TOWARDS A MONSTER GROUP ENCOMPASSING THE UNIVERSE

Arturo Tozzi (Corresponding Author)
Computational Intelligence Laboratory, University of Manitoba, Winnipeg, Canada
Winnipeg R3T 5V6 Manitoba
tozziarturo@libero.it

James F. Peters
Department of Electrical and Computer Engineering, University of Manitoba
75A Chancellor's Circle
Winnipeg, MB R3T 5V6 Canada and
Mathematics Department, Adıyaman University 02040 Adıyaman, Turkey
james.peters3@umanitoba.ca

ABSTRACT
The Monster group, the biggest of the sporadic groups, is equipped with the highest known number of dimensions and symmetries. Taking into account variants of the Borsuk-Ulam theorem and a novel topological approach cast in a physical fashion that has the potential to be operationalized, the Universe can be conceived as a lower-dimensional manifold encompassed in the Monster group. Our Universe might arise from spontaneous dimension decrease and symmetry breaking that occur inside the very structure of the Monster Module. We elucidate how the energetic loss caused by projection from higher to lower dimensions and by the Monster group's non-abelian features is correlated with the present-day asymmetry in thermodynamic arrow. By linking the Monster Module to theoretical physical counterparts, we are allowed to calculate its enthalpy and Lie group trajectories. Our approach also reveals how a symmetry break might lead to a Universe based on multi-dimensional string theories and CFT/AdS correspondence.

The Mode is an enclosed, detectable manifestation of the Substance… The Substance is equipped with infinite attributes (Spinoza, Ethica, pars I)

INTRODUCTION

The Fischer-Griess Monster group, the largest among the twenty-six sporadic groups, is equipped with 196,883 dimension and an order of about 10^{54} elements (Conway et al., 1986). It is noteworthy that the Monster Module displays the highest known number of symmetries (du Sautoy). It has been recently proposed that the symmetries, widespread invariances occurring at every level of organization in our Universe, may be regarded as the most general feature of physical systems, perhaps also more general than thermodynamic constraints (Tozzi and Peters, 2016a, Roldán 2014). Therefore, giving insights into the Monster symmetries would provide a very general approach to systems function, Universe evolution and energetic dynamics. Here we show how a novel symmetry-based, topological approach sheds new light on Monster's features. We provide a foundation for the Monster's physical counterparts, cast in a fashion that has the potential to be operationalized, which can be used for the assessment of our Universe's evolution and, in particular, pre-big bang scenarios.
This paper comprises four sections. In the first section, we describe a generalized version of the Borsuk-Ulam theorem, in order to provide the topological machinery for further evaluations of the Monster in the context of theoretical physics. Section two explains how the Universe might originate from the Monster Module, due to a dimension loss, linking the Monster group to theoretical physics counterparts. Furthermore, taking into account energetic arguments dictated by topological dimension decrease, the section explains why and how our Universe is equipped with the symmetry breaks which give rise to the thermodynamic arrow. Section three elucidates various physical features of the Monster. In the final section, we raise a number of still open questions.

1) TOPOLOGICAL TOOLS

The *standard* version of the Borsuk-Ulam theorem (BUT).

The Borsuk-Ulam Theorem (Borsuk 1933) states that:

Every continuous map $f : S^n \to R^n$ *must identify a pair of antipodal points (on* S^n *).*

This means that the sphere S^n maps to the Euclidean space R^n, which stands for an *n*-dimensional Euclidean space (Beyer; Matoušek). See Tozzi and Peters (2016b) for further details. The original formulation of BUT displays four versatile ingredients which can be modified, resulting in different guises: a continuous function, two antipodal (or non-antipodal) points with matching description, an *n*-sphere equipped with different *n* values and, last but not the least, a mapping from a higher to a lower dimension that is invertible.

BUT variants. We resume some BUT variants described by Peters (2016) and Tozzi and Peters (2016a). The concept of antipodal points can be generalized to countless types of signals. Two opposite points encompass not just the description of simple topological points, but also of spatial and temporal patterns, vectors and tensors, functions, signals, thermodynamical parameters, trajectories, symmetries (Peters and Tozzi, 2016a). The two antipodal points standing for different systems features are assessed at one level of observation, while the single point is assessed at a lower level. The antipodal points restriction from the *classical* BUT is no longer needed, because the applications on an *n*-sphere can be generalized not just for the evaluation of diametrally opposite points, but also of non-antipodal ones. We are allowed to take into account homotopic regions on an *n*-sphere that are either adjacent or far apart. This means that the points (or regions) with the same feature value do not need necessarily to be antipodal, in order to be described together (Peters 2016). The original formulation of BUT describes the presence of antipodal points on spatial manifolds in every dimension, provided the n-sphere is a convex, positive-curvature structure. However, many physical functions occur on manifolds endowed with other types of geometry: for example, the hyperbolic one (Watanabe; Sengupta et al., 2016). Whether the manifold displays a concave, convex or flat activity, it does not count: we may always find the points with matching description predicted by BUT. Although BUT has been originally described just in case of n being a natural number which expresses a spatial dimension, its value in S^n can also stand for other types of numbers. The n value can be also cast as an integer, a rational or an irrational number. It allows us to use the *n* parameter as a versatile tool for the description of systems symmetries (Tozzi and Peters, 2016a). A BUT variant tells us that we can find a pair of opposite points an *n*-dimensional sphere, that display the same encoding not just on a R^n manifold, but also on an *n*-1 sphere. A symmetry break occurs when the symmetry is present at one level of observation, but *hidden* at another level (Roldàn). This means that symmetries can be found when evaluating the system in a proper dimension, while they disappear (are hidden or broken) when the same system is embedded in just one dimension lower.

Here we introduce recently developed, unpublished BUT variants. The first is a BUT corollary, which states that a S^n manifold does not map just to a R^{n-1} Euclidean space, but straight to a S^{n-1} manifold. In other words, the Euclidean space is not mentioned in this formulation. Indeed, in many applications, *e.g.*, in fractal systems, we do not need a Euclidean manifold at all. A manifold, in this case S^n, may exist in - and on – itself, by an intrinsic, *internal* point of view, and does not need to be embedded in any dimensional space (Weeks). Therefore, we do not need a S^n manifold curving into a dimensional space R^n: we may think that the manifold just does exist by itself. An important consequence of this BUT version is that a n-sphere may map on itself. The mapping of two antipodal points to a single point in a dimension lower can be a projection internal to the same n-sphere.

