Abstract

In present paper we evaluate the fine structure constant variation which should take place as the Universe is expanded and its curvature is changed adiabatically. This changing of the fine structure constant is attributed to the energy lost by physical system (consist of baryonic component and electromagnetic field) due to expansion of our Universe. Obtained ratio $\dot{\alpha}/\alpha = 1 \cdot 10^{-18}$ (per second) is only five times smaller than actually reported experimental limit on this value. For this reason this variation can probably be measured within a couple of years. To argue the correctness of our approach we calculate the Planck constant as adiabatic invariant of electromagnetic field, from geometry of our Universe in the framework of the pseudo- Riemannian geometry. Finally we discuss the double clock experiment based on Al^+ and Hg^+ clocks carried out by T. Rosenband et al. (Science 2008). We show that in this particular case there is an error in method and this way the value $\dot{\alpha}/\alpha$ can not be measured if the fine structure constant is changed adiabatically.

Variation of the fine structure constant

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1 Introduction

The question regarding the changes of the fundamental constants attracted wide attention of the scientific community over the last decades. Every year a lot of papers on this subject are published: both in theory and measurement methods (see [1,2] and references therein).Such interest is due to the huge importance of the problem of variation of fundamental constants for understanding fundaments of physics. Particular attention is paid for the variation of the fine structure constant, because it is a basic parameter for QED. It is known, the search for such variations are conducted in laboratories [3-7] and with the cosmological data - from observed spectra of distant quasars [1,2,8,9,10].Until now, such variations have not been detected yet, but it is important to note that in the last decade, the accuracy of laboratory measurements approached closely to the limit of variation of fundamental constants that must take place through the adiabatic change in the geometry of our universe. For this reason, clearly there is a need to carry out such calculations.

In this paper we fill this gap and suggest the calculation of the fine structure constant variation over time, which must take place due to the adiabatic changes of scalar curvature determined by expansion of our universe. Besides this we also evaluate the adiabatic invariant for free electromagnetic field (propagating on the manifold characterized by adiabatically changed curvature) which actually is the Planck constant. All calculations are carried out in the framework of pseudo - Riemannian geometry and for this reason obtained values are slightly differ (by factor 3/2) of their real values. Finally we explain why this variations are not detected in the experiments based on comparison of two different frequencies like those discussed in recent papers [6,7].

2 Changing of the fine structure constant due to expansion of the Universe

Let us consider a system consists of a classical field on Riemannian manifold characterized by the adiabatically changed curvature. In this case as it was shown in 2014 [11] (see also the next part of this paper) such a system is characterized by an adiabatic invariant, which for the electromagnetic field is actually the Planck constant. Moreover, this adiabatic invariant depends on the scalar curvature of the Universe in the point of observation and for this reason is varied over time [11]. The fine structure constant in turn depends on h ($\alpha = e^2/hc$) and for this reason its value also must change over time. It should be stressed here, this consideration is applied not only to the classical fields (particularly to the electromagnetic field), but also to any adiabatically isolated system consisting of fields and baryonic matter interacting with these fields. In this case, the system is depending adiabatically on the geometry of manifold by means of the fields involved in the system. So as the Universe expands, any physical system which contains fields (for example an atom) will lose its energy adiabatically.

How large is this variation in energy? To begin with let us make very preliminary estimation of the effect we are interested in. Consider a system consist of the classical field and characterized by energy E distributed over volume V (we can put $V = 1cm^3$). In this case the changing of the energy due to expansion of the Universe is

$$\frac{\delta E}{E} = -\frac{\delta V}{V} = -3\frac{\delta l}{l} \tag{1}$$

But in consistence with Hubble relation

$$\delta l = Hl\delta t \tag{2}$$

So, we can evaluate

$$\frac{\delta E}{E} \approx \frac{\delta \alpha}{\alpha} \approx -3H\delta t = -7 \cdot 10^{-18}\delta t \tag{3}$$

This very simple estimation gives us an idea about the value of variation we should expect in general case.

