BENCZE MIHÁLY
FLORIN POPOVICI
FLORENTIN SMARANDACHE

The Solution of OQ.I02

In Florentin Smarandache: "Collected Papers", vol. III. Oradea (Romania): Abaddaba, 2000.

The Solution Of OQ.102[1]

In the "Octogon" Vol.5, No.2, Zoltán Blázik, in the open problem OQ.102, asked if there exists a polynomial P(x,y) of at most second degree such that on the set {1,2,3}x{1,2,3} it takes the values 1, 2, 3, 4, 5, 6, 7, 8, 10 , each of them exactly once. We show that doesn't exist such a polynom. Let $P(x,y)=Ax^2+Bxy+Cy^2+Dx+Ey+F$ be a such polynom. It results that P(1,1)-2P(1,2)+P(1,3)-2P(2,1)+4P(2,2)-2P(2,3)+P(3,1)--2P(3,2)+P(3,3)=0. In this sum there are only integer numbers, and each coefficient divided by 3 give one remainder. From this one gets that

0=P(1,1)-2P(1,2)+P(1,3)-2P(2,1)+4P(2,2)-2P(2,3)+P(3,1)-2P(3,2)+P(3, 3) ≡ P(1,1) +P(1,2) +P(1,3)+P(2,1)+P(2,2)+P(2,3)+P(3, 1)+P(3,2)+P(3,3)= 1 +2+3+4+5+6+7+8+10≡46(*mod3*) and this is a contradiction.

Next we propose the following open question:

Is there a polynomial $P(x_1,x_2,...,x_n)$ of at most degree n such that on the set {1,2,...,n,n+1}x{1,2,...,n,n+1}x...x{1,2,...,n,n+1} (the braces are repeated n times) it takes the values $1,2,3, ...,n,...,(n+1)^2-2,(n+1)^2-1,(n+1)^2+1$ exactly once?

["Octogon", Vol.6, No.1, 81, 1998.]

[1]Together with Mihály Bencze and Florin Popovici