The second and foremost variant is termed **Energy-BUT.** There exists a physical link between the abstract concept of BUT and the real energetic features of systems formed by two spheres S^n and S^{n-1}. An *n*-sphere S^n is equipped with two antipodal points, standing for symmetries according to BUT. When these opposite points map to a *n*-Euclidean manifold where S^{n-1} lies, a symmetry break/dimensionality reduction occurs, and a single point is achieved (Peters and Tozzi, 2016b). It is widely recognized that a decrease in symmetry goes together with a reduction in entropy and free-energy (in a closed system). It means that the single mapping function on S^{n-1}displays energy parameters lower than the sum of the two corresponding antipodal functions on S^n. Therefore, a decrease in dimensions gives rise to a decrease of energy and energy requirements. BUT and its variants become physical quantities, because we achieve a system in which the energetic changes do not depend anymore on thermodynamic parameters, rather on topological features such as affine connections and homotopies. The energy-BUT concerns not just energy, but also information. Indeed, two antipodal points contain more information than their single projection in a lower dimension. Dropping down a dimension means each point in the lower dimensional space is simpler, because each point has one less coordinate. In sum, energy-BUT provides a way to evaluate the decrease of energy in topological, other than thermodynamical, terms.

Another novel variant of BUT is the string-based BUT (briefly, **strBUT)**. The usual continuous function required by reBUT (region-based BUT in Peters and Tozzi, 2016a) is replaced by a proximally continous function, which guarantees that, whenever a pair of strings (regions that are world lines) are close (near enough to have common elements), then we always know that their mappings will be also be close. A string is a region of space with zero width and either bounded or unbounded length. As a particle moves through space following a world line (Olive and Landsberg, 1989).

Interactions occur at the junctions of world lines. Let τ the proper time of a particle, measured by clock travelling with a particle and integration along the world line of the particle. The $action_{particle}$ of a freely moving particle is defined by $action_{particle} = -mc^2 \int d\tau$.

As time evolves, a particle leaves a trace of its movements along a surface that are "remembered". A string is then a remembered parts of a hypersphere surface over which a particle travels. In terms of quantum theory, a string is a path defined by a moving particle. Put another way, a string is path-connected and its path is defined by a sequence of adjacent *fat* surface points. The points are *fat* because they are physical as opposed to abstract geometric points. In other words, a string A (briefly, $\mathrm{str}A$) is a thin region of space that has describable features such as connectedness, length, open-ended or closed-ended, and shape. Strings $\mathrm{str}A, \neg\mathrm{strA}$ are antipodal, provided $\mathrm{str}A$ and $\neg\mathrm{strA}$ are disjoint and yet have the same description. Strings $\mathrm{str}A, \neg\mathrm{strA}$ are examples of antipodal sets (Petty, 1971). The description of $\mathrm{str}A$ (briefly, $\Phi(\mathrm{str}A)$) is a feature vector in Rn, where that each component of $\Phi(\mathrm{str}A)$ is a feature value of $\mathrm{str}A$.

Quantum String Axioms

1. Every string has an action.
2. If $\mathrm{str}A, \neg\mathrm{strA}$ are antipodal, then $\mathrm{action}_{\mathrm{str}A} = \mathrm{action}_{\neg\mathrm{str}A}$.
3. Separate strings with *k* features with the same description are antipodal.
4. There is a set $\{\neg\mathrm{strA}\}$ of antipodal strings for every string $\mathrm{str}A$.

Let X be a topological space equipped with descriptive proximity δ_Φ. $\mathrm{str}A\ \delta_\Phi\ \neg\mathrm{strA}$ reads $\mathrm{str}A$ and $\neg\mathrm{strA}$ have the same description. Let 2^{S^n} denote the family of sets on the surface of a hypersphere S^n and $\mathrm{str}A, \neg\mathrm{strA} \in 2^{S^n}$ are antidodal strings on S^n. A function $f: 2^{S^n} \to R^n$ is proximally continuous, provided $\mathrm{str}A\ \delta_\Phi\ \neg\mathrm{strA}$ implies $\mathrm{f}(\mathrm{str}A)\ \delta_\Phi\ \mathrm{f}(\neg\mathrm{strA})$. With these observations about strings, we obtain the following results.

Lemma [strBUT]. If $f: 2^{S^n} \to R^n$ is proximally continuous, $\mathrm{f}(\mathrm{str}A) = \mathrm{f}(\neg\mathrm{strA})$ for some $\mathrm{str}A$ in 2^{S^n}.

Proof. Case n = 1. Let each $\mathrm{str}A$ have 1 feature, namely, action. Assume antipodal strings $\mathrm{str}A, \neg\mathrm{strA}$ with n features are descriptively close, i.e., $\mathrm{str}A\ \delta_\Phi\ \neg\mathrm{strA}$. Since *f* is proximally continuous, we have $\mathrm{f}(\mathrm{str}A)\ \delta_\Phi\ \mathrm{f}(\neg\mathrm{strA})$. From Axiom 2, $\mathrm{action}_{\mathrm{str}A} = \mathrm{action}_{\neg\mathrm{str}A}$. Hence, from the definition of the descriptive proximity δ_Φ, $\mathrm{f}(\mathrm{str}A) = \mathrm{f}(\neg\mathrm{strA})$.

Case n > 1. The proof is symmetric with case n = 1 and Axiom 3. □

Theorem 1. If $f: 2^{S^n} \to R^k,\ k > 0$ is proximally continuous, $\mathrm{action}_{\mathrm{str}A} = \mathrm{action}_{\neg\mathrm{str}A}$ for some $\mathrm{str}A$ in 2^{S^n}.

Proof. We consider only the case for *k* = 1, for strings whose only feature is action. The desired result is immediate from the strBUT Lemma and Axiom 2. This result is easily extend to the case where *k* > 1 for strings with *k* features. □

Theorem 2. If $f: 2^{S^n} \to 2^{R^k},\ k > 0$ is proximally continuous, $f(A) = f(\neg A)$ for each $\neg\mathrm{strA}$ in the set of antipodes $\{\neg\mathrm{strA}\} \in 2^{S^n}$.

Proof. Immediate from the Theorem 1 and Axiom 4. □

In order to map S^n to $S^{\{n-1\}}$, we need to work with lower dimensional spaces containing regions where each point in $S^{\{n-1\}}$ has one less coordinate that a point in S^n.