Now let M be an 3-dimensional C^∞ manifold characterized by scalar curvature $\mathcal{R} = 2/R^2$, where R is the curvature radius, x be a local coordinate on an open subset $U \subset M$. $T_p(M)$ and $T_p^*(M)$ are respectively tangent and cotangent bundles on M, where $P_\alpha \in T_p(M)$ and $P^\alpha \in T_p^*(M)$ are covariant and contravariant components of corresponding 4-momentum.

We are interested in variation of the 4-momentum components P as functions of the Universe radius R and, consequently, time t. By taking into account relation $x = R\varphi$, we can write projection of x on tangent and cotangent bundles on M as

$$P^\alpha = \xi R \sin\varphi \tag{4}$$

$$P_\alpha = \xi R \tan\varphi \tag{5}$$

Where coefficients $\xi = \frac{2c}{\kappa}$ (here $\kappa = 8\pi G/c^2$ is the coupling constant for the Einstein field equations) are written to comply $\mathcal{R} = \frac{\kappa}{c^2}T$ in classical limit, and factor 2 appears from relation $\mathcal{R} = 2/R^2$. In this case the momentum can be written as

$$P = \sqrt{P_\alpha P^\alpha} = \frac{2c}{\kappa} R \frac{\sin\varphi}{\sqrt{\cos\varphi}} \tag{6}$$

where R is for the local (effective) radius of curvature of the universe in the point in which our system is localized.

By taking into account that $\varphi \ll 1$ for any reasonable laboratory system, we can restrict our consideration by first and second terms of the expansion of $\sin\varphi$ and $\sqrt{\cos\varphi}$, then we get

$$P = \xi R \left(\varphi + \frac{\varphi^3}{12}\right) \tag{7}$$

As our manifold M expands, the value of P is changing and taking into account that $x = R\varphi$, we immediately obtain

$$\delta P = -\frac{c^3}{24\pi G R^3}\delta R \tag{8}$$

It should be stressed here - we write this expression for propagating electromagnetic field localized within a unit volume. Actually it is the momentum loses by system due to adiabatic changing of the manifold's curvature.

To evaluate this expression, we need reduce R to a measurable parameter. Actually we have such a parameter, named as Hubble constant H. But H give us relation for passing trajectory l: $\delta l = H l \delta t$.

To establish relation between R and l let us imagine a fly walking over globe with velocity c, whereas we inflate the globe such that $\dot{R} = c$ too. It is easy to show that in this simple case the integrated length l is $l = 2R$. Actually this is the length which pass a photon when it is propagating on manifold while its curvature is changing due to expansion.

So in this case our expression can be rewritten as:

$$\delta P = -\frac{cH^3}{6\pi G}\delta t \tag{9}$$

To evaluate variation of the fine structure constant α, let's remember that historically it was introduced by Sommerfeld as $\alpha = v/c$, where v is the electron velocity at the first Bohr orbit for the hydrogen atom. By taking into account that a loss of momentum by electron on the first Bohr orbit is equal to the loss of momentum by the bounded electromagnetic field due to adiabatically changing curvature governed by expansion of our universe (see also [12]) we can write

$$P = \frac{m\alpha c}{\sqrt{1-\alpha^2}} \tag{10}$$

and varying it obtain

$$\delta P = \frac{mc}{\left(1-\alpha^2\right)^{3/2}}\delta\alpha \tag{11}$$

Substituting this expression into (9), we find

$$\delta\alpha = -\frac{\left(1-\alpha^2\right)^{3/2} H^3}{6\pi G m}\delta t \tag{12}$$

This is variation of the fine structure constant on the time due to adiabatically changed curvature of the Riemannian manifold.

It should be stressed here, this expression for $\delta\alpha$ coincide well with that obtained in [11], within Einstein-Cartan geometry framework, if we write it for the Riemannian manifold (when $\Lambda = 0$).

