Physics on the adiabatically changed Finslerian manifold and cosmology

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Abstract

In present paper we confirm our previous result [4] that Planck constant is adiabatic invariant of electromagnetic field propagating on the adiabatically changed Finslerian manifold. Direct calculation from cosmological parameters gives value h=6x10(-27) (erg s). We also confirm that Planck constant (and hence other fundamental constants which depend on h) is varied on time due to changing of geometry.

As an example the variation of the fine structure constant is calculated. Its relative variation ((da/dt)/a) consist 1.0x10(-18) (1/s).

We show that on the Finsler manifold characterized by adiabatically changed geometry, classical free electromagnetic field is quantized geometrically, from the properties of the manifold in such manner that adiabatic invariant of field is ET=6x10(-27)=h.

Electrodynamic equations on the Finslerian manifold are suggested. It is stressed that quantization naturally appears from these equations and is provoked by adiabatically changed geometry of manifold. We consider in details two direct consequences of the equations: i) cosmological redshift of photons and ii) effects of Aharonov – Bohm that immediately follow from equations. It is shown that quantization of system consists of electromagnetic field and baryonic components (like atoms) is obvious and has clear explanation.

1 Introduction

The problem of formulation of general theory which could naturally unify General Relativity (GR) and Quantum Theory (QT) is of most fundamental and actual problem of modern physics. But to resolve this problem we should first to know what is the Planck constant. This is the question which open the door and allow us to find unified theory for all branches of physics. To obtain this key we have first to remember some important facts.

1) Planck constant in quantum mechanics always appears together with derivatives and in equations them appears in the same power. This fact clearly points out on the possible relation between Planck constant and geometry.

2) Einstein [1] and later Debye [2] have shown from thermodynamical approach that electromagnetic field is quantized alone, without any assumption on the nature of oscillators. So this is another hint that it should be quantized from geometry.

3) Recently the cornerstone result was announced [3] in respect to the quantization at very small scales. It was found that at the small scales the behavior (movements) of quantum system becomes classical one (see paper [3] for details). This result also argue that QT is pure geometrical phenomenon and it disappears at small scales when geometry becomes to be approximately Euclidean.

4) Recently we have publish result which clearly point out into geometrical origin of the Planck constant [4]. In this paper we have shown that the Planck constant is adiabatic invariant of electromagnetic field on the adiabatically changed Finslerian manifold. From this fact the variation of the Planck constant on the time directly follows (and hence variations of fundamental constants, for example variation of the fine structure constant, due to adiabatically changed geometry [4,5]).

On the one hand we have these serious arguments for the geometric origin of quantization, but on the other hand we have also serious problems with Riemannian geometry. General relativity was created as theory on (pseudo-) Riemannian manifold (we will use farther in this paper "Riemannian" instead of "pseudo-Remannian"). Such choice was not favored by some serious arguments but only by the fact the real geometry is curved. Metrical function $L(x)$ in this case depends only on the coordinates and this fact applies some restrictions on the theory, and lead to the serious problem with singularity. However, as it follows from experiments on measurement of the cosmological constant, our Universe is expanding with acceleration and for this reason complete geometry of Finsler should be favoured. On the Finslerian manifold the metric function depends not only of coordinates, but also on velocities $L = L(x, \dot{x})$ and (as it will be shown in this paper) this fact allows us to introduce the cosmological constant by natural way, calculate from geometry correct value of Planck constant and unify gravity, electrodynamics and QT. From the Finsler geometry in natural way follows Hubble constant, cosmological constant, quantization (as we will show in this paper) and much more and we can conclude that Finsler geometry naturally comply with all observational data.

In this paper we obtain classical equations of motion for a system on the manyfold with adiabatically changed geometry and supplied by Finslerian metrical function $L(x, \dot{x})$. We show how the Planck's constant naturally appears from geometry and within (3+1) formalism write exact relation between h on the one hand and scalar curvature and cosmological constant on the other hand.

We show how the classical electromagnetic field is naturally quantized due to existence of adiabatic invariant of the field on adiabatically changed Finslerian manifold.