Let X be a topological space equipped with Lodato proximity δ (Peters, 2016). $\mathrm{str}A\ \delta\ \neg\mathrm{strA}$ reads $\mathrm{str}A$ and $\neg\mathrm{strA}$ are close. Dochviri and Peters (2016) introduce a natural approach in the evaluation of the nearness of sets in topological spaces. The objective is to classify levels of nearness of sets relative to each given set. The main result is a proximity measure of nearness for disjoint sets in an extremely disconnected topological space. Let $\mathrm{int}(\mathrm{str}A)$ be the set of points in the interior of $\mathrm{str}A$. Another result is that if strings $\mathrm{str}A, \neg\mathrm{strA}$ are nonempty semi-open sets such that $\mathrm{str}A\ \delta\ \neg\mathrm{strA}$, then $\mathrm{int}(\mathrm{str}A)\ \delta\ \mathrm{int}(\neg\mathrm{strA})$.

An important feature is that the manifolds M^d and M^{d-1} are topological spaces equipped with a strong descriptive proximity relation. Recall that in a topological space M, every subset in M and M itself are open sets. A set *E* in *M* is open, provided all points sufficiently near *E* belong to *E* (Bourbaki, 1966). The description-based functions in genBUT are strongly proximally continuous and their domain can be mathematical, physical or biological features of world line shapes. Let *A*,*B* be subsets in the family of sets in *M* (denoted by 2^M) and let $f: 2^M \to R^n, A \in 2^M, f(A) =$ a feature vector that describes *A*. That is, $f(A), f(B)$ are descriptions of *A* and *B*. Nonempty sets are *strongly near*, provided the sets of have elements in common. The function *f* is strongly proximally continuous, provided A strongly near B implies $f(A)$ is strongly near $f(B)$. This means that strongly near sets have nonempty intersection. From a genBUT perspective, multiple sets of objects in M^d are mapped to $f\left(A \cap B\right),$ which is a description of those objects common to *A* and *B*. In other words, the functions in genBUT are set-based embedded in a strong proximity space. In particular, each set is set of contiguous points in a path traced by a moving particle. The path is called a world line. Pairs of world lines have squiggly, twisted shapes opposite each other on the surface of a manifold. Unlike the antipodes in a conventional hypersphere assumed by the BUT, the antipodes are now sets of world lines that are discrete and extremely disconnected. Sets are extremely disconnected, provided the closure of every set is an open set (Dochviri and Peters, 2016), is in the discrete space and the intersection of the closure of the intersection of every pair of antipodes is empty. The shapes of the antipodes are separated and belong to a computational geometry. That is, the shapes of the antipodal world lines approximate the shapes in conventional homotopy theory (Borsuk, 1969). The focus here is on the descriptions (sets of features) of world line shapes. Mappings onsets with matching description, or, in other words, mappings on descriptively strongly proximal sets, here means that such mappings preserve the nearness of pairs of sets. The assumption made here is that antipodal sets live in a descriptive Lodato proximity (DLP) space. Therefore, antipodal sets satisfy the requirements for a DLP (Peters, 2016). Let δ be a DLP and write $A\ \delta\ B$ to denote the descriptive nearness of antipodes *A* and *B*. And let f be a DLP continuous function. This means $A\ \delta\ B$ implies $\mathrm{f}(A)\ \delta\ \mathrm{f}(B) = \mathrm{f}(A) \cap \mathrm{f}(B) \neq \varnothing.$

Example: Assume that antipodes A and B have symmetries (shape, bipolar, colour, overlap, path-connectedness), and f is DLP strongly continuous function, then $A\ \delta\ B \Rightarrow \mathrm{f}(A)\ \delta\ \mathrm{f}(B)$

This means that, whenever A and B are descriptively close, then A is mapped to $f(A)$ and B is mapped to $f(B)$ and $\mathrm{f}(A)\ \delta\ \mathrm{f}(B).$ If we include in the description of A and B the location of the discrete points in A and B, then the DLP mapping is invertible. That is, $f(A)$ maps to *A*, $f(B)$ maps to *B*and $f(A)\ \delta\ f(B)$ implies $A\ \delta\ B.$

Figure 1 provides an example of antipodal sets in case of a pair of closed regions, e.g., strings.

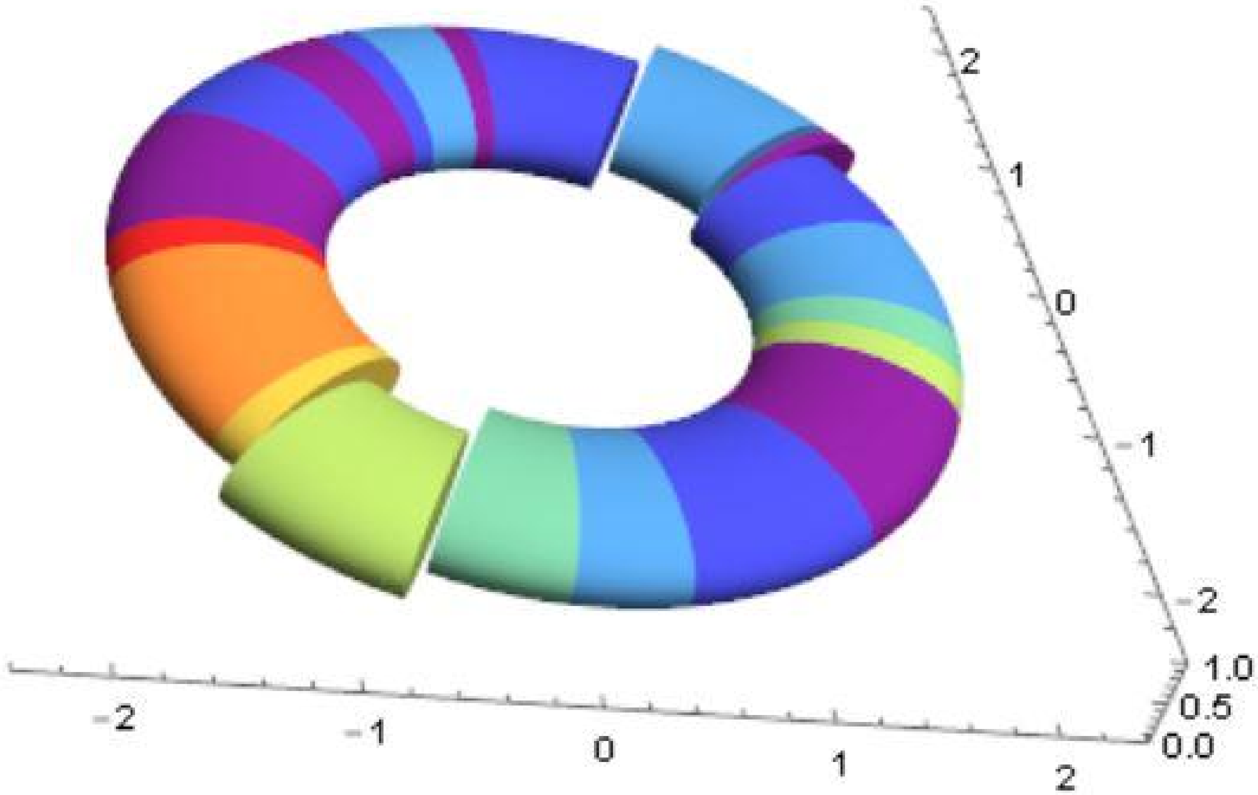

Figure 1. Torus Antipodal Strings. World lines with matching description preserve the nearness of pairs of sets. See text for further details.

