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A Solution of OQ. 128

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A Solution of OQ. 128[1]

In "Octogon", vol. 6, Nr. 1, April 1998, Mihály Bencze proposed the following open question:

"Let $A_1 A_2 \ldots A_n$ be a convex polygon and $\{B_1\} = A_1A_3 \cap A_2A_n$, $\{B_2\} = A_2A_4 \cap A_1A_3, \ldots, \{B_n\} = A_1 A_{n-1} \cap A_2A_n$. Prove that

$$\frac{A_1B_1}{B_1A_2} \cdot \frac{A_2B_2}{B_2A_3} \cdot \ldots \cdot \frac{A_{n-1}B_{n-1}}{B_nA_1} = 1".$$

Let:

$x_1 = m(A_2A_1B_1)$, $x_2 = (A_1A_2B_1)$, $x_3 = m(A_3A_1A_2)$,
$x_4 = m(A_2A_3B_2), \ldots, x_{k+1} = m(A_{k+1}A_kB_k)$,
$x_{k+2} = m(A_kA_{k+1}B_k), \ldots, x_{2n-1} = m(A_1A_nB_n)$,
$x_{2n} = m(A_nA_1B_n)$ and $A_kA_{k+1} = a_k$ $(k = 1, 2, \ldots, n)$.

Using the sinus theorem in the triangle $A_kB_kA_{k+1}$, we obtain:

$$(1) \qquad \frac{A_kB_k}{B_kA_{k+1}} = \frac{\sin x_{k+1}}{\sin x_k}.$$

Using again the sinus theorem in the triangle $A_kA_{k+1}A_{k+2}$, we obtain:

$$(2) \qquad \frac{A_kA_{k+1}}{A_{k+1}A_{k+2}} = \frac{a_k}{a_{k+1}} = \frac{\sin x_{k+3}}{\sin x_k}.$$

From (1) and (2), by multiplication, we obtain the proposed relation.

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[1] Together with Marian Dinca, Mihály Bencze, and Szilárd András.