The PMNS Neutrino-Mixing Matrix in the Scale-Symmetric Theory

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Abstract: The Pontecorvo, Maki, Nakagawa and Sakata (PMNS) neutrino-mixing matrix is in the Scale-Symmetric Theory (SST) the unitary mixing matrix which contains information on states of neutrinos when they collide with the neutrinos in the Cosmic Neutrino Background (CNB). Since in SST there are only 3 Dirac neutrino flavours (there are not in existence Majorana neutrinos) and because the CP phase is equal to 180 degrees, the PMNS matrix is parameterized by three mixing angles only that are calculated from three fundamental weak phenomena that took place in the nuclear plasma at the beginning of the expansion of the Universe - it fixed the abundances of different flavours in CNB. Calculated mixing angles A(ij) are A(13) = 8.2654, A(12) = 33.0616 and A(23) = 41.3250 degrees. We showed that the mixing angle A(13) follows from internal structure of pions and from the atom-like structure of baryons. The ratios of the mixing angles are as 1 : 4 : 5 and they represent respectively one stable flavour (i.e. electron- or muon-neutrino), two entangled flavours (i.e. electron- or muon- plus tau-neutrino i.e. four entangled stable neutrinos), and three entangled flavours (i.e. electron- plus muon- plus tau-neutrino i.e. five entangled stable neutrinos). We as well calculated the degenerate masses of the three different neutrinos - calculated sum is 0.287 eV (it is consistent with experimental data). We calculated mass of tauon (1777.2 MeV), which is an analog to muon. We answered following question: Why neutrinos are left-handed whereas antineutrinos are right-handed? SST shows that sterile neutrinos are not in existence. The matter-antimatter asymmetry follows from the left-handedness of the Higgs-field components.

1. Introduction

Here, applying the Scale-Symmetric Theory (SST) [1] we described the PMNS neutrino-mixing matrix. At the beginning of the expansion of the Universe, due to the three fundamental weak processes in nuclear plasma (they are described in this paper), there appeared the cosmic neutrino background (CNB) [2] which is crucial for the neutrino substitutions/“oscillations” in “vacuum”. In collisions of the initial experimental neutrinos with the neutrinos in CNB and in the next succeeding collisions, there is transferred energy so in such collisions flavour of neutrino can change. Such changes in flavour do not require different masses for the different neutrino flavours. We will show that they are the degenerate masses i.e. masses of the electron-neutrino and muon-neutrino are the same whereas mass of tau-neutrino is only 3 times greater. Here we calculated the sum of masses of the 3 flavours – this result is consistent with experimental data. The neutrino substitutions/”oscillations” can be in very good approximation described via the quantum formalism for the $K^o - K^o_{anti}$ and $B^o - B^o_{anti}$ mixing and oscillations [3]. We can assume that in order for neutrinos “to oscillate”, each of their three SST flavours can be in a good approximation a superposition of the three masses. Such quantum superposition evolves over time so from time to time there appear pure flavours that represent the substitutions of neutrinos for other neutrinos.

It is obvious that the mixing angles in the neutrino mixing matrix should depend on abundances of different neutrinos in the CNB.

We as well showed that the CPT symmetry is conserved [4]. The insignificantly broken T symmetry (so CP symmetry as well) follows from internal helicity of the main torus in each fermion – such tori produce jets in the spacetime [4]. The jets cause that a single phase δ_{CP} that relates to CP violations (i.e. relates to differences in the rates of oscillation between two states with opposite starting points – it makes the order in time in which events take place i.e. there is a sequence of tenses) is δ_{CP} $[^o] = 180$ i.e. $exp(i\ \delta_{CP}) = -1$. Within SST, under CPT symmetry, the PMNS neutrino-mixing matrix for antineutrinos is identical to the matrix for neutrinos.

We must distinguish the internal helicity of the main torus in fermions from their external helicity that defines orientation (parallel or antiparallel) of velocity and spin. Here, we explained why all neutrinos are externally left-handed whereas antineutrinos are externally right-handed. Here we showed as well that properties of the SST fermions connect fields of opposite internal helicity (chirality) so the SST neutrinos are the Dirac neutrinos, not Majorana neutrinos that connect fields of the same internal helicity (chirality). The conservation of the SST weak charge of neutrinos [1A] allows only Dirac-type mass terms. SST shows that sterile neutrinos are not in existence.

The above remarks lead to conclusion that we can simplify the mixing matrix U. Moreover, within the SST we can calculate the mixing parameters/angles A_{ij} for the “oscillations”/substitutions in “vacuum”. But notice that the additional interactions of neutrinos with matter lead to very complex analyse. Here we do not take into account processes that can lead to a strong enhancement of the oscillation probability in “vacuum”.

