Comparative Studies of Law of Gravity and General Relativity

—No.1 of Comparative Physics Series Papers

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Abstract: As No.1 of comparative physics series papers, this paper discusses the same points of law of gravity (including improved law of gravity) and general relativity: they are all the effective theories of gravitational interaction, and can be used to solve the problems of advance of planetary perihelion and deflection of photon around the Sun; and the different points of them: general relativity can conform to general relativity principle while law of gravity cannot conform to this principle, and law of gravity can solve the constraint problems in gravitational field (such as a small ball rolls along the inclined plane) while general relativity cannot solve these problems. These two theories can also overcome their own weak points by learning from each other's strong points, and jointly present the new explanation that the advance of planetary perihelion is the combined result of two motions: the first elliptical motion decided by law of gravity creates the perihelion, and the second vortex motion creates the advance of perihelion in which the advance parameters can be decided by general relativity. The common defect of these two theories is that they cannot take into account the principle of conservation of energy, comparing with the result that applying the principle of conservation of energy to derive Newton's second law and law of gravity, the further topic is applying the principle of conservation of energy to derive the related formulae and equations of general relativity.

Key words: Comparative physics, law of gravity, improved law of gravity, general relativity, same point, different point, principle of conservation of energy

Introduction

In reference [1], the concept of comparative physics is proposed. As one of the series papers of comparative physics, this paper discusses the comparative studies of law of gravity and general relativity. Mainly include: two theories' same points and different points; two theories' joint application; and two theories' common defect and the way of improvement.

1 Same points of law of gravity (including improved law of gravity) and general relativity

The first same point: they are all the effective theories of gravitational interaction.

Law of gravity is the first effective theory of gravitational interaction expressed by mathematical formula. It reveals the rules of the motions of celestial bodies, and it is widely used in the fields of astronomy and astrodynamics. The discoveries of Comet Halley, Neptune and Pluto in the history of science are all the examples of the successful applications of law of gravity. Newton also explains that the tide phenomenon is caused by gravitational pull of the Moon and the Sun. For most of the phenomena of gravitation, law of gravity is sufficiently accurate.

Many scholars believe that general relativity is the more accurate effective theory of gravitational interaction than law of gravity; however after a series of improvements to law of gravity, this claim has been questioned more and more; but in any case, general relativity has had a substantial impact in history.

The second same point: all of them can be used to solve the problems of advance of planetary perihelion and deflection of photon around the Sun.

The initial two experimental tests and verifies of general relativity are the problems of advance of planetary perihelion and deflection of photon around the Sun. Especially during the total solar eclipse of May 29, 1919, the measured deflection of light is very good agreed with the theoretical prediction of general relativity, for this reason, there had been the sensational effect in the world.

Many scholars also believe that these two problems cannot be solved with law of gravity, however, the real fact is that both problems can also be solved with law of gravity.

As well-known, by means of law of gravity, the field equations of general relativity can be derived. Similarly, by means of general relativity, the improved law of gravity can also be derived.

In references [2-4], jointly applying law of gravity and general relativity, the improved formula of universal gravitation can be derived, which may be rewritten as follows.

As discussing the problem of planet's movement around the sun according to the general relativity, the following equation can be given

$$u''+u=\frac{1}{p}+\frac{3GMu^2}{c^2} \qquad (1)$$

where, $u=\frac{1}{r}$; G – gravitational constant; M – mass of the Sun; c – velocity of light; p - half normal focal chord.

Due to the central force, the orbit differential equation (Binet's formula) reads

$$h^2u^2(u''+u)=-\frac{F}{m} \qquad (2)$$

where, h^2 is a constant.

Substituting Eq.(1) into Eq.(2), we have

$$F=-mh^2u^2\left(\frac{1}{p}+\frac{3GMu^2}{c^2}\right) \qquad (3)$$

The original law of gravity reads

$$F=-\frac{GMm}{r^2}=-GMmu^2 \qquad (4)$$

For Eq.(3) and Eq.(4), comparing the terms including u^2, we have

$$h^2=GMp$$

Substituting h^2 into Eq.(3), it gives

$$F=-GMmu^2-\frac{3G^2M^2mpu^4}{c^2} \qquad (5)$$

Substituting $u = \frac{1}{r}$ into Eq.(5), the improved law of gravity reads

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2mp}{c^2r^4} \qquad (6)$$

In reference [5], by using this improved formula, the classical mechanics can be used to solve the problem of advance of planetary perihelion and the problem of gravitational deflection of photon around the Sun, while these solutions are the same as given by general relativity.