Finally we write equations of electrodynamics, which classically (geometrically) describe quantization of electromagnetic field. We clearly show two important particular cases for these equations: i) case of free electromagnetic field, when quantization appears from geometry and leads to losses of energy by propagating photon (so called cosmological redshift), and ii) the Aharonov - Bohm effects, which immediately follows from equations.

All-around in this paper we suppose that Latin indexes $i, j, k, l, m = 1, 2, 3$ and greek $\alpha, \beta, ...\mu, \nu... = 0, 1, 2, 3$. Signature of metric is $(1, -1, -1, -1)$.

2 Adiabatic invariant and general formalism

let M be an 3-dimensional, class C^3 manifold characterized by scalar curvature $\mathcal{R} = 2/R^2$, where R is the curvature radius in point x^k, where x^k is a local coordinate on an open subset $U \subset M$. Let suppose M be supplied by Finsler metric $L(x, \dot{x})$ and write a 1-parameter family of hypersurfaces on the X_n defined by equation

$$S_m(x^k, \dot{x}^k) = S_M(x^k, \dot{x}^k) \tag{1}$$

Here S_m stay for matter action and S_M corresponds to a 1 - parameter family of hypersurfaces on the M.

Our aim is to write the geodesic equations (Hamilton or Lagrange - Euler equations) for this general case. In classical physics right hand part is constant or zero and we get usual classical Hamilton (or Lagrange - Euler) equations. In the case of General Relativity we put there the only invariant we have in pseudo-Riemann geometry - scalar curvature of manifold $\mathcal{R}$. Let us consider what happens in general case, when the right-hand term consist of the adiabatically changed parameters of the Finslerian manifold. In this case by varying (1)

$$\delta S_m = \int \delta \mathcal{L}_m(x^k, \dot{x}^k) dt = \int \delta(p_k dx^k - H dt) = \delta S_M = \int \delta \mathcal{L}_M(x^k, \dot{x}^k) dt \tag{2}$$

we immediately obtain Hamilton - like equations

$$\frac{dp_k}{dt} = -\frac{\partial H}{\partial x^k} - \frac{\partial \mathcal{L}_M}{\partial x^k} \tag{3}$$

and

$$\frac{dx^k}{dt} = \frac{\partial H}{\partial p_k} + \frac{\partial \mathcal{L}_M}{\partial p_k} \quad . \tag{4}$$

One can see there appears an additional force in right - hand term of equation (3) and an additional velocity in (4), which we naturally can attribute to cosmological constant (acceleration) and to the Hubble constant ($v = Hx$). It actually corresponds to the fact that there no exist absolutely closed systems and these additional terms appear due to adiabatic changes of geometry (tensor of metric) because of expansion of the Universe.

Absolutely the same way we can write equations of Lagrange - Euler varying (1):

$$\frac{\partial \mathcal{L}_m}{\partial x^k} - \frac{d}{dt}\frac{\partial \mathcal{L}_m}{\partial \dot{x}^k} = \frac{\partial \mathcal{L}_M}{\partial x^k} - \frac{d}{dt}\frac{\partial \mathcal{L}_M}{\partial \dot{x}^k} \quad . \tag{5}$$

As one can see in right part of this equation again appear two additional terms due to expansion of the Universe (due to changing of geometry of the manifold, as the system under consideration is moving).

3 Exact Planck's constant value and quantization of electromagnetic field

Let us calculate value of the Planck constant from the parameters which characterize the Finslerian manifold. Consider a generalized system distributed over volume. Let $T_p(M)$ and $T_p^*(M)$ be respectively tangent and cotangent bundles on M, where $p_\alpha \in T_p(M)$ and $p^\alpha \in T_p^*(M)$ are covariant and contravariant components of corresponding impulse.

Total momentum density of our system, in unit volume summed over all directions is given by expression

$$p = \frac{c^3}{8\pi G}\mathcal{R} = \frac{c^3}{4\pi G}\frac{1}{R^2} \quad . \tag{6}$$

Here, as usually, $\mathcal{R}$ =$2/R^2$ is the scalar curvature and R is the radius of curvature.

