The misuse of the formalism in the No-communication Theorem

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Abstract

This short note seeks to draw together and clarify the author's early papers on the matter of an error in use of the formalism of quantum mechanics in the No-communication theorem (NCT). The enquiry occurred from the investigation of two interferometer based communication systems: one two photon entanglement, the other single photon path entanglement. The state vector treatment confirmed the communication protocol but the NCT, couched in the density matrix treatment forbade it. Since both state vector and density matrix formalisms contain the same treatment of quantum mechanics, which of course, has been extensively tested, the only conclusion is the NCT has an error in the use of the formalism of density matrices. Here we clarify that error cogently and with brevity.

1. Introduction

The author investigated two schemes using entangled communication [1, 2][†] and is currently seeking partners to corroboration the latter path-entangled method. Mathematically the two entangled polarised photon system is identical to the one photon path-entangled system, as both are forms of the Bell states[3].

$$\begin{aligned}\left|\Phi^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}|0\rangle_1|0\rangle_2 \pm \frac{1}{\sqrt{2}}|1\rangle_1|1\rangle_2 \\ \left|\Psi^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}|0\rangle_1|1\rangle_2 \pm \frac{1}{\sqrt{2}}|1\rangle_1|0\rangle_2\end{aligned} \qquad \text{eqn. 1}$$

The author has directly interacted with two of the founders of the No-communications theorem (NCT), Michael Hall (Australian patent office, whom granted a patent) and Giancarlo Ghirardi, whom offered a repost[4]. In this note, Ghirardi used the density matrix treatment. We seek to counter those arguments coherently and concisely.

2. The state vector approach

We are interested in two endpoints of communication, so the joint evolution of a two particle system is used[‡]. The state vector formalism gives this as:

$$|\psi_1'\rangle \otimes |\psi_1'\rangle = O_1|\psi_1\rangle \otimes O_2|\psi_1\rangle \qquad \text{eqn. 2}$$

Where the operators O_1 and O_2 (which themselves may be several operators) act on their respective quantum states, be they unitary or non-unitary.
The ensuing bone of contention, as we shall see, arises when the states can't be factorised and we shall discuss the first apparatus[1] in this context[§]. The evolution (not writing explicitly the tensor product symbol) is then:

$$|\psi_{12}'\rangle = O_1 O_2 |\psi_{12}\rangle \qquad \text{eqn. 3}$$

For the first apparatus[1], if the input is:

$$\psi_{12} = \frac{1}{\sqrt{2}}\left(|H_1\rangle \otimes |V_2\rangle + |V_1\rangle \otimes |H_2\rangle\right) \qquad \text{eqn. 4}$$

Then the evolution is: $O_1 O_2 |\psi_{12}\rangle \to |\psi_{12}'\rangle$

$$|\psi_{12}'\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \hat{U}_1|H_1\rangle \otimes e^{-i\theta} e^{-i\frac{\pi}{4}} \hat{U}_{PBS}|V_2\rangle \\ + \hat{U}_1|V_1\rangle \otimes e^{+i\frac{\pi}{4}} \hat{U}_{PBS}|H_2\rangle \end{pmatrix} \qquad \text{eqn. 5}$$

Where,

- The first photon travels through free space: $\hat{U}_1$
- The polarising beam-splitter is the projection: $\hat{U}_{PBS} = |H_2\rangle\langle H_2| + |V_2\rangle\langle V_2|$
- The Faraday rotators (there can be just one with angle $\pi/2$) are shown: $e^{-i\frac{\pi}{4}}$ and $e^{+i\frac{\pi}{4}}$
- Then the phase plate to adjust the interference fringe is: $e^{-i\theta}$

Before measurement, the state is (D represents the diagonal basis) :

$$|\psi_{12}\rangle = \frac{1}{\sqrt{2}}\left(|H_1\rangle \otimes e^{-i\theta}|D_2\rangle + |V_1\rangle \otimes |D_2\rangle\right) \text{eqn. 6}$$