Namely we have in [11] ($\Lambda = 0$):

$$\alpha = \frac{c^2}{32\pi^2 G m}\mathcal{R} \tag{13}$$

By varying this expression we immediately obtain

$$\delta\alpha = -\frac{H^3}{2\pi^2 G m}\delta t \tag{14}$$

that perfectly agree with above obtained expression (12).

Direct calculation for $H = 73\ kms^{-1}Mpc^{-1} = 2.4 \cdot 10^{-18}s^{-1}$ give us value $\dot{\alpha}/\alpha = -1.7 \cdot 10^{-18}$ (for 1 second).

This value is about 5 times smaller if compared with reported sensitivity $\dot{\alpha}/\alpha < 5 \cdot 10^{-18}$ [3], but the difference is not so large and we hope the required sensitivity will be achieved within a couple of years.

3 Planck constant from the first principles

Einstein [13] and later Debye [14] at the beginning of XX century show from thermodynamics that electromagnetic field is quantized and this fact do not depends of the oscillators properties (properties of baryonic matter). Unfortunately this result was not paid duly attention and historically it is baryonic matter that was quantized first whereas the electromagnetic field was quantized much later in 1950 by Gupta [15] and Bleuler [16].

In this part of paper we show how the electromagnetic field is quantized on the pseudo - Riemannian manifold with adiabatically changed scalar curvature. Namely we obtain from the geometry of our Universe the adiabatic invariant for Electromagnetic field (which is well known as Planck constant).

As we have seen from the first part of this paper, the momentum P and energy of electromagnetic field on the manifold with adiabatically changed curvature are changed on the time. This variation proceeds adiabatically and can be considered as lineal function i.e.

$$\frac{\delta E}{E} = -\frac{\delta t}{t} \tag{15}$$

From this expression we can immediately write the adiabatic invariant we are interested in

$$Et = -\frac{\delta E}{\delta t} t^2 \tag{16}$$

But for free electromagnetic field we have

$$\delta E = c\delta P \tag{17}$$

By substituting δP obtained before into this expression we can write finally for energy in 1 $cm.^{-3}$

$$Et = \frac{c^2 H^3}{6\pi G} t^2 = 9.93 \cdot 10^{-27}\,(ergs \cdot s.) \tag{18}$$

for one second. It is a very good coincidence with real value $h = 6.6 \cdot 10^{-27}\,(ergs \cdot s.)$ for such a simple model we have considered here within the framework of the Riemannian geometry which is differ of the Riemann-Cartan geometry by the presence of the cosmological constant. It should be stressed, we do not include the cosmological constant Λ into consideration because on the one hand it naturally appears only in the Riemann-Cartan (and more complete Finsler) geometry. On the other hand this paper is dedicated mainly to the problem of the fine structure constant variation, and it is difficult discus here all details of real geometry of our Universe and nature of cosmological constant. We just note here that if $\Lambda = 1.7 \cdot 10^{-56}$ taken into account, the obtained by us value of the Planck constant will decrease slightly and reach actually measured $h = 6 \cdot 10^{-27}\,(ergs \cdot s.)$. We will show this calculation in our next paper, but also reader can see these details in our previous paper [11].

To conclude this part we stress again we prove geometrically the fact that the electromagnetic field is quantized alone. To do this we need not oscillators and baryonic matter. The only we need for free electromagnetic field to be quantized is adiabatically changed curvature of manifold.

4 The Hg^+ and Al^+ optical clocks experiment

In first part of the paper we have shown that the fine – structure constant variation due to adiabatically changed curvature of manifold is $\dot{\alpha}/\alpha = 1.7 \cdot 10^{-18}\,(s^{-1})$. As it was mentioned above, at present time the experimental constrain on the $\dot{\alpha}/\alpha$ is very close to calculated value and consist $\dot{\alpha}/\alpha < 5 \cdot$

$10^{-18}\left(s^{-1}\right)$ [3], so, probably within a couple of years experimental facilities will be able to measure the variation of fine structure constant discussed above, caused by expansion of the universe .