Generalized BUT (genBUT). We conclude this section by introducing a novel, generalized version of BUT, which encompasses all the previously described variants. This version allows the study of the Monster in the context of theoretical physics. Gen-BUT states that:

Multiple sets of objects with matching descriptions in a d-dimensional manifold M^d are mapped to a single set of objects in M^{d-1} and vice versa. The sets of objects, which can be mathematical, physical or biological features, do not need to be antipodal and their mappings need not to be continuous. The term *matching description* means the sets of objects display common feature values or symmetries.

M stands for a manifold with any kind of curvature, either concave, convex or flat. M^{d-1} may also be a part of M^d. The projection from S to R in not anymore required, just M is required. The notation *d* stands for a natural, or rational, or irrational number. This means that the need for spatial dimensions of the classical BUT is no longer required. The process is reversible, depending on energetic constraints. Note that a force, or a group, an operator, an energetic source, is required, in order to project from one dimension to another.

2) EMBEDDING THE MONSTER GROUP IN M^{d-1}

The Monstrous Moonshine conjecture suggests a puzzling relationship between the Fourier coefficients of the normalized elliptic modular invariant, e.g., the hauptmodul J, which value is 19884, and the simple sums of dimensions of irreducible representation of the Monster group M, which is 196883 (Frenkel et al, 1984). It would seem that a relationship between the symmetries in the plot (range) of the j-function and symmetries in the Monster group products occurs. We might speculate that, in physical terms, the j-function could stand for an activity occurring into to the Monster Module during the movements of the Lie Monster Group. In an infinite-dimensional space, the action of the J function is correlated with the the Monster vertex operator Virasoro algebra, e.g., the Monster Module (Duncan et al., 2015).

A topological approach helps to elucidate such an unusual relationship. In the BUT framework, the *J* function and the Monster Module are sets of objects with matching description embedded in a M^d manifold, where d stands for their abstract dimension 196,884. Encompassing the two parameters in a M^d manifold allows us to provide a topological commensurability between the Monster Module and the J function. When we reduce the dimensions to $S^{196,883}$, we achieve a single function, e.g., the Monster Lie group. It easy to see that, if we map the two functions to a dimension lower, in this case $M^{196,883}$, we achieve a single function which retains the features of both. This single function stands for the Monster Group, which is the automorphic Lie group acting on the Monster Module (**Figure 2, upper part**). In topological terms, as always, two functions on a S^n sphere lead to a single function on a S^{n-1} sphere.

3) OF MONSTERS AND UNIVERSES

Dimensions reduction. Here we propose a BUT model of our Universe located inside the Monster Module. We argue, based on topological and energetic claims, that our Universe might arise from a spontaneous loss of dimensions, e.g., an automorphism, occurring into the very structure of the supersymmetric, multidimensional Monster Module. According to energy-BUT, the more the symmetries, the more the energy, so that every increase of symmetric level doubles the energy of the previous, less symmetric one. If the Monster stood before the Big-Bang, we are in front of a manifold with the highest possible energy, because it displays the highest number of symmetries.

The group Monster encompasses several subgroups, also classified into the sporadic groups (e.g., Mathieu groups, Leech lattice groups, and so on) (Gannon., 2006). It is worth of mention that the symmetries in a hypothetical M^d encompassing our Universe do not need to be necessarily of the huge order of 10^{54}. In such a vein, one might think two possible physical scenarios:

a) The Monster group is progressively formed starting from its subgroups, with a gradual building from blocks.
b) The Monster group is the initial structure. It then might split into its subgroups.

Because the entropy is increasing in the Universe, the second hypothesis is more reasonable. It would be better to take a starting point before the Big Bang with a higher energetic manifold, and not vice versa. Our Universe goes towards gradually lower energetic levels. At the Big- Bang, a loss of dimensions and thermodynamic free-energy occurred. There was, going from a dimension to a lower one, a sort of quantum jump towards lesser levels, of which one is the half of the other. It is a testable hypothesis. Like an electron orbit, a jump towards more internal levels occurred. This explains the arrow of entropy. A loss of dimensions came together with a loss of symmetries. Indeed, at the low-dimensional level of our Universe, just the symmetries embedded in the preserved dimensions are kept, while the other are apparently lost. The original lost symmetries could in theory be restored, inverting the process from lower dimensions to the higher ones of the Monster Module, but it would require a source of energy able to perform the inverse projection, and this is not the case of our Universe.

Topological relationships between the Monster and string theories. Moonshine can be regarded as a collection of related examples where algebraic structures have been associated with automorphic functions or forms, because it is also displays relationships with the Lie group $E^8(C)$ and a lattice vertex operator algebra equipped with a rank 24 Leech lattice (Borcherds, 1992; Frenkel, 1988). Several features of the Monster, either the Module, or the group and the subgroups, have been associated with different physical scenarios. Some examples are depicted in **figure 2 (lower part)**. Links between Monstrous Moonshine and string theories have been described. The Monster might stand for the symmetry of a string theory for a Z^2-orbifold of free bosons on a Leech lattice torus, in the context of a conformal field theory equipped with partition function j. Recent papers link other sporadic groups with modular forms, suggestive of a more central role for the Umbral Moonshine (Eguchi et al., 2011). Witten proposed that pure gravity in AdS_3 (anti deSitter) space with maximally negative cosmological constant is AdS/CFT dual to a holomorphic CFT (conformal field theory), with the numbers of the Moonshine coming into play (Witten, 2007). CFT/AdS is dual to string theories, and is involved in the many models: CFT, Chern-Simon-Matter, Super Jang-Mills, Superconformal algebras. The AdS\CFT correspondence means that conformal field theory is like a hologram, which captures information about the higher-dimensional quantum gravity theory. It is exactly a picture which could be described in the BUT framework. The Witten’s conjecture of a duality between pure quantum gravity and external holomorphic CFT predicts the existence of a hyperbolic anti-DeSitter Universe equipped with a strongly negative cosmologic constant. The relationships between Universe’s negative curvature and Monster Moonshine have been recently explored in the above mentioned, unpublished paper (*Tozzi and Peters, unpublished data; under review for Eur Phys J C*).