And one final operation gives the effect of the detector by the number operator $\hat{n}_2 = \hat{a}_2^{\dagger}\hat{a}_2$ projecting into the number basis:

$$\begin{aligned}&|\psi_{12}'\rangle = \hat{U}_1 \hat{n}_2 |\psi_{12}\rangle \\ &\Rightarrow \\ &|\psi_{12}'\rangle = \hat{U}_1|\psi_1\rangle \otimes \frac{1}{\sqrt{2}}\left(1 - e^{-i\theta}\right)|1_2\rangle\end{aligned} \qquad \text{eqn. 7}$$

It is easy to then trace out system one to see system two:

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(1 - e^{-i\theta}\right)|1_2\rangle \qquad \text{eqn. 8}$$

Measurement on either system and the spectral theory yield the mixed state:

$$\frac{e^{-iRand}}{\sqrt{2}}\left(|H_1\rangle \otimes e^{-i\theta}|D_2\rangle\ \right)$$
$$\text{or} \quad \frac{e^{-iRand}}{\sqrt{2}}\left(|V_1\rangle \otimes |D_2\rangle\ \right)$$

Where e^{-iRand} is the random phase relation between them. The number operator and trace out yields (the

[†] Diagrams/schematics of the apparatus:
http://webspace.qmul.ac.uk/rocornwall/protocol.jpg
http://webspace.qmul.ac.uk/rocornwall/Flyer_QSE1.gif

[‡] Though for the second apparatus, one particle evolution is sufficient, although one could model the absence of a particle, the vacuum state, as a quasi-particle.

[§] Indeed, Michael Hall said to Cornwall in private communication, "You don't think that one of the pair of photons goes through the interferometer as though it is in the diagonal basis?" We find it does.

phase of individual events is not important nor is there superposition) at the detector:

$$\left|\psi_2\right\rangle = \frac{1}{\sqrt{2}}\left|1_2\right\rangle \qquad \text{eqn. 9}$$

The state vector approach permits discernment of distant measurement on an entangled system (not least a single photon path entangled[2]) and this should give identical results to density matrix analysis.

<u>3.</u> <u>The density matrix approach</u>

The state vector and density matrix formulations of quantum mechanics are of course identical[5]. If we can write:

$$\left|\psi'\right\rangle = U\left|\psi\right\rangle \text{ and } \left\langle\psi'\right| = \left\langle\psi\right|U^\dagger$$

Then it is correct to write:

$$\begin{aligned} &\left|\psi'\right\rangle\left\langle\psi'\right| = U\left|\psi\right\rangle\left\langle\psi\right|U \\ &or \\ &\rho' = U\rho U^\dagger \end{aligned} \qquad \text{eqn. 10}$$

Considering only two particles (though the results are extensible to any number of particles), if the states were separable, we'd write the evolution as:

$$\rho_1' \otimes \rho_2' = U_1\rho_1U_1^\dagger \otimes U_2\rho_2U_2^\dagger \qquad \text{eqn. 11}$$

If the states cannot be factorised, the evolution is:

$$\rho_{12}' = U_{12}\rho_{12}U_{12}^\dagger \qquad \text{eqn. 12}$$

These results, eqn. 11 and eqn. 12 are no different than eqn. 2 or eqn. 3, respectively. Let us now explore the NCT which is couched in terms of the density matrix treatment.

<u>4.</u> <u>The NCT and the flaw it contains</u>

The NCT[6, 7] essentially performs this mathematical sleight-of-hand**:

$$\begin{aligned} \rho_2' &= tr_1\left[\left(U_2\otimes\mathbf{I}\right)\rho_{12}\left(U_2\otimes\mathbf{I}\right)^\dagger\right] \\ &= tr_1\left(\rho_1\otimes U_2\rho_2U_2^\dagger\right) \\ &= tr_1\left(\rho_1\otimes\rho_2'\right) \\ &= \rho_2' \end{aligned} \qquad \text{eqn. 13}$$

The implication is: a system can only perform local operations and these are not affected by a remote system. The operator U_2 can be any type of operator. The partial trace *procedure* renders any entangled non-factorisable state to a mixed state.

