{\|V\rangle_1}{\sqrt{2}}$ | Not entangled <=> Mixed state $\frac{\|H\rangle_2}{\sqrt{2}}$ or $\frac{\|V\rangle_2}{\sqrt{2}}$ | Mixed state gives no interference |

## Appendix 2 – Single photon path entangled communication scheme

Single photon source (SPS) incident on Mach-Zehnder type interferometer with 50:50 beamsplitters. Alice's measurements discerned over space-like seperations by Bob at his detectors C (constructive) or D (destructive)




| Alice | Bob |
|---|---|
| 0: No measurement | No signal, destructive interference from pure state |
| 1: Measurement | Signal from mixed state |

$$P(\text{Measurement, bit 1}) = \left|\frac{i}{\sqrt{2}}\right|^2 + \left|\frac{i}{\sqrt{4}}\right|^2$$
$$= 0.5 + 0.25$$
$$= 0.75$$

$$P(\text{No-measurement, bit 0}) = \left|\frac{i}{\sqrt{2}}\right|^2 + \left|\frac{e^{i\theta}}{\sqrt{4}}\right|^2 + 2\left|\frac{i}{\sqrt{2}}\right|\left|\frac{e^{i\theta}}{\sqrt{4}}\right|\cos(\arg\theta)$$
$$= 0.5 + 0.25 + \frac{1}{\sqrt{2}}\cos(\arg\theta)$$
$$= 0.75 \pm 0.707\cos(\arg\theta)$$
$$= 0.043 \text{ minimum}$$