2 Different points of law of gravity (including improved law of gravity) and general relativity

The first different point: general relativity can conform to general relativity principle while law of gravity cannot conform to this principle.

In fact, for law of gravity, mathematics is only a tool, rather than the starting point and basic principle; while for general relativity, mathematics is not only a tool, but also the starting point and basic principle.

The second different point: law of gravity can solve the constraint problems in gravitational field (such as a small ball rolls along the inclined plane) while general relativity cannot solve these problems.

The ordinary scholars agree that all the problems that can be solved by law of gravity, can also be solved by general relativity; while, some problems that can be solved by general relativity, may not be solved by law of gravity. However, this is not the real case, some problems that can be solved by law of gravity, may not be solved by general relativity. The reason for this is that general relativity can only solve the free-particle motion in a gravitational field, so general relativity cannot solve the constraint problems in gravitational field, and at present these problems can only be solved by using law of gravity or improved law of gravity.

In reference [4, 5], for the example of a small ball rolls along the inclined plane, the improved law of gravity and improved Newton's second law are derived with principle of conservation of energy.

The results suitable for this example with the constant dimension fractal form is as follows.

The improved law of gravity reads

$$F = -\frac{GMm}{r^{1.99989}}$$

The improved Newton's second law reads

$$F = ma^{1.01458}$$

The results suitable for this example with the variable dimension fractal form are as follows.

Supposing that the improved Newton's second law and the improved law of gravity with the form of variable dimension fractal can be written as follows: $F = ma^{1+\varepsilon}$, $\varepsilon = k_1 u$;

$F = -GMm/r^{2-\delta}$, $\delta = k_2 u$; where: u is the horizon distance that the small ball rolls ($u = x + H$) .

After the values of k_1, k_2 are determined, the results are as follows

$$\varepsilon = 8.85 \times 10^{-8} u \text{ , } \delta = 2.71 \times 10^{-13} u$$

The results of variable dimension fractal are much better than that of constant dimension fractal.

3 Jointly present the new explanation of the advance of planetary perihelion with these two theories

After comparing we can find that these two theories can also overcome their own weak points by learning from each other's strong points, and jointly present the new explanation that the advance of planetary perihelion is the combined result of two motions.

This problem has been basically solved in reference [6], and it can be rewritten as follows.

Many scholars believe that general relativity does not end the studying for problem of advance of planetary perihelion, because there are many factors affecting the advance of planetary perihelion, therefore it still needs to continue to study this issue.

Although the explanation of general relativity for the advance of planetary perihelion is reasonably consistent with the observed data, because its orbit is not closed, whether or not it is consistent with the law of conservation of energy has not been verified. For this reason, jointly applying law of gravity and general relativity, a new explanation is presented: The advance of planetary perihelion is the combined result of two motions. The first elliptical motion creates the perihelion, and the second vortex motion creates the advance of perihelion. In the motion of planet-sun system, under the action of gravity, the planetary orbit is a closed ellipse, and consistent with the law of conservation of energy. Meanwhile, the planet also participates in the vortex motion of solar system taking the Sun as center; the long-term trend of the vortex is the further topic, but in the short-term may be considered that due to the inertia the planetary perihelion will run circular motion in vortex and lead to the advance of perihelion, thus also without acting against the law of conservation of energy. Based on the result of general relativity, the approximate angular velocity of advance of perihelion is given.

According to general relativity, the value of advance of planetary perihelion reads

$$\varepsilon = \frac{24\pi^3 a^2}{T^2 c^2 (1-e^2)} \qquad (7)$$

where: c is the speed of light; T, a, and e are orbital period, semi-major axis and eccentricity respectively.

According to Eq.(7), taking the Sun as center, the angular velocity of advance of planetary perihelion is as follows

$$\omega = \frac{\varepsilon}{T} = \frac{24\pi^3 a^2}{T^3 c^2 (1-e^2)} \qquad (8)$$

According to Kepler's third law, it gives

$$\frac{T^2}{a^3}=\frac{4\pi^2}{GM}$$

where: G is the gravitational constant, and M is the solar mass.

Then Eq. (8) can be rewritten as

$$\omega=\frac{3G^{3/2}M^{3/2}}{a^{5/2}c^2(1-e^2)} \qquad (9)$$

According to this expression we can see that, the angular velocity of advance of planetary perihelion is inversely proportional to $a^{5/2}$, and the velocity of advance of planetary perihelion is inversely proportional to $a^{3/2}$.