{ In some people's minds, the partial trace *procedure* has become synonymous with measurement *operators* (Appendix 1). }

However the NCT and all that cite the NCT, implicate the <u>separable</u> density matrix evolution form eqn. 11 at step 2, in eqn. 13. **The irony and glaring inconsistency** here is that, the very thing the NCT was set up to work with, entangled systems, <u>can't be factored</u>.

** This particularly trite example is due to sources such as Wikipedia and what they cite or summarise or is the "vernacular" summary of it in current research, with the proof handed down generations nth-hand.

Source papers[6, 7] do better than use the steps depicted in eqn. 13 and use the joint evolution (eqn. 12) but then do the following approach with their operators, to ensure they operate on their respective spaces:

$$\rho_{12}' = \left(U_1\otimes\mathbf{I}\right)\left(\mathbf{I}\otimes U_2\right)\rho_{12}\left(U_1\otimes\mathbf{I}\right)^\dagger\left(\mathbf{I}\otimes U_2\right)^\dagger \qquad \text{eqn. 14}$$

Even though the density matrix isn't "slyly" factorised, the partial trace can only lead to the result:

$$\begin{aligned} \rho_2' &= tr_1\left[\left(U_2\otimes\mathbf{I}\right)\rho_{12}\left(U_2\otimes\mathbf{I}\right)^\dagger\right] \\ &= tr_1\left[\left(U_2\otimes\mathbf{I}\right)\rho_2\left(U_2\otimes\mathbf{I}\right)^\dagger\right] \end{aligned}$$

This, of course also applies to system 1 too. The argument is a *truism and fallacy* resulting from the operators being setup at the start to commute, to then lead to the desired result or inference - eqn. 13.

The system before measurement and during measurement evolves jointly. The *procedure* of taking the partial trace should then look like this:

$$\rho_2' = tr_1\left(U_{12}\rho_{12}U_{12}^\dagger\right) \qquad \text{eqn. 15}$$

We shall prove that the density matrix formulation gives the same result as the state vector approach considered in section 2 – it must; this showed the distant system 1 could affect system 2.

The foundation of the NCT has a flaw for this reason (step 2 eqn. 13) :-

$$\rho_2' = tr_1\left(U_{12}\rho_{12}U_{12}^\dagger\right) \neq tr_1\left(U_1\rho_1U_1^\dagger\otimes U_2\rho_2U_2^\dagger\right) \qquad \text{ineqn. 16}$$

Where $U_{12} = U_1\otimes U_2$, and is factored, as it must, because systems 1 and 2 have independent apparatus. It is quite apparent that if all the matrices are unitary the trace of either form in ineqn. 16 is unity.

However if we write for the 1st system operator NU_1, a non-unitary operator to simulate measurement, the situation for the ineqn. 16 is:

$$\rho_2' = tr_1\left(\left(NU_1\otimes U_2\right)\rho_{12}\left(NU_1\otimes U_2\right)^\dagger\right) \neq 1$$

And

$$\rho_2' = tr_1\left(\cancel{NU_1\rho_1NU_1^\dagger}\otimes U_2\rho_2U_2^\dagger\right) = 1$$

This glaring error, effectively implying a separable system when the mandate was for an entangled system, is in the NCT proofs.

Furthermore it is obviously apparent that the distant system affects the local measurement with the measurement/no-measurement protocol[1, 2] to send digital data over a quantum channel:

$$\begin{aligned} &{\scriptstyle NO\ MEASUREMENT}\ \rho_2' = tr_1\left(\left(U_1\otimes U_2\right)\rho_{12}\left(U_1\otimes U_2\right)^\dagger\right) = 1 \\ &\neq \\ &{\scriptstyle MEASUREMENT}\ \rho_2' = tr_1\left(\left(NU_1\otimes U_2\right)\rho_{12}\left(NU_1\otimes U_2\right)^\dagger\right) \end{aligned} \qquad \text{ineqn. 17}$$

For the incredulous reader, we list out the components for a two state, two particle system in Appendix 2.