However there is another type of experiments based on comparison of frequencies variation of two optical clocks. Most precise measurements of this kind were reported by Rosenband et al in 2008 [6] (see also paper [7] for the same problem) Al^+ and Hg^+ single-ion optical clocks. In this paper the preliminary constraint on the temporal variation of the fine-structure constant $\dot{\alpha}/\alpha < 5 \cdot 10^{-17}\left(yr^{-1}\right)$ were suggested, that actually corresponds to variation $\dot{\alpha}/\alpha < 3 \cdot 10^{-25}\left(s^{-1}\right)$. In this case a reasonable question arises: why this variation $\dot{\alpha}/\alpha = 10^{-18}\left(s^{-1}\right)$ was not measured, whereas (as we have seen before) it inescapably should appears due to expansion of the Universe? The answer on this question is simple: because the changing proceeds adiabatically. Let us consider this issue in details by taking as an example the paper [6] (the same way one can explain the result reported in [7]). The authors of paper [6] reported that they were measuring variation of ratio of frequencies, i.e. $\delta\left(\nu_{Al^+}/\nu_{Hg^+}\right)$.

To make our expressions more clear, let us write 1 for Al^+ and 2 for Hg^+ In this case the measured variation can be written as:

$$\delta\left(\frac{\nu_1}{\nu_2}\right) = \frac{E_1}{E_2}\left(\frac{\delta E_1}{E_1} - \frac{\delta E_2}{E_2}\right) \tag{19}$$

where E_1 and E_2 are the energies of transitions $i \to f$ for Al^+ and Hg^+ respectively.

$$\left(\frac{\delta E_1}{E_1} - \frac{\delta E_2}{E_2}\right) = \frac{\delta\left(E_{1i} - E_{1f}\right)}{E_{1i} - E_{1f}} - \frac{\delta\left(E_{2i} - E_{2f}\right)}{E_{2i} - E_{2f}} = \tag{20}$$

$$= -\frac{\delta E_{1i}}{E_{1i}}\frac{E_{1i}}{E_{1f}}\frac{1}{1-\frac{E_{1i}}{E_{1f}}} + \frac{\delta E_{1f}}{E_{1f}}\frac{1}{1-\frac{E_{1i}}{E_{1f}}} + \frac{\delta E_{2i}}{E_{2i}}\frac{E_{2i}}{E_{2f}}\frac{1}{1-\frac{E_{2i}}{E_{2f}}} - \frac{\delta E_{2f}}{E_{2f}}\frac{1}{1-\frac{E_{2i}}{E_{2f}}}$$

but for adiabatic changing we have $\frac{\delta E_{1i}}{E_{1i}} = \frac{\delta E_{1f}}{E_{1f}} = \frac{\delta E_{2i}}{E_{2i}} = \frac{\delta E_{2f}}{E_{2f}}$, thus

$$\frac{\delta E_{1i}}{E_{1i}} - \frac{\delta E_{1i}}{E_{1i}} = 0 \tag{21}$$

therefore

$$\delta\left(\frac{\nu_{Al^+}}{\nu_{Hg^+}}\right) = 0 \tag{22}$$

So one can conclude that adiabatic changes cannot be observed in such experiments, when the frequencies of two single-ion optical clocks are compared.

5 Conclusions

We calculate variation of the fine structure constant which must take place due to expansion of our Universe. For the pseudo – Riemannian manifold it consist $\dot{\alpha}/\alpha = 1.7 \cdot 10^{-18} \left(s^{-1}\right)$ that only 5 time smaller than currently established constrains on this value $\dot{\alpha}/\alpha < 5 \cdot 10^{-18} \left(s^{-1}\right)$ [3].

We also show that even on the pseudo – Riemannian manifold there exist adiabatic invariant for electromagnetic field which depend on the curvature and has a value very close (it differ by factor 3/2) to the laboratory measured Planck constant. Exact value for the Planck constant, as function of curvature and cosmological constant, can be calculated only within the framework of the complete Finslerian geometry and can be found in [11].

It is shown that double clock experiment is not appropriate for measurement of adiabatically changed values (particularly for the measurements of the fine structure constant variation).

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