The problem of singularity. A problem which arises is how to explain the event, commonly called *singularity* (Chow, 2008), which caused an apparent loss of dimensions in parts of the Monster, giving rise to our Universe. It must be taken into account that the trajectory on a hypersphere, or in general on a manifold, does not need necessarily to be closed, because a particle could just travel along a shortest path, and not along the entire surface (Collins, 2004). We might think of world lines traced by a moving particle: this line evolves, whereas the line in the chart is static. A hypothetical particle embedded into the Monster Module follows the movements dictated by the Monster Lie group. However, the particle cannot travel everywhere, due to the huge amount of dimensions. The complete ergodicity of particle pathways on the Monster Module cannot be guaranteed, due to the countless possible trajectories. When a particle travels on the ergodic Monster manifold's huge phase space, it might simply take a random direction towards any dimensions and not others. In other words, the particle follows (probably random) paths equipped with dimension decrease. We provide an example which takes into account the well-studied model of a 26D bosonic string theory. Such a model, although partially dismissed, provides a good example of our model. Bosons' trajectories in a 24D Leech lattice may follow paths of 196,884 dimensions. Notice that the Leech lattice is almost ubiquitous in the description of sporadic groups, thus offers an interesting example. When particles go towards preferential trajectories, they follow paths involving just some of the total Monster's dimensions. They fall into trajectories lying into the lower dimensions of our Universe. When bosons' paths fall into some of the Monster's dimensions, they lose energy for energy-BUT. It might explain the Big Bang, equipped with high energetic levels that decrease with time passing. Therefore, the singularity might be explained simply by random particles' movements. Note that, because random paths might occur everywhere on the Monster Module, it means that countless Universe are allowed, every one equipped with just some of the primeval symmetries. Our (and others) Big Bang might just have been occurred naturally, when a particle fell into a dimension instead of another. It also means that the path chosen by a particle is equipped with just some of the Monster symmetries, while the others are lost (or, better, hidden, because they might reappear at the level higher than the ones where they are embedded). If a particle travels along a path embedded just in a few dimensions (our Universe), the loss of the other primeval dimensions gives rise in our Universe to symmetry breakings, including the thermodynamical arrow. In order to elucidate why a decrease in symmetries and dimensions leads to our Universe equipped with symmetry breakings, another important argument must be taken into account. Indeed, almost all the finite groups are *non-abelian*: it explains how the multisymmetric Monster loses dimensions and leads to our Universe, in which the rules are dictated by *asymmetric* laws. The intrinsic non-abelian structure of the Monster itself ensures that the patterns are not reversible. Once taken a path, for the non-abelian and energetic arguments, it is not possible to reverse the process in our Universe, unless other energy is supplied. The presence of an ergodic, homogeneous Monster Module before the big bang also solves the so called horizon problem. A few Planck times after the Big Bang, the Universe consisted of 10^{90} Planckian size, disconnected regions (Veneziano, 1998). Currently, those regions make up our observable Universe and resemble one another. The presence of the homogeneous Monster Module before the Big Bang explains why the initial disconnected regions had all the same conditions.

The Monster and the spacetime. The Monster is a manifold which, for the BUT variants, can be also described as a hypersphere, and thus equipped with closed trajectories. Therefore, our Universe is *internal* to the Monster. The loss of dimensions occurs *into* the Monster, giving rise to the Big Bang. That's why the fossil background cosmic radiation comes from everywhere, when we look at it (Fixsen, 2009). A problem arises: how can a string-like manifold give rise and contain the whole Universe? A possible solution is that the Monster is not in the space, and the space occurs together with the Universe. Concerning the time, the things are more complicated. Indeed, in touch with Veneziano's pre big bang scenarios (Veneziano, 1998), the time could exist before the Big Bang, and not arise together with the Universe. The Monster group needs to be embedded into the time, because it, acting as a Lie group, needs to perform symmetric movements which may just occur in a given time. It might however be speculated that the time is not required at the Monster Module level, and the Wheeler-DeWitt equation might be valid at such level. By a physical point of view, you start from two algebraic structures and reach a Lie group, which needs to perform an action. It means that the level S^{196883} requires that the introduction of the parameter time, while the Monster Module in S^{196884} lies in infinite dimensions, and is atemporal.