5. Conclusion

The No-communication theorem since its inception has been extensively cited. The simple and obviously correct proof herein shows a glaring flaw in it and the slavish, unthinking citing of it must cease.

Single particle path-entanglement experiments alone[2], without all the machinery of multi-particle quantum systems, show obvious known experimental fact; yet if one transforms it into a two particle system by considering the vacuum state as a quasi-particle and uses the incorrectly applied density matrix rationale of the NCT, we arrive at a result not just in abeyance of experimental fact *and* not just in abeyance of the state vector treatment for a multi-particle system *but also* in abeyance of the state vector treatment for a single particle system too. This is a ridiculous situation.

Appendix 1. Hermitian operators, the partial trace and measurement

Hermitian operators deliver real values that reflect measurement and that inevitably *leads to* the trace. Using the state vector approach, if M is our measurement operator then:

$$\left|\psi'\right\rangle = M\left|\psi\right\rangle and \left\langle\psi'\right| = \left\langle\psi\right|M^{\dagger}$$

But $M = M^{\dagger}$ and hence the norm is real.

$$\left\langle M^{\dagger}M\right\rangle = \left\langle\psi'\middle|\psi'\right\rangle = \left\langle\psi\right|M^{\dagger}M\left|\psi\right\rangle \quad \text{eqn. 18}$$

Whereupon the experimental outcomes, by spectral decomposition, express the expectation of the operator; it is inevitably synonymous with the trace because of the inner product. Thus the partial trace *procedure* in the main text, "acts like a measurement" operator.

The density matrix form of measurement is just:

$$\left\langle M^{\dagger}M\right\rangle = tr\left(M\left|\psi\right\rangle\left\langle\psi\right|M^{\dagger}\right) = tr\left(\rho M^{\dagger}M\right) \quad \text{eqn. 19}$$

Appendix 2. Proof by listing out matrix components

In a nutshell for the density matrix approach before measurement,

$$\left(\hat{\mathbf{U}}_A \otimes \hat{\mathbf{U}}_B\right)\left|\psi_{AB}\right\rangle\left\langle\psi_{AB}\right|\left(\hat{\mathbf{U}}_A \otimes \hat{\mathbf{U}}_B\right)^{\dagger} \quad \rightarrow \quad \left|\psi'_{AB}\right\rangle\left\langle\psi'_{AB}\right|$$

Whereupon unitary operators for system A will have no effect on system B, other than a global phase when the partial trace is performed.

Let us show the evolution of a 2 state to particle system to grasp this intuitively.

$$\rho'_{AB} = \begin{pmatrix} A_{11}B_{11} & A_{11}B_{12} & A_{12}B_{11} & A_{12}B_{12} \\ A_{11}B_{21} & A_{11}B_{22} & A_{12}B_{21} & A_{12}B_{22} \\ A_{21}B_{11} & A_{21}B_{12} & A_{22}B_{11} & A_{22}B_{12} \\ A_{21}B_{21} & A_{21}B_{22} & A_{22}B_{21} & A_{22}B_{22} \end{pmatrix} \times \begin{pmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{12}b_{11} & a_{12}b_{12} \\ a_{11}b_{21} & a_{11}b_{22} & a_{12}b_{21} & a_{12}b_{22} \\ a_{21}b_{11} & a_{21}b_{12} & a_{22}b_{11} & a_{22}b_{12} \\ a_{21}b_{21} & a_{21}b_{22} & a_{22}b_{21} & a_{22}b_{22} \end{pmatrix} \times \begin{pmatrix} A_{11}B_{11} & A_{11}B_{12} & A_{12}B_{11} & A_{12}B_{12} \\ A_{11}B_{21} & A_{11}B_{22} & A_{12}B_{21} & A_{12}B_{22} \\ A_{21}B_{11} & A_{21}B_{12} & A_{22}B_{11} & A_{22}B_{12} \\ A_{21}B_{21} & A_{21}B_{22} & A_{22}B_{21} & A_{22}B_{22} \end{pmatrix}^{\dagger}$$