But on the Finsler manifold the tensor of metric depends on x and $\dot{x}$ for this reason the momentum $p^2 = g^{\mu\nu}(x,\dot{x})p_\mu p_\nu$ also depends on x and $\dot{x}$. Let's fix the spatial part of the coordinate system x_0. In this case we have $p(t,\dot{x})$ and, hence $R(t,\dot{x})$ too. So we can write

$$\delta p = \frac{c^3}{2\pi G}\frac{1}{R^3}\delta R(t,\dot{x}) = \frac{c^3}{2\pi G}\frac{1}{R^3}\left(\delta R(t) + \frac{\partial R(H)}{\partial H}\delta H\right) \quad , \tag{7}$$

here we note that $R(t,\dot{x}) = R(t, Hx_0)$ where H is Hubble constant.

But by taking into account that

$$\frac{\partial R}{\partial t} = \frac{\partial}{\partial t}\frac{c}{2H} = -\frac{c}{2H^2}\frac{\partial H}{\partial t} \tag{8}$$

we can write

$$\frac{\partial R}{\partial H} = \frac{\partial R}{\partial t}\frac{\partial t}{\partial H} = -\frac{c}{2H^2} \quad . \tag{9}$$

Let us consider variation of Hubble constant.

From the relation $\dot{x} = Hx$ we can find

$$\delta \dot{x} = x\delta H + H\delta x \tag{10}$$

or

$$x\delta H = \ddot{x}\delta t - H\delta x \tag{11}$$

But the only cosmological acceleration we have experimentally measured is associated with the cosmological constant Λ , so we can write for this variation

$$\delta H = \left(c^2\Lambda - H^2\right)\delta t \tag{12}$$

Substituting these expressions into (7) we find

$$\delta p = \frac{2c^3 H}{\pi G}\left(\frac{2H^2}{c^2} - \Lambda\right)\delta t \quad , \tag{13}$$

or taking into account that $\mathcal{R} = 2/R^2$ and by changing volume from the spherical coordinates to the euclidean ones, we obtain

$$\delta p = \frac{3c^3 H}{8\pi^2 G}\left(\mathcal{R} - 4\Lambda\right)\delta t \quad . \tag{14}$$

This is variation of momentum (in unit volume in 3 directions) of our generalized system located on the Finslerian manifold, due to adiabatically changed geometry.

Now we are ready to write complete adiabatic invariant for a free propagating electromagnetic field. The components of the 4 - momentum p of free electromagnetic field propagating on the Finslerian manifold with adiabatically changed geometry are varied on time. This variation proceeds adiabatically and can be considered as lineal function i.e. for energy ε of the field, for example, we have

$$\frac{\delta\varepsilon}{\varepsilon} = -\frac{\delta t}{t} \tag{15}$$

so, the adiabatic invariant we are interested in is

$$\varepsilon t = -\frac{\delta\varepsilon}{\delta t}t^2 \quad . \tag{16}$$

But

$$\delta\varepsilon = c\delta p_k \quad . \tag{17}$$

By substituting (14) into (16) we can write finally (we divide δp by factor 3 because we are interesting only in one direction of the momentum)

$$\varepsilon t = -ct^2\frac{\partial p}{\partial t} = -\frac{c^4 H}{8\pi^2 G}\left(\mathcal{R} - 4\Lambda\right)t^2 = \eta_0 \tag{18}$$

so, for one second and measured values of $H = 73\ kms^{-1}Mpc^{-1} = 2.4 \cdot 10^{-18}s^{-1}$ and $\Lambda = 1.7 \cdot 10^{-56}cm^{-2}$ [6] we have for this adiabatic invariant $\eta_0 = h = 6 \cdot 10^{-27}\,(ergs \cdot s.)$ as it should be. In the same way we can obtain similar relations for other components of 4-momentum:

$$p_\gamma x^\gamma = \eta_\gamma \tag{19}$$

(there is no summation over γ in this relation and for the photon propagating in direction x^3 the components $p_1 = p_2 = 0$). Introduced here 4-vector η_γ has components in unit volume:

$$\eta_\gamma = (h, h, h, h) \quad . \tag{20}$$

Here the adiabatic invariant for electromagnetic field (Planck constant), which depends clearly on the parameters of the manifold $\mathcal{R}$ and Λ (and consequently depends on time) is:

$$h = -\frac{c^4 H}{8\pi^2 G}\,(\mathcal{R} - 4\Lambda)\,t^2 \tag{21}$$

that give for unit time at present epoch $h = 6 \cdot 10^{-27}\,(ergs \cdot s.)$ as it was mentioned above. From this relation it is easy to see the Planck constant depends on time as $h \sim 1/t$.