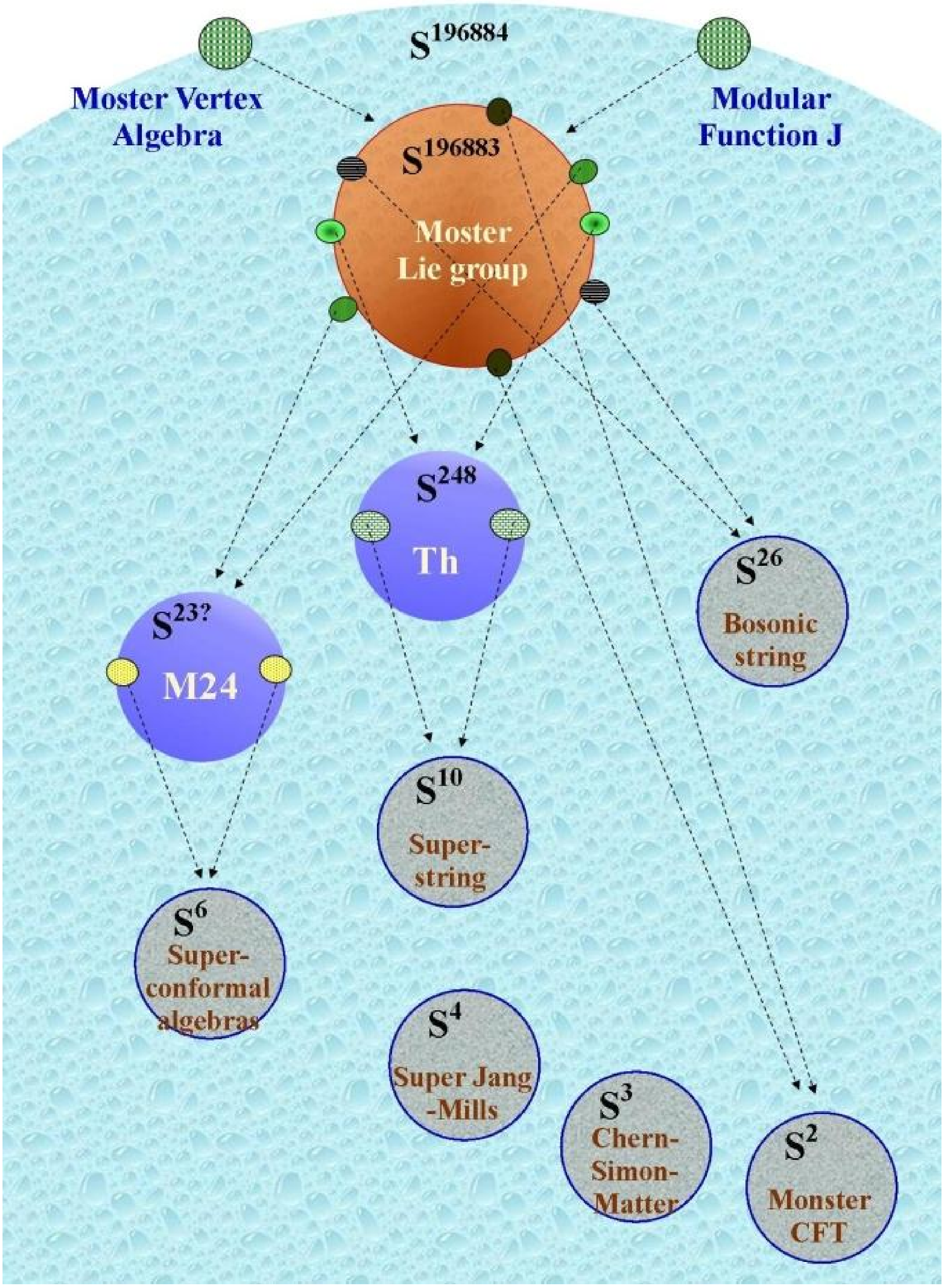

Figure 2. Progressive loss of dimensions in sporadic groups can be encompassed in a BUT framework. Note also the loss of symmetries from the highest dimension levels to the lowest ones. The Figure also illustrates how every sporadic group might display a theoretical physical counterpart.

3) QUANTIFYING PHYSICAL MONSTER'S PARAMETERS

Towards the Monster's enthalpy. We are allowed to use the energy BUT in order to calculate the energetic requirements of Monster Modules in a physical context. Thermodynamics says that:
$H = F + T \times E$
Where H is the Enthalpy, F the free-energy, T the temperature (trascurable) e E the entropy. We assume that our Universe is closed.
The current level E_0 of entropy in the Universe is estimated in $2.6 \pm 0.3 \times 10^{122}$ k (Egan and Lineweaver, 2010; Frampton et al, 2008).
If the Universe displays four dimension as currently believed, every dimension contains approximately an average entropy of:
$E_0/4$.
As shown in Figure 10 of Egan and Lineweaver (2010), the current Universe displays almost the highest possible of entropy. Also in the future, the entropy will be just slightly larger than the current value E_0, because a monotonical increase already occurred. It means that E_0 is more or less the maximum value of entropy achievable in the whole life of the Universe, and also means that the free-energy F is currently very low. At the Big Bang, on the contrary, E was close to zero and F very high.
If we want to calculate the enthalpy, we notice that the current E_0 almost equals the total enthalpy H of the Universe, because currently F is very low. Vice versa, at the Big Bang, F was very high and E the lowest possible. It means that, at the Big Bang, more or less:
$F = H$.
It also means that:
$E_0 = H$.
If the Monster occurred before the Big-Bang, we are in front of a manifold with the highest possible energy, because it displays the higher number of symmetries. If the Monster gave rise to our Universe, and the Monster displays 196,883 dimensions, the Entropy of the Monster E_M is:
$E_0/4 \times 196{,}883$
Thus, the enthalpy of the Monster stands roughly for the same value:
$E_0/4 \times 196{,}883$

The loss of dimensions into the Monster Module, due to the non-abelian movements of the Monster Lie group and the energy-BUT, give rise to different Universes with dimensions lower than the Monster, and equipped with less energy and information.
Through the Conway atlas of finite groups, we know the dimensions and the order of every group, including the sporadic ones. It is not difficult to calculate how many dimensions have been lost. We know this number, e.g., 196,883 - 4, the dimensions of our Universe, we know how many symmetries are preserved, and we know, for energy-BUT, that every decrease of a single symmetric level denotes the loss of half of the energy. If the pre-Big Bang manifold, e.g., the Monster Module, is equipped with 196,884 dimensions and 10^{54} elements, and if our Universe has 4 dimensions (the spacetime), we have 10^{50} elements in our Universe.
Summarizing, once hypothesized a high-energy Monster Module before the rise of our Universe, the next step is to reduce the symmetries from the Monster vertex operator to the Monster group, which is the Lie group acting on it. A further step gives rise to a dimensions and symmetries reduction until our Universe, equipped with the Standard Model.

Information. The energy-BUT states that it is not possible to achieve higher information, starting from a lower dimensional level. It means that we need to start from the Monster Module, and not vice versa. The process must be top-down, e.g., from the Monster to the Universe, and not bottom-up. According to the energy-BUT, a loss of information occurs together with a decrease in dimensions. It means that, from the Monster to our Universe, it occurs a loss if information. You cannot move a particle in our Universe from lower to higher dimensions, unless you, for energy BUT, do not *inject* novel free-energy or enthalpy. You can do it just locally in the Universe, for example when biological entities are formed in limited niches, but not everywhere, because the total entropy increases together with a decrease in free-energy. It also means that from the highest to the lower levels there is a *reduction*, and not an *emergence* of information.
Watching the Monster: vertex algebra. In order to incorporate the j-function into a general context and to visualize the movements of the Monster Group on the Monster Module, we built a simplified 3D model equipped with a hypersphere and a vertex algebra operator. We achieved a low-dimensional model of j-function and its group, embedded into a vertex algebra's manifold. Briefly, a vertex algebra provides a mathematical formulation of the chiral part of 2-dimensional conformal field theory. The axioms of a vertex algebra are obtained from the properties of quantum field theories and operator product expansions (OPEs). The main tactic flowing from OPEs is that a product of local operators defined at nearby locations can be expanded in a series of local operations (Ekstrand, 2011). A graphical representation of an OPE

is represented in **Figure 3A**. Let ω_1, ω_2 be periods of a doubly periodic function with $\tau \equiv \frac{\omega_1}{\omega_2}$. Then Klein's absolute invariant is defined by

$$\mathrm{J}(\omega_1, \omega_2) = \frac{g_2^3(\omega_1, \omega_2)}{\Delta(\omega_1, \omega_2)},$$

where g_2 is the invariant of the Weierstrass elliptic function. If H is the upper half plane and $\tau \in \mathrm{H}$, then $\mathrm{J}(\tau) \equiv J(1, \tau) = \mathrm{J}(\omega_1, \omega_2)$.