The left-hand 4x4 is the tensor product of:

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \otimes \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

for joint evolution of the individual operators for system A and B. The right-hand matrix is the complex transpose. The middle matrix is the density matrix.

$$\rho'_{AB} = \begin{pmatrix} a'_{11}b'_{11} & a'_{11}b'_{12} & a'_{12}b'_{11} & a'_{12}b'_{12} \\ a'_{11}b'_{21} & a'_{11}b'_{22} & a'_{12}b'_{21} & a'_{12}b'_{22} \\ a'_{21}b'_{11} & a'_{21}b'_{12} & a'_{22}b'_{11} & a'_{22}b'_{12} \\ a'_{21}b'_{21} & a'_{21}b'_{22} & a'_{22}b'_{21} & a'_{22}b'_{22} \end{pmatrix}$$

$$\boxed{\text{and so} \quad \rho'_B = \begin{pmatrix} a'_{11}b'_{11} + a'_{22}b'_{11} & a'_{11}b'_{12} + a'_{22}b'_{12} \\ a'_{11}b'_{21} + a'_{22}b'_{21} & a'_{11}b'_{22} + a'_{22}b'_{22} \end{pmatrix} \quad \text{eqn. 20}}$$

If one or both of the evolution matrices is non-unitary (this implies a measurement), the joint state function will collapse and the evolution is not trace preserving. Even when the partial trace is taken to isolate a sub-system that has evolved unitarily, $\rho_B = Tr_A \rho_{AB}$, it is still affected.

Let's see this intuitively by writing out the previous system with a non-unitary evolution on one or the other matrix by zeroing the A_{11} and A_{22} elements in A, as follows:

A non-unitary process in system A:

$$\mathit{CASE}\ \mathrm{A}_{11}=0$$

$$\rho'_{AB}=$$
$$\begin{pmatrix} 0 & 0 & A_{12}B_{11} & A_{12}B_{12} \\ 0 & 0 & A_{12}B_{21} & A_{12}B_{22} \\ A_{21}B_{11} & A_{21}B_{12} & A_{22}B_{11} & A_{22}B_{12} \\ A_{21}B_{21} & A_{21}B_{22} & A_{22}B_{21} & A_{22}B_{22} \end{pmatrix}\times$$
$$\begin{pmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{12}b_{11} & a_{12}b_{12} \\ a_{11}b_{21} & a_{11}b_{22} & a_{12}b_{21} & a_{12}b_{22} \\ a_{21}b_{11} & a_{21}b_{12} & a_{22}b_{11} & a_{22}b_{12} \\ a_{21}b_{21} & a_{21}b_{22} & a_{22}b_{21} & a_{22}b_{22} \end{pmatrix}\times$$
$$\begin{pmatrix} 0 & 0 & A_{12}B_{11} & A_{12}B_{12} \\ 0 & 0 & A_{12}B_{21} & A_{12}B_{22} \\ A_{21}B_{11} & A_{21}B_{12} & A_{22}B_{11} & A_{22}B_{12} \\ A_{21}B_{21} & A_{21}B_{22} & A_{22}B_{21} & A_{22}B_{22} \end{pmatrix}^{\dagger}$$