3.1 Hilbert integral

Now let's consider integral of Hilbert for particular case of the free electromagnetic field propagating along geodesic on the adiabatically changed Finslerian manifold:

$$\int p_\alpha dq^\alpha = \Delta S_M \quad , \tag{22}$$

where p_α is 4-momentum of the field, q^α is generalized coordinate and right hand term δS_M, as before, corresponds to the changing of the 1-parameter family of hypersurfaces due to adiabatic variation of geometry as system under consideration is moving on M . For electromagnetic field with Lagrangian $\mathcal{L}_m = F_{\mu\nu}F^{\mu\nu}/16\pi$ we have from (22)

$$\frac{1}{c}\int \frac{\partial \mathcal{L}_m}{\partial A_{\mu,\nu}} A_{\mu,\sigma} dx^\sigma = \Delta S_M \tag{23}$$

By taking into account that tensor of energy - momentum is

$$T^\nu_\sigma = \frac{\partial \mathcal{L}_m}{\partial A_{\mu,\nu}} A_{\mu,\sigma} - \delta^\nu_\sigma \mathcal{L}_m \tag{24}$$

we can write for propagating classical electromagnetic field

$$\frac{1}{c}\int\left(T^{\nu}_{\sigma}+\delta^{\nu}_{\sigma}\mathcal{L}_m\right)dx^{\sigma}=\Delta S_M \tag{25}$$

If the field propagate in the direction x_3, than electric field $\mathbf{E}=E_1$ and magnetic field $\mathbf{H}=H_2$. So the only non-zero components of the field tensor $F_{\mu\nu}$ are $F_{01}=-F_{10}=E_1$; $F_{13}=-F_{31}=-H_2$ and hence $T^{00}=T^{33}=T^{30}=(E^2+H^2)/8\pi$.

In this case for the 00 - component, for example, by taking into account (18) we have

$$\frac{E^2+H^2}{8\pi}=\frac{h}{T}=h\nu \tag{26}$$

and similar relations one can write for other components.

So, as one can see the classical electromagnetic field is quantized due to adiabatic variation of the Finslerian manifold and we do not need some artificial methods to quantize it.

3.2 Variation of the fine structure constant

As it was shown before [5] even on the Riemannian manifold the value of the fine structure constant is changed adiabatically on time. In the case of Finslerian manifold this variation is smaller by factor 2/3 due to presence of the cosmological constant. To show this let's start from (14). For one direction (divided by factor 3) we have from (14):

$$\delta p=\frac{c^3H}{8\pi^2G}\left(\mathcal{R}-4\Lambda\right)\delta t \quad . \tag{27}$$

But the fine structure constant is $\alpha=V/c$ where V is electron velocity at the first Bohr orbit. Momentum in this case we can write as

$$P=\frac{m\alpha c}{\sqrt{1-\alpha^2}} \tag{28}$$

so

$$\delta P=\frac{mc}{\left(1-\alpha^2\right)^{3/2}}\delta\alpha \tag{29}$$

and

$$\delta\alpha=\frac{\left(1-\alpha^2\right)^{3/2}Hc^3}{mc8\pi^2G}\left(\mathcal{R}-4\Lambda\right)\delta t \tag{30}$$

that give us value $\dot{\alpha}/\alpha=-1.03\cdot10^{-18}$ (for 1 second), $\mathcal{R}=2/R^2$, $R=c/2H$ (see [5]), $H=73\ kms^{-1}Mpc^{-1}=2.4\cdot10^{-18}s^{-1}$ and $\Lambda=1.7\cdot10^{-56}cm^{-2}$).