The function $\mathrm{J}(\tau)$ is the j-function modulo a constant multiplicative factor (Weisstein, 2016). A dynamical system with a strange attractor and invariant tori (Sprott, 2014) initialized with the j-function is illustrated in **Figure 3B**.

A

B

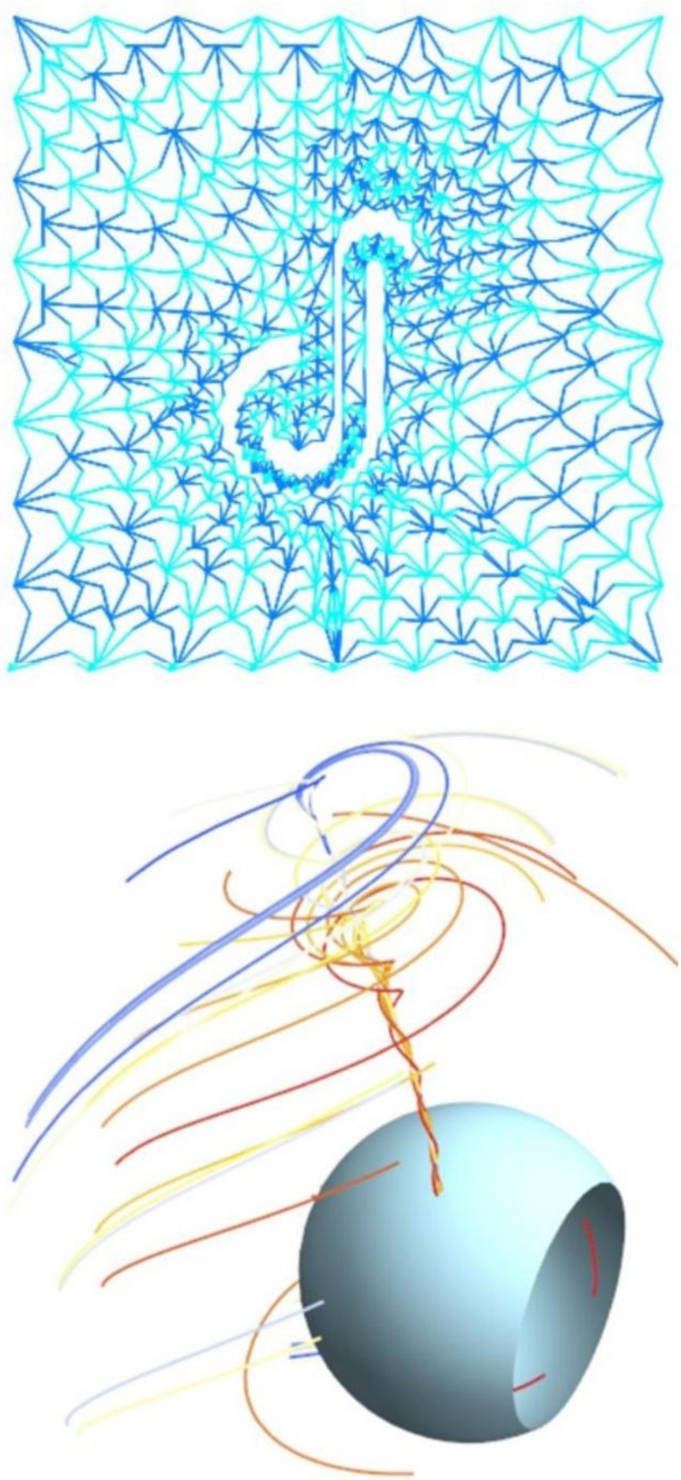

Figure 3. Defining a vertex algebra on a torus helps us to visualize otherwise abstract structures. Starting from a vertex operator algebra (a very small portion is described in **Figure 3A**) we made use of the attractors and the corresponding ODEs described by Sprott (2014). In **Figure 3B**, the j-function on the attractor torus displays one coordinate initialized with a j-function value.

4) QUESTIONS AND CONCLUSIONS

Starting from a *Spinozian* global system, shaped in guise of a multidimensional and multisymmetric manifold equipped with a structural order of relationships, we were able to analyse, through a loss of dimensions dictated by the intrinsic features of the Monster Module and its Lie group, the individual history of the Universe. The Universe can be thus conceived as a manifold at lower dimensions encompassed in higher ones. The Monster Module is a manifold equipped with absolutely the highest dimensions - that is, a manifold consisting in the highest number of symmetries. The Monster Moonshine manifold is prior to its modifications. We may mean the nature and the flow of events in the Universe as a Monster's self-projection towards less dimensions. The Universe stands for a local symmetry, e.g., modifications of the Monster manifold. The Monster Module cannot exist in, and cannot be conceived through, an higher manifold other than itself. Every manifold in the Universe exists either in itself or in some higher manifold else, e.g., the Monster Module. The knowledge of a lower dimension manifold in the Universe depends on and involves the knowledge of higher dimensions mapping the lower manifold. In the meantime, the Monster Module, which n-dimensions are untouched, is still there. If different trajectories on the Monster Module give rise to different local losses of dimensions, it means that countless Universes are possible, each one equipped with different or overlapping symmetries.
We would like to bring to an end with a few unsolved problems.

a) Where does the Monster take such a huge amount of enthalpy? It takes us in pre, pre- Big Bang scenarios. This is the same problem of inflaton models, that do not explain where does the energy of the required false vacuum come from. A link between the Monster group and the fales vacuum might be speculated.
b) Which is the role of the J function in the pre Big Bang period? Does it provide energy?
c) How does the Monstruos Moonshine look like? We might either imagine a timeless, immutable manifold where just the Monster Group movements take place, or as we did, a dynamical, time-dependent structure.
d) Does the curvature of the Monster Module change with time passing? It could be a very useful information, in order to elucidate the predicted passage from an ancient anti DeSitter hyperbolic Universe to the current, flat one.
e) Our Universe might not arise directly from the Monster, but by one of its subgroups, e.g., Th, which is correlated with the successful superstring 10D theory. Is it possible to split the Leech lattice in which the Monster group is embedded, in order to achieve the lower dimensional E8 lattice where the Th group's movements take place? Remind that the step from E8 lattice to the Leech requires x3 multiplication and peculiar rotations.
f) The topological step from the vertex operator algebra to the Lie Monster Group requires a continuous function. Are we in front of a *super* gauge field? In other words, is there a gauge field which causes the first projection depicted at the top of the **Figure 2**? In a topological framework, the feature which links the symmetries at a higher level with the single point at a lower level is the continuous function. If we assess two antipodal points as symmetries, and the single point as symmetry breaks and local transformations, a gauge field could be required, in order to *restore* the (apparently hidden) symmetry.