For the results of Eq.(7), there are small differences compared with accurate astronomical observations, so we say that results of Eq.(8) and Eq.(9) are the approximate angular velocities of advance of perihelion based on the related results of general relativity.

Now the rotate transformation in Cartesian coordinate system is applied to derive the planetary orbit equation including the advance of perihelion.

In the planet-sun system, taking the solar center as the origin of coordinate, the planetary orbit equation reads

$$\frac{(x-k)^2}{a^2}+\frac{y^2}{b^2}=1$$

where: k is the semi-focal length of ellipse.

According to the rotate transformation in Cartesian coordinate system, it gives

$$x=x'\cos\theta-y'\sin\theta$$

$$y=x'\sin\theta+y'\cos\theta$$

where: θ is the angle of rotation (namely the angle of advance), $\theta=\omega t$.

Thus, after considering the vortex motion, the planetary rotation orbit equation is as follows

$$\frac{(x'\cos\theta-y'\sin\theta-k)^2}{a^2}+\frac{(x'\sin\theta+y'\cos\theta)^2}{b^2}=1$$

4 The common defect of these two theories

Principle (law) of conservation of energy is the most important principle (law) in physics.

After comparing we can find that the common defect of these two theories is that they cannot take into account the principle of conservation of energy, while many scholars have not paid attention to this defect.

The way to eliminate this common defect is to derive law of gravity and related formulas and equations of the general relativity with the principle of conservation of energy.

The comparing with the result that applying the principle of conservation of energy to derive Newton's second law and law of gravity, the further topic is applying the principle of conservation of energy to derive the related formulae and equations of general relativity.

Now we derive the original Newton's second law and law of gravity.

Above all, in this section only Newton's second law can be derived, but we have to apply law of gravity at the same time, so we present the general forms of Newton's second law and law of gravity contained undetermined constants firstly.

Assuming that for law of gravity, the related exponent is unknown, and we only know the form of this formula is as follows

$$F = -\frac{GMm}{r^D}$$

where: D is an undetermined constant, in the next section we will derive that its value is equal to 2.

Similarly, assuming that for Newton's second law, the related exponent is also unknown, and we only know the form of this formula is as follows

$$F = ma^{D'}$$

where: D' is an undetermined constant, in this section we will derive that its value is equal to 1.

As shown in Figure 1, supposing that circle O' denotes the Earth, M denotes its mass; m denotes the mass of the small ball (treated as a mass point P), A O' is a plumb line, and coordinate y is parallel to AO'. The length of AC is equal to H, and O'C equals the radius R of the Earth.

We also assume that it does not take into account the motion of the Earth and only considering the free falling of the small ball in the gravitational field of the Earth (from point A to point C).

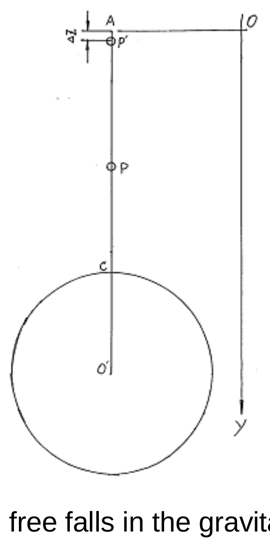

Figure 1 A small ball free falls in the gravitational field of the Earth

For this example, the value of v_{P}^2 which is the square of the velocity for the small ball located at point P will be investigated. To distinguish the quantities calculated by different methods, we denote the value given by law of gravity and Newton's second law as v_{P}^2, while v'^2_{P} denotes the value given by law of conservation of energy.

Now we calculate the related quantities according to law of conservation of energy.

From law of gravity contained undetermined constant, the potential energy of the small ball located at point P is as follows

$$V = -\frac{GMm}{(D-1)r_{O'P}^{D-1}}$$

According to law of conservation of energy, we can get

$$-\frac{GMm}{(D-1)r_{O'A}^{D-1}} = \frac{1}{2}mv'^2_P - \frac{GMm}{(D-1)r_{O'P}^{D-1}}$$

And therefore

$$v'^2_P = \frac{2GM}{D-1}[\frac{1}{r_{O'P}^{D-1}} - \frac{1}{(R+H)^{D-1}}]$$

Now we calculate the related quantities according to law of gravity and Newton's second law.