The SST shows that the succeeding phase transitions of the superluminal non-gravitating Higgs field during its inflation (the initial big bang) lead to the different scales of sizes/energies [1A]. Due to a few new symmetries, there consequently appear the superluminal binary systems of closed strings (entanglons) responsible for the quantum entanglement (it is the quantum-entanglement scale), stable neutrinos and luminal neutrino-antineutrino pairs which are the components of the luminal gravitating Einstein spacetime (it is the Planck scale), cores of baryons (it is the electric-charge scale), and the cosmic-structures/protoworlds (it is the cosmological scale) that evolution leads to the dark matter, dark energy and expanding universes (the “soft” big bangs) [1A], [1B]. The electric-charge scale leads to the atom-like structure of baryons [1A].

The SST neutrinos acquire their masses due to the Higgs mechanism described within SST [1A].

Among a thousand of calculated quantities within SST, which are consistent with experimental data, we calculated quantities that we use in this paper, i.e. the fundamental physical constants, mass of neutral pion $m_{Pion(o)} = 134.97$ MeV, of charged pion $m_{Pion(+,-)} = 139.57$ MeV, mass of stable lightest neutrino (this mass is not measured in experiments) $m_{Neutrino(e,\mu)} = 3.3349306 \cdot 10^{-67}$ kg (we will call this mass the particle mass to distinguish this mass from the wave mass of neutrinos that is calculated in this paper; we will show that in the mainstream theory of neutrinos we use the geometric mean of the particle mass and wave mass of neutrinos), equatorial radius of stable neutrino $r_{Neutrino(e,\mu)} = 1.1184555 \cdot 10^{-35}$ m, mass of electron $m_{Electron} = 0.5109989$ MeV, radiation mass of electron $m_{em(electron)} = 0.0005919$ MeV, fine-structure constant $\alpha_{em} = 1 / 137.0360$, coupling constant for nuclear weak interactions of the nucleons $\alpha_{w(proton)} = 0.0187228615$, coupling constant for strong-weak interactions inside the baryons for the $d = 1$ state $\alpha_{swW(+-),d=1} = 0.762594$, relativistic mass of the charged pion in the $d = 1$ state $m_{W(+-),d=1} = 215.760$ MeV and the typical energy of neutrinos produced in nuclear plasma $E_{o,neutrino} = m_{LL} / 2 = 33.7722$ MeV, where $m_{LL} = 67.54441$ MeV is the mass of two loops a neutral pion consists of, mass of the Einstein-spacetime condensate in the centre of the baryons $Y = 424.1245$ MeV, and mass of hyperon Λ: $m_\Lambda = 1115.3$ MeV [1A]. We as well described the four-neutrino/four-particle symmetry – number of entangled objects in a system is quantized [1B]

$$D_n = 4^{\mathrm{d}} \text{ (for single objects),} \tag{1a}$$

or

$$D_n = 2 \cdot 4^{\mathrm{d}} \text{ (for binary systems),} \tag{1b}$$

where $d = 0, 1, 2, 4, 8, \ldots = 0, 2^{\mathrm{n}}$, where $\mathrm{n} = 0, 1, 2, 3, 4, 5, \ldots$ We can see that a basic multiple structure contains 4 objects/particles.

2. The SST flavours of neutrinos, why the SST neutrinos are the Dirac neutrinos, why the tau-neutrinos are associated with tauon and why neutrinos are left-handed (it concerns the external helicity)

According to SST, the tremendous non-gravitating energy inside the four stable neutrinos (i.e. the electron-neutrino, muon-neutrino and their antiparticles) causes that they are indestructible in the present-day inner Cosmos [1B], [1A].

The charged pions can decay as follows

$$\pi^+ \rightarrow e^+ + \nu_e + \nu_{\mu,anti} + \nu_\mu \,, \tag{2a}$$

$$\pi^- \rightarrow e^- + \nu_{e,anti} + \nu_\mu + \nu_{\mu,anti} \,. \tag{2b}$$

There is a possibility that the three neutrinos that appear in the decays of charged pions will be entangled and will carry half-integral spin. We claim that such objects composed of three entangled stable neutrinos are the tau-neutrinos

$$\nu_\tau \equiv \nu_e \left(\nu_{\mu,anti} \, \nu_\mu \right) , \tag{3a}$$

$$\nu_{\tau,anti} \equiv \nu_{e,anti} \left(\nu_\mu \, \nu_{\mu,anti} \right) . \tag{3b}$$

The pair of neutrinos in the parentheses has the total weak charge and total internal helicity both equal to zero so the tau-neutrinos behave as the electron-neutrinos with shifted mass (mass is three times greater).