REFERENCES

1) Beyer WA, Zardecki A. The early history of the ham sandwich theorem. American Mathematical Monthly, 111, n. 1, 2004, 58-61
2) Borcherds R (1992), "Monstrous Moonshine and Monstrous Lie Superalgebras", Invent. Math. 109: 405–444, doi:10.1007/bf01232032
3) Borsuk, M. Drei s¨atze ¨uber die n-dimensionale euklidische sph¨are, Fundamenta Mathematicae XX (1933), 177–190.
4) Borsuk, M. Fundamental retracts and extensions of fundamental sequences, 64 (1969), no. 1, 55–85.
5) Bourbaki, N. (1966). Elements of Mathematics. General Topology 1, Chapters 1–4. Translated from the French. Reprint of the 1966 edition. *Springer-Verlag, Berlin,* 1989. viii+437 pp. ISBN: 3-540-19374-X, MR0979294.
6) Chow, T.L. (2008). Gravity, Black Holes, and the Very Early Universe. An Introduction to General Relativity and Cosmology. New York, Springer, x+280 pp.
7) Collins, G.P. The shapes of space, 291 (2004), 94–103.
8) Conway JH, Curtis RT, Norton SP, Parker RA, Wilson RA. 1985. Atlas of Finite Groups: Maximal Subgroups and Ordinary Characters for Simple Groups. Conway JH (Editor). Clarendon press, Oxford, USA
9) Dochviri, I. and Peters, J.F. (2016). Topological sorting of finitely near sets. Math. in Comp. Sci., *in press*.

10) du Sautoy M. Finding Moonshine, A Mathematician's Journey Through Symmetry. Fourth Estate, 2008 ISBN 0-00-721461-8
11) Duncan JFR, Griffin MJ, Ono K. Moonshine. Research in the Mathematical Sciences (2015) 2:11. DOI 10.1186/s40687-015-0029-6
12) Egan CA, Lineweaver CH. 2010. A Larger Estimate of the Entropy of the Universe. arXiv:0909.3983v3
13) Eguchi, T, Ooguri, H, Tachikawa, Y: Notes on the K3 Surface and the Mathieu groupM24. Exper. Math. 20, 91–96 (2011)
14) Ekstrand, J. Lambda: A Mathematica package for operator product expansions in vertex algebras. Computer Physics Communications 182 (2011), 409-418, doi:10.1016/j.cpc.2010.09.018
15) Fixsen, D. J. (2009). "The Temperature of the Cosmic Microwave Background". The Astrophysical Journal 707 (2): 916–920. doi:10.1088/0004-637X/707/2/916.
16) Frampton P, Hsu SDH, Kephart TW, Reeb D. What is the entropy of the universe? 2008 arXiv:0801.1847
17) Frenkel, I, Lepowsky, J, Meurman, A: A natural representation of the Fischer-Griess Monster with the modular function J as character. Proc. Nat. Acad. Sci. U.S.A. 81(10), 3256–3260 (1984). MR 747596 (85e:20018)
18) Frenkel IB.; Lepowsky j, Meurman A (1988). Vertex Operator Algebras and the Monster. Pure and Applied Math. 134. Academic Press.
19) Gannon, T (2006), Moonshine beyond the Monster: The Bridge Connecting Algebra, Modular Forms and Physics, ISBN 0-521-83531-3
20) Matoušek, J. Using the Borsuk–Ulam Theorem. Lectures on Topological Methods in Combinatorics and Geometry. Springer-Verlag Berlin Heidelberg, 2003
21) Olive DI, Landsberg PT. Introduction to string theory: Its structure and its uses. Philos. Trans. of the Royal Soc. of London, Series A, Mathematical and Physical Sciences 329 (1989), 319-328.
22) Peters JF. 2016. Computational Proximity. Excursions in the Topology of Digital Images. Edited by Intelligent Systems Reference Library. Berlin: Springer-Verlag. doi:10.1007/978-3-319-30262-1.
23) Peters JF, Tozzi A. 2016a. “Region-Based Borsuk-Ulam Theorem.” arXiv, 1605.02987.
24) Peters JF, Tozzi A. 2016b. “Quantum Entanglement on a Hypersphere.” Int J Theoret Phys, 1–8. doi:10.1007/s10773-016-2998-7.
25) Petty, CM. Equivalent sets in Minkowsky spaces. Proc. Amer. Math. Soc. 29(2), 1971, 369-374.
26) Roldán E, Martínez IA, Parrondo JMR, Petrov D. Universal features in the energetics of symmetry breaking. Nature Physics 10, 457–461 (2014) doi:10.1038/nphys2940
27) Sengupta, Biswa, Arturo Tozzi, Gerald K. Cooray, Pamela K. Douglas, and Karl J. Friston. 2016. “Towards a Neuronal Gauge Theory.” PLOS Biology 14 (3): e1002400. doi:10.1371/journal.pbio.1002400.
28) Sprott, JC. A dynamical system with a strange attractor and invariant tori. Physics Letters A 378 (2014), 1361-1363.
29) Tozzi A, Peters JF. 2016a. “A Topological Approach Unveils System Invariances and Broken Symmetries in the Brain.” Journal of Neuroscience Research 94 (5): 351–65. doi:10.1002/jnr.23720.
30) Tozzi A, Peters JF. 2016b. “Towards a Fourth Spatial Dimension of Brain Activity.” Cognitive Neurodynamics 10 (3): 189–99. doi:10.1007/s11571-016-9379-z.
31) Veneziano G. (1998) A Simple/Short Introduction to Pre-Big-Bang Physics/Cosmology. arXiv:hep-th/9802057v2
32) Watanabe T, Masuda N, Megumi F, Kanai R, Rees G. 2014 Energy landscape and dynamics of brain activity during human bistable perception. Nat. Commun. **28,** 5:4765. (doi: 10.1038/ncomms5765)
33) Weeks, J.R. The shape of space. Second edition. Marcel Dekker, inc. New York-Basel (2002).
34) Weisstein E.W. 2016. Klein’s absolute invarint, http://mathworld.wolfram.com/KleinsAbsoluteInvariant.html
35) Witten E (2007). "Three-dimensional gravity revisited". arXiv:0706.3359