4 Electrodynamics on the Finslerian manifold

In Riemannian geometry the first pair of equations of electrodynamics is follows directly from the properties of the field tensor.

$$F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu} = A_{\nu,\mu} - A_{\mu,\nu} \tag{31}$$

where

$$A_{\mu;\nu} = A_{\mu,\nu} - \Gamma^{\sigma}_{\mu\nu} A_{\sigma} \tag{32}$$

And for this reason ($\Gamma^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu}$) the first pair of equations on Riemannian manifold with constant scalar curvature is

$$\partial_{\sigma} F_{\mu\nu} + \partial_{\mu} F_{\nu\sigma} + \partial_{\nu} F_{\sigma\mu} = 0 \tag{33}$$

On the Finsler manifold we can obtain the first pair in the same way, but in this case the field tensor is

$$\breve{F}_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu} \tag{34}$$

where covariant differentials DA_{μ} include now terms with the Cartan connections $C_{\mu\nu\sigma} = \frac{1}{2}\frac{\partial g_{\mu\nu}}{\partial \dot{x}^{\sigma}}$ like this $C_{\mu\ \nu}^{\ \sigma} A_{\sigma} d\dot{x}^{\nu}$ and also $\Gamma_{\mu\ \nu}^{\ \sigma} A_{\sigma} dx^{\nu}$. In the most important case, we are interested in, when the scalar curvature is small and the tensor of metric has spatial structure described by the Robertson-Walker metric, the additional terms $C^{\sigma}_{\mu\nu} A_{\sigma} d\dot{x}^{\nu}$ and $\Gamma^{\sigma}_{\mu\nu} A_{\sigma} dx^{\nu}$ in covariant derivative can be evaluated easily as $\approx A_{\sigma} dx^{\nu}/R$ (here $1/R = 2H/c = 5 \cdot 10^{-29}$ is the inverse radius of scalar curvature) so we have

$$\breve{F}_{\mu\nu} = A_{\nu\mu} - A_{\mu\nu} - t_{\mu\nu} \tag{35}$$

where our estimation for the small components consist $t_{\mu\nu} \approx A_{\mu} dx^{\nu}/R \approx A_{\mu} \cdot 10^{-29}$.

As one can see these components probably will be significant only in the vicinity of black holes and can be omitted in our present consideration.

For this reason the first pair of the equations can be written as

$$\tilde{F}_{\mu\nu;\sigma} + \tilde{F}_{\nu\sigma;\mu} + \tilde{F}_{\sigma\mu;\nu} = 0 \tag{36}$$

or, by taking account our estimations

$$\partial_{\sigma} F_{\mu\nu} + \partial_{\mu} F_{\nu\sigma} + \partial_{\nu} F_{\sigma\mu} = O(t_{\mu\nu,\sigma}) \tag{37}$$

So one can see that the first pair of electrodynamic equations remains to be the first pair of the Maxwell equations with high precision.

The second pair of equations of electrodynamics follows directly from variation of functional (1) if we consider a charge characterized by 4-current j^{α}, and the electromagnetic field on the Finslerian manifold:

$$S_m = S_M \tag{38}$$

Here S_M as before corresponds to family of the hypersurfaces on the expanded manifold. By varying S_m we have

$$\delta S_m = -\frac{1}{c}\int\limits_\Omega \left[\frac{1}{c}j^\alpha \delta A_\alpha + \frac{1}{16\pi}\delta(F_{\mu\nu}F^{\mu\nu})\right] d\Omega \tag{39}$$

here we put $F_{\mu\nu}$ instead of $\overset{\frown}{F}_{\mu\nu}$ because, as we have seen, small additional terms, corresponding to small components of $t_{\mu\nu}$ are insignificant in the case of Robertson-Walker metric. Integrating the second term by parts, we obtain

$$\delta S_m = -\frac{1}{c}\int\limits_\Omega \left[\frac{1}{c}j^\mu + \frac{1}{4\pi}\frac{\partial F^{\mu\nu}}{\partial x^\nu}\right] A_{\mu,\sigma}\delta x^\sigma d\Omega \tag{40}$$

By varying S_M we have (see eq. (16 - 20))

$$\delta S_M = \int\limits_\Omega \frac{\eta_\sigma}{(x^\sigma)^2}\delta x^\sigma d\Omega \tag{41}$$

where, as it was shown before, $\eta_\sigma = (h, h, h, h)$ in unit volume (here h is the Planck constant). The equations under discussion one can write as follows

$$\frac{1}{c}\int\limits_\Omega \left[\frac{1}{c}j^\mu + \frac{1}{4\pi}\frac{\partial F^{\mu\nu}}{\partial x^\nu}\right] A_{\mu,\sigma}\delta x^\sigma d\Omega = -\int\limits_\Omega \frac{\eta_\sigma}{(x^\sigma)^2}\delta x^\sigma d\Omega + O(\eta_\sigma^2) \tag{42}$$

or finally

$$\frac{1}{c}\left[\frac{1}{c}j^\mu + \frac{1}{4\pi}\frac{\partial F^{\mu\nu}}{\partial x^\nu}\right] A_{\mu,\sigma} = -\frac{\eta_\sigma}{(x^\sigma)^2} + O(\eta_\sigma^2) \tag{43}$$

(there is no summation over σ here).