We can see that there are three flavours of neutrinos ν_a, where $a = e, \mu, \tau$.

The internal helicities and charges of fermions that follow from the SST [1A] are collected in Table 1.

The neutrinos interact with the condensate of electron [1A]. They are all fermions so their physical states should be different. Neutrinos and electrons can differ by internal helicity (which dominates inside the muon) and, if not by it, by the sign of the electric charge and the weak charge (it is for the third neutrino inside charged pion). The possible bound states are as follows

$$\mu^-{}_R \equiv e^-{}_R\, \nu_{e,anti,L+}\, \nu_{\mu,L-},$$
$$\mu^+{}_L \equiv e^+{}_L\, \nu_{eR-}\, \nu_{\mu\,(anti)R+},$$
$$\pi^-{}_R \equiv e^-{}_R\, \nu_{e(anti)L}\, (\nu_{\mu,L-}\, \nu_{\mu\,(anti)R+})\, L_L\, L_{LA} \rightarrow \mu^-{}_R\, \nu_{\mu\,(anti)R+},$$

where L_{LA} denotes the large loop [1A] with the left helicity and antiparallel spin.

$$\pi^+{}_L \equiv e^+{}_L\, \nu_{eR-}\, (\nu_{\mu\,(anti)R+}\, \nu_{\mu,L-})\, L_R\, L_{RA}.$$

According to SST, the two neutrinos in muon and the two first neutrinos in charged pion interact with the condensate in electron and the internal helicities (chiralities) of such neutrinos are opposite to the electron or positron – it means that they are the Dirac particles, not Majorana particles. The third neutrino in the charged pions does not couple to electron but to the first muon-neutrino – their internal helicities are opposite so the third neutrino is the Dirac neutrino as well. The same concerns the tau-neutrinos.

Table 1 *New symbols* [1A]

Particle	**Internal helicity (chirality)**	**Electric charge**	**Weak charge**	**New symbol**
$\nu_{e(anti)}$	L (left)		+	$\nu_{e(anti)L+}$
ν_e	R (right)		–	ν_{eR-}
$\nu_{\mu\,(anti)}$	R		+	$\nu_{\mu\,(anti)R+}$
ν_μ	L		–	$\nu_{\mu L-}$
e^-	R	–		$e^-{}_R$
e^+	L	+		$e^+{}_L$
p^+	L	+		$p^+{}_L$
p^-	R	–		$p^-{}_R$
n	L[1)]		+	n_L
$n_{(anti)}$	R[1)]		–	$n_{(anti)R}$
μ^-	R[1)]	–		$\mu^-{}_R$
μ^+	L[1)]	+		$\mu^+{}_L$
π^-	R[1)]	–	+	$\pi^-{}_R$
π^+	L[1)]	+	–	$\pi^+{}_L$

[1)] The resultant internal helicity is the same as the internal helicity of the torus having greatest mass.

Why the tau-neutrinos are associated with tauon?

According to SST, the tau lepton consists of an electron and massive particle, created inside a baryon, which interacts with the condensate of the electron.

The charged relativistic W_d pion in the $d = 1$ state of baryons is responsible for the properties of the proton [1A]. What should be the mass of a lepton in order to the mass of such charged relativistic pion was the radiation mass of the lepton for the strong-weak interactions in the $d = 1$ state? From formula

$$\alpha_{swW(+,-),d=1}\, m_{tau,d=1} / m_{W(+,-),d=1} = \alpha_{em}\, m_{electron} / m_{em(electron)}, \tag{4a}$$

we obtain the approximate mass of the tau lepton

$$m_{tau}{}^{*} = 1782.5 \text{ MeV}. \tag{4b}$$

But it is not the whole story. According to SST, muon consists of two energetic neutrinos interacting with its central condensate. Tauon should have similar structure. At first notice that a mass of a loop with circumference $2\pi r$ that transforms into condensate with wave length equal to the radius of the loop has mass 2π times higher than the loop (such processes cool the nuclear plasma) – notice, for example, that the ratio of the baryon central condensate and the large loop created in the baryonic core is $Y / m_{LL} \approx 2\pi$. The structure of charged pions suggests that due to the $2\pi r \rightarrow r$ process, a charged pion can transform into charged condensate with a mass 2π times higher and into tau-neutrino. There can be the electroweak interaction of such condensate, $C_{Pion(+,-)} = 2\pi\, m_{Pion(+,-)} = 876.95$ MeV, with electron – the binding energy is frozen inside the resultant particle. The coupling constant for the nuclear electroweak interactions is the arithmetic mean of the fine structure constant and the coupling constant for the nuclear weak interactions