OR

$$\mathit{CASE}\ \mathrm{A}_{22}=0$$

$$\rho'_{AB}=$$
$$\begin{pmatrix} A_{11}B_{11} & A_{11}B_{12} & A_{12}B_{11} & A_{12}B_{12} \\ A_{11}B_{21} & A_{11}B_{22} & A_{12}B_{21} & A_{12}B_{22} \\ A_{21}B_{11} & A_{21}B_{12} & 0 & 0 \\ A_{21}B_{21} & A_{21}B_{22} & 0 & 0 \end{pmatrix}\times$$
$$\begin{pmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{12}b_{11} & a_{12}b_{12} \\ a_{11}b_{21} & a_{11}b_{22} & a_{12}b_{21} & a_{12}b_{22} \\ a_{21}b_{11} & a_{21}b_{12} & a_{22}b_{11} & a_{22}b_{12} \\ a_{21}b_{21} & a_{21}b_{22} & a_{22}b_{21} & a_{22}b_{22} \end{pmatrix}\times$$
$$\begin{pmatrix} A_{11}B_{11} & A_{11}B_{12} & A_{12}B_{11} & A_{12}B_{12} \\ A_{11}B_{21} & A_{11}B_{22} & A_{12}B_{21} & A_{12}B_{22} \\ A_{21}B_{11} & A_{21}B_{12} & 0 & 0 \\ A_{21}B_{21} & A_{21}B_{22} & 0 & 0 \end{pmatrix}^{\dagger}$$

Then the partial trace to obtain system B after (respectively) zeroing the A_{11} and A_{22} elements in A is:

$$\mathit{CASE}\ \mathrm{A}_{11}=0$$

$$\rho'_{AB}=\begin{pmatrix} a'''_{11}b'''_{11} & a'''_{11}b'''_{12} & a''_{12}b''_{11} & a''_{12}b''_{12} \\ a'''_{11}b'''_{21} & a'''_{11}b'''_{22} & a''_{12}b''_{21} & a''_{12}b''_{22} \\ a'_{21}b'_{11} & a'_{21}b'_{12} & a'_{22}b'_{11} & a'_{22}b'_{12} \\ a'_{21}b'_{21} & a'_{21}b'_{22} & a'_{22}b'_{21} & a'_{22}b'_{22} \end{pmatrix}$$

OR

$$\mathit{CASE}\ \mathrm{A}_{22}=0$$

$$\rho'_{AB}=\begin{pmatrix} a'_{11}b'_{11} & a'_{11}b'_{12} & a'_{12}b'_{11} & a'_{12}b'_{12} \\ a'_{11}b'_{21} & a'_{11}b'_{22} & a'_{12}b'_{21} & a'_{12}b'_{22} \\ a''_{21}b''_{11} & a''_{21}b''_{12} & a'''_{22}b'''_{11} & a'''_{22}b'''_{12} \\ a''_{21}b''_{21} & a''_{21}b''_{22} & a'''_{22}b'''_{21} & a'''_{22}b'''_{22} \end{pmatrix}$$

The apostrophes denote, firstly: that the density matrix has transformed, secondly: that a row suffered the effect of multiplication with a row containing zeros and thirdly: that it suffered two such multiplications.

A non-unitary process (a measurement) happened on system A and so we trace out system A to see the effect on system B:

$$\mathit{CASE}\ \mathrm{A}_{11}=0$$

$$\rho'_{B}=\begin{pmatrix} a'''_{11}b'''_{11}+a'_{22}b'_{11} & a'''_{11}b'''_{12}+a'_{22}b'_{12} \\ a'''_{11}b'''_{21}+a'_{22}b'_{21} & a'''_{11}b'''_{22}+a'_{22}b'_{22} \end{pmatrix}$$

or

$$\mathit{CASE}\ \mathrm{A}_{22}=0$$

$$\rho'_{B}=\begin{pmatrix} a'_{11}b'_{11}+a'''_{22}b'''_{11} & a'_{11}b'_{12}+a'''_{22}b'''_{12} \\ a'_{11}b'_{21}+a'''_{22}b'''_{21} & a'_{11}b'_{22}+a'''_{22}b'''_{22} \end{pmatrix}$$

eqn. 21

Compare this (eqn. 21) with eqn. 20. So system B has been affected by system A's measurement which can force it into a mixed state before B's measurement.

References

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