This is the second pair of equations of electrodynamics on the adiabatically changed Finslerian manifold. The bounded electromagnetic field (second term) in this case is explicitly included into consideration, as it take place in the case of Bohmian formalism when this field appears in QT as quantum potential (see [7] for details and also results of paper [8]). Here we consider two important cases which immediately follow from these equations.

4.1 Cosmological redshift

It is well known as the photon propagate through expanding universe its frequency (or wave length) is changed. This loss of energy by free electromagnetic field, named as cosmological redshift, appears in our equations by natural way as losses of the energy by photon due to adiabatically changed geometry of manifold.

$$\frac{1}{4\pi c}\frac{\partial F^{\mu\nu}}{\partial x^\nu}A_{\mu,\sigma}=\frac{1}{8\pi c}\frac{\partial(F_{\mu\nu}F^{\mu\nu})}{\partial x^\sigma}=-\frac{\eta_\sigma}{(x^\sigma)^2}+O(\eta_\sigma^2) \tag{44}$$

(there is no summation over σ).

4.2 The Aharonov - Bohm effects

Another important case that follows directly from the second pair of equations is the Aharonov - Bohm effects. As is known, a necessary condition for the existence of the Aharonov-Bohm (AB) effects is the presence, in the overall structure of the equations, of the "zero field" potentials which cannot be removed by gauge transformations and do not create electromagnetic fields [9,10]. These "zero - potentials" are the result of "non-trivial topology" of the area on which the particle moves [9,11,12]. Such a situation arises in electrodynamics of anisotropic media where the structure of Maxwell's equations eliminates the possibility of satisfying the boundary conditions. To satisfy regularly the boundary conditions in anisotropic media, usually introduce the zero - potential, which do not create electromagnetic fields (see [9] and references therein). In the case of adiabatically expanding Finslerian manifold, the anisotropy of space occurs for any moving body automatically as right part of eq. (43).Therefore, it is safe to say that in the case of AB effects we are dealing directly with the anisotropy of space, due to adiabatically changed Finslerian manifold as the particle moves along its trajectory. In this case, the role of the zero potentials (which do not generate electromagnetic fields) performs variable geometry as it follows directly from (43).

In absence of electric and magnetic fields on the path of propagation of the particle under consideration, the second term disappears (but it still take place inside of the solenoid and affects our particle: "In spite of the fact that the magnetic field vanishes out of the solenoid, the phase shift in the wave functions is proportional to the corresponding magnetic flux inside of the solenoid" [13]) and we obtain (we neglect here by the small term $O(\eta_\sigma^2)$)

$$\frac{1}{c^2}j^\mu A_{\mu,\sigma}=-\frac{\eta_\sigma}{(x^\sigma)^2} \tag{45}$$

but $j^\mu=(\rho c,\rho V^k)$ (here ρ is charge density and V^k is 3-velocity) so if we put $\rho=e$ and remember that $\delta j^\mu=0$ (for this reason $A_\mu\partial_\sigma j^\mu=0$ and $A_{\mu,\sigma}j^\mu=\partial_\sigma(A_\mu j^\mu)$) we obtain by using the Gauss theorem

$$\frac{e}{c}A_0=-\frac{\eta_0}{(x^0)}\quad,\quad\frac{e}{c}A_kV^k=-\frac{\eta_k}{(x^k)}\quad. \tag{48}$$

These equations describe the electric and magnetic effects of Aharonov - Bohm (here x^0 and x^k are fixed). Namely for $\mu=0$ we have for the phase variation $\Delta\Phi$

$$e\int_t\varphi dt=-h\frac{\Delta t}{1\,\mathrm{sec.}}=-h\Delta\Phi \tag{49}$$

electric effect of Aharonov - Bohm, and when $\mu = k$ (here $k = 1, 2, 3$) we have relation

$$\frac{e}{c}\int\limits_l A_k dx^k = -h\frac{\Delta x}{1cm.} = -h\Delta\Phi \tag{50}$$

describes magnetic effect of Aharonov - Bohm.