$$\alpha_{ew} = (\alpha_{em} + \alpha_{w(proton)}) / 2 = 0.0130101. \tag{5}$$

Consider mass of particle composed of electron, two charged-pion condensates and tau neutrino-antineutrino pair interacting due to electroweak interactions with the central condensate of the electron – such structure is an analogue to the muon structure [1A]

$$m_{tau^-} = m_{electron} + (C_{Pion(+)} + C_{Pion(-)})\,(1 + \alpha_{ew}) + \nu_{\tau,R-}\, \nu_{\tau,anti,L+} = 1777.2 \text{ MeV}. \tag{6}$$

It means that the structure that follows from formula (4a) can transform into the tauon defined by formula (6) (resultant mass is about 5 MeV smaller). We can see that in the decay of tauon can appear electron, tau-neutrino and tau-antineutrino – it is the reason that tau-neutrinos are associated with the tau lepton as well.

Why neutrinos are left-handed (it concerns the external helicity)?

Generally, due to the jet produced by each neutrino in spacetime [4], neutrino should move in opposite direction to the jet. Direction of jet follows from internal helicity – it leads to conclusion that neutrino with left internal helicity should have right-handed external helicity – it is valid for neutrinos that are weakly coupled to electron i.e. for muon-neutrinos. Electron-neutrinos and tau-neutrinos (they behave as the electron-neutrinos with shifted mass) are stronger coupled to electrons so their behaviour depends on behaviour of electrons that dominates because of much greater mass. Electron has the spin parallel to electron-antineutrino or tau-antineutrino (the Einstein spacetime consists of the neutrino-antineutrino pairs with parallel spins so it prefers vector particles) and has right-handed internal helicity so its external helicity should be left-handed. On the other hand, the antineutrino should move in opposite direction to electron. It means that in decays the antineutrino has right-handed external helicity. It leads to conclusion that all neutrinos have left-handed external helicity whereas antineutrinos are right-handed.

3. The experimental masses of neutrinos

There are two different masses of stable neutrinos i.e. the particle mass [1A] and wave mass. Calculate the wave mass of stable neutrino – it is associated with its rotation i.e. the wavelength is equal to the equatorial circumference $2\pi r_{Neutrino(e,\mu)}$

$$M_{Neutrino(e,\mu)} = h\ \nu / c^{2} = 2\ \pi\ \hbar / (2\ \pi\ r_{Neutrino(e,\mu)}\ c) = 3.1451 \cdot 10^{-8}\ \text{kg}\,. \qquad (7)$$

In the experiments we should observe the geometric mean

$$M_{Neutrino(e,\mu),exp} = (M_{Neutrino(e,\mu)}\ m_{Neutrino(e,\mu)})^{1/2} = 1.0241 \cdot 10^{-37}\ \text{kg} = \underline{0.057451\ \text{eV}}. \qquad (8)$$

Calculate the sum of masses of the three degenerate neutrinos with different flavours

$$M_{3,exp} = \Sigma_{a=e,\mu,\tau}\, M_{a} = 5\ M_{Neutrino(e,\mu),exp} = \underline{0.28726\ \text{eV}}. \qquad (9)$$

This value is consistent with experimental data [5]: 0.320 ± 0.081 eV.

4. The neutrino mixing matrix

We can parameterize the PMNS neutrino-mixing matrix U within the SST via the 3 mixing angles A_{ij} only.

The mixing angles we can calculate from three fundamental weak phenomena that took place in the nuclear plasma at the beginning of the expansion of the Universe – it fixed the abundances of different flavours in CNB.

A.

The first fundamental phenomenon in the early nuclear plasma was the production of the charged pion-antipion pairs $F = (\pi^{-}\pi^{+}) = 279.14$ MeV and next their decays to, first of all, electron-neutrinos and tau-neutrinos with the characteristic energy $E_{o,neutrino} = m_{LL} / 2 = 33.7722$ MeV. It means that the ratio of these masses can define the A_{13} angle which is a powerful discriminator of the Neutrino Theory [3] because it appears in all three definitions of the mixing angles within the SST. We obtain

$$A_{13}\ [^{o}] = F / E_{o,neutrino} = \underline{8.2654}. \qquad (10)$$

Notice that the first mixing angle is as well equal to the ratio of the masses of the lightest hyperon (i.e. hyperon Λ that decays due to the nuclear weak interactions) and lightest meson (i.e. neutral pion): $m_{\Lambda} / m_{Pion(o)} = 8.2657$.

B.