For the small ball located at any point P, we have

$$dv/dt = a$$

We also have

$$dt = \frac{dy}{v}$$

Therefore

$$vdv = ady$$

According to law of gravity contained undetermined constant, along the plumb direction, the force acted on the small ball is as follows

$$F_a = \frac{GMm}{r_{O'P}^D}$$

From Newton's second law contained undetermined constant , it gives

$$a = (\frac{F_a}{m})^{1/D'} = (\frac{GM}{r_{O'P}^D})^{1/D'}$$

Then we have

$$vdv = \{\frac{GM}{(R+H-y)^D}\}^{1/D'} dy$$

For the two sides of this expression, we run the integral operation from A to P, it gives

$$v_P^2 = 2(GM)^{1/D'} \int_0^{y_p} (R+H-y)^{-D/D'} dy$$

$$v_P^2 = 2(GM)^{1/D'} \{ -\frac{1}{1-D/D'} [(R+H-y)^{1-D/D'}] \Big|_0^{y_p} \}$$

$$v_P^2 = \frac{2(GM)^{1/D'}}{(D/D')-1} [\frac{1}{r_{O'P}^{(D/D')-1}} - \frac{1}{(R+H)^{(D/D')-1}}]$$

Let $v_P^2 = v_P'^2$, then we should have: $1 = 1/D'$, and $D-1 = (D/D')-1$; these two equations all give: $D' = 1$, this means that for free falling problem, by using law of conservation of energy, we strictly derive the original Newton's second law $F = ma$.

Here, although the original law of gravity cannot be derived (the value of D may be any constant, certainly including the case that D=2), we already prove that the original law of gravity is not contradicted to the law of conservation of energy.

Now, we derive the original law of gravity by using law of conservation of energy.

In order to really derive the original law of gravity for the example of free falling problem, we should consider the case that a small ball free falls from point A to point P' (point P' is also shown in Figure1) through a very short distance ΔZ (the two endpoints of the interval ΔZ are point A and point P').

As deriving the original Newton's second law, we already reach

$$v'^2_{P'} = \frac{2GM}{D-1} [\frac{1}{(R+H-\Delta Z)^{D-1}} - \frac{1}{(R+H)^{D-1}}]$$

where: $R+H-\Delta Z = r_{O'P'}$

For the reason that the distance of ΔZ is very short, and in this interval the gravity can be considered as a linear function, therefore the work W of gravity in this interval can be written as follows

$$W = F_{av} \Delta Z = \frac{GMm}{(R+H-\frac{1}{2}\Delta Z)^D} \Delta Z$$

where, F_{av} is the average value of gravity in this interval ΔZ, namely the value of gravity for the midpoint of interval ΔZ.

Omitting the second order term of ΔZ ($\frac{1}{4}(\Delta Z)^2$), it gives

$$W = \frac{GMm\Delta Z}{(R^2+H^2+2RH-R\Delta Z-H\Delta Z)^{D/2}}$$

As the small ball free falls from point A to point P', its kinetic energy is as follows

$$\frac{1}{2} m v'^2_{P'} = \frac{GMm}{D-1} [\frac{(R+H)^{D-1} - (R+H-\Delta Z)^{D-1}}{(R^2+H^2+2RH-R\Delta Z-H\Delta Z)^{D-1}}]$$

According to law of conservation of energy, we have

$$W = \frac{1}{2} m v'^2_{P'}$$

Substituting the related quantities into the above expression, it gives

$$\frac{GMm}{D-1}\left[\frac{(R+H)^{D-1} - (R+H-\Delta Z)^{D-1}}{(R^2+H^2+2RH-R\Delta Z-H\Delta Z)^{D-1}}\right]$$

$$= \frac{GMm\Delta Z}{(R^2+H^2+2RH-R\Delta Z-H\Delta Z)^{D/2}}$$

To compare the related terms, we can reach the following three equations

$$D-1=1$$

$$D/2=D-1$$

$$\Delta Z = (R+H)^{D-1} - (R+H-\Delta Z)^{D-1}$$

All of these three equations will give the following result

$$D=2$$

Thus, we already derive the original law of gravity by using principle of conservation of energy.

Comparing with the result that applying the principle of conservation of energy to derive Newton's second law and law of gravity, the further topic is applying the principle of conservation of energy to derive the related formulae and equations of general relativity.

5 Conclusions

In comparative physics, based on the comparative method, we can discuss the same points and different points of different physical laws; and then discuss how different physics laws can learn from each other; and the common defet of some physics laws can be eliminated with the effective way of comparative method. Thus, it seems that the comparative physics will have good development prospects.

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