To conclude this part we would like to stress again that whereas the bounded field (second term in (43)) do not appears in these relations, it actually affects the moving particle through potentials A_μ [13] and this bounded field corresponds to quantum potential in the Bohmian formalism [7,8].

5 Complete theory

In previous part we have obtained electrodynamic equations. They are applied in the case when the movement of charge (or j^μ) is defined. To construct self-consistent theory we should treat j^μ as variable from the beginning.

We consider now a charge characterized by 4-current j^α, and the electromagnetic field on the Finslerian manifold:

$$S_m = S_M \tag{51}$$

Here S_M as before corresponds to family of the hypersurfaces on the expanded manifold. In complete form, when $\delta j^\mu \neq 0$, we have the action

$$-\sum\int mcds - \frac{1}{c}\int\limits_\Omega \left[\frac{1}{c}j^\alpha A_\alpha + \frac{1}{16\pi}(F_{\mu\nu}F^{\mu\nu})\right] d\Omega = S_M \tag{52}$$

which describes quantum properties of our system. It is clear there are a lot of different systems and applications. For this reason it is impossible to write here a general theory, but as an example let us consider the hydrogen atom. In order to coincide with quantum mechanical calculations we should neglect by third term in (52) which corresponds to quantum potential [7,8] and gives the zero energy correction (for example in the case of harmonic oscillator it gives term 1/2 in expression for energy [4,7]). In this case by varying (52) we have

$$-\delta\int(mcds + \frac{e}{c}A_\alpha dx^\alpha) = \delta S_M \tag{53}$$

and we can write

$$\int\left[mc\frac{du_\mu}{ds} - \frac{e}{c}\left(\frac{A_\nu}{\partial x^\mu} - \frac{A_\mu}{\partial x^\nu}\right)u^\nu\right]\delta x^\mu ds = -\int\frac{\eta_\mu}{(x^\mu)^2}\delta x^\mu ds \quad . \tag{54}$$

So, finally we obtain equation of movement

$$mc\frac{du_\mu}{ds} - \frac{e}{c}F^{\mu\nu}u_\nu = -\frac{\eta_\mu}{(x^\mu)^2} \quad . \tag{55}$$

By taking into account that classical period for orbital movement of electron is

$$T = \pi e^2\sqrt{\frac{m}{2\left|E\right|^3}} \ , \tag{56}$$

in classical limit $v << c$ the straightforward calculations give the energy for first Bohr orbit, obtained from classical electrodynamic on Finslerian manifold:

$$E_1 = \frac{me^4}{2\hbar^2} \tag{57}$$

that coincide with quantum calculations. Relativistic corrections are obvious.

6 Conclusions

In this paper we confirm our previous result [4] that Planck constant is adiabatic invariant of electromagnetic field propagating on the adiabatically changed Finslerian manifold. Direct calculation of the Planck constant value made from cosmological parameters gives $h = 6 \cdot 10^{-27}$ $(ergs \cdot s.)$ that is in great agreement with measured value. We also confirm that Planck constant (and hence other fundamental constants which depend on h) is varied on time due to changing of geometry of the manifold.

As an example we suggest calculation of the fine structure constant variation. Obtained value consist $\dot{\alpha}/\alpha = -1.03 \cdot 10^{-18}$ (for 1 second) and this variation is expected to be measured in nearest future.

We show that on the Finslerian manifold characterized by adiabatically changed geometry, classical free electromagnetic field is quantized geometrically, from the properties of the manifold.

Equations for electrodynamics on the Finslerian manifold are suggested. It is shown that quantization naturally appears from these equations and is provoked by adiabatically changed geometry of manifold. We consider in details two direct consequences of the equations - cosmological redshift of photons and effects of Aharonov - Bohm.

Finally we show how quantization of systems which consist of electromagnetic field and baryonic components (like atoms and molecules) appears.

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