The second fundamental phenomenon in the early Universe was and is the four-neutrino symmetry (there are the four stable neutrinos) that can lead to the transition of four protons into atomic nucleus of helium-4. We can call such mixing angle the solar angle because it is associated with the solar neutrinos – it is the mixing angle A_{12}. We can define the solar mixing angle using formula

$$A_{12}\,[^{o}] = 4\,A_{13} = \underline{33.0616}. \qquad (11)$$

C.

The third fundamental phenomenon in the early Universe was creation of objects composed of three entangled neutrinos with different flavours – such objects were composed of 5 stable neutrinos (see formula (9)) so the mixing angle A_{23} we can define using formula

$$A_{23}\,[^{o}] = 5\,A_{13} = \underline{41.3250}. \qquad (12)$$

$$s_{ij} = \sin A_{ij} \qquad c_{ij} = \cos A_{ij}$$

$$U_{SST} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & -s_{13} \\ -s_{12}c_{23} + c_{12}s_{23}s_{13} & c_{12}c_{23} + s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} + c_{12}c_{23}s_{13} & -c_{12}s_{23} + s_{12}c_{23}s_{13} & c_{23}c_{13} \end{bmatrix}$$

$$U_{SST} = \begin{bmatrix} 0.8294 & 0.5399 & -0.1438 \\ -0.3301 & 0.6812 & 0.6535 \\ 0.4507 & -0.4945 & 0.7432 \end{bmatrix}$$

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Calculated here mixing angles are consistent with experimental data [6], [7].
We obtain

$$\sin^{2}(2\,A_{13}) = 0.0810, \qquad (13a)$$

$$\sin^{2}(2\,A_{12}) = 0.8362, \qquad (13b)$$

$$\sin^{2}(2\,A_{23}) = 0.9836. \qquad (13c)$$

Obtained values are consistent with experimental data [6]

$$\sin^{2}(2\,A_{13})_{experiment} = 0.085 \pm 0.005, \qquad (14a)$$

$$\sin^{2}(2\,A_{12})_{experiment} = 0.846 \pm 0.021, \qquad (14b)$$

$$\sin^{2}(2\,A_{23})_{experiment} = 0.999^{+0.001}_{-0.018} \text{ (normal mass hierarchy)}. \qquad (14c)$$

5. The matter-antimatter asymmetry

The matter-antimatter asymmetry follows from the initial left-handedness of the Higgs-field components [1B]. Such handedness preferred creation of the Protoworld with left-handed internal helicity. It means that there appeared more pairs in which the left-handed torus had bigger mass i.e. there was more p^+e^- than p^-e^+. It leads to the matter-antimatter asymmetry.

On the other hand, the baryonic matter, which has left internal helicity, produced and produces more π^+e^- than π^-e^+ pairs. It is the reason that there appears in experiments more electron-neutrinos than electron-antineutrinos [7].

6. Summary

The PMNS neutrino-mixing matrix is in the Scale-Symmetric Theory the unitary mixing matrix which contains information on states of neutrinos when they collide with the neutrinos in the Cosmic Neutrino Background (CNB). We can assume that in order for neutrinos “to oscillate”, each of their three SST flavours can be in a good approximation a superposition of the three masses. Such quantum superposition evolves over time so from time to time there appear pure flavours that represent the succeeding substitutions of the neutrinos for the CNB neutrinos.

Since in SST there are only 3 Dirac neutrino flavours (there are not in existence Majorana neutrinos) and because the CP phase is equal to 180 degrees, the PMNS matrix is parameterized by three mixing angles only that are calculated from three fundamental phenomena that took place in the nuclear plasma at the beginning of the expansion of the Universe – it fixed the abundances of different flavours in CNB. Calculated mixing angles A_{ij} are $A_{13} = 8.2654$, $A_{12} = 33.0616$, and $A_{23} = 41.3250$ degrees (they are consistent with experimental data). Due to entanglement, ratios of the mixing angles are as $1 : 4 : 5$. We proved that the A_{13} mixing angle is a powerful discriminator of the Neutrino Theory because it appears in all three definitions of the mixing angles within the SST. We showed that the mixing angle A_{13} follows from internal structure of pions and from the atom-like structure of baryons.

We as well calculated the degenerate masses of the three different neutrinos – calculated sum is 0.287 eV – it is consistent with experimental data.

We calculated the mass of tauon (1777.2 MeV) which is an analog to muon.

We answered following question: Why neutrinos are left-handed whereas antineutrinos are right-handed?

SST shows that sterile neutrinos are not in existence.

The matter-antimatter asymmetry follows from the left-handedness of the Higgs-field components.

We solved as well a few other basic problems that are unsolved within the mainstream Theory of Neutrinos.

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