The superluminal signal in quantum billiard and in the Casimir configuration

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Abstract

The quantum energy levels of electron inside the box with the infinite barriers at point 0 and l is considered. The situation is then extended to the thee dimensions. Quantum mechanics of such so called quantum billiard does not involve the retarded wave functions (the retarded Green functions) and it means that the quantum pressure is instantaneous at the walls of the box. The instantaneous process is equal to the action at a distance, or to the existence of the superluminal signals inside of the quantum box. The similar situation is in case of the Casimir effect between two capacitor plates.

Billiard is the two-dimensional stadium where the balls are confined inside the stadium. The three-dimensional billiard dynamics of particles is only the generalization and extension of the two-dimensional billiard.

Let us firstly, consider quantum mechanics with the electron confined in the box with the infinite barriers at point 0 and l. Then, the energy levels of electron inside the box is (Sokolov et al. 1962)

$$E_n = \frac{\pi^2\hbar^2 n^2}{2ml^2} \tag{1}$$

and the corresponding wave function is

$$\psi_n = \sqrt{\frac{2}{l}} \sin\left(\pi n \frac{x}{l}\right). \tag{2}$$

The quantum pressure caused by the quantum mechanical motion of particle is obtained by the same operation as in the Casimir effect. Or,

$$F = -\frac{\partial E_n}{\partial l} = \frac{\pi^2\hbar^2 n^2}{2ml^3}. \tag{3}$$

The physical interpretation of eq. (3) is, that if some pressure p is at point 0, then the same pressure is instantaneously without retardation in point l,

In case that the thermal box is three-dimensional, we get (Sokolov et al., 1962)

$$E_{n_1,n_2,n_3} = \frac{\pi^2\hbar^2}{2m}\left[\left(\frac{n_1}{l_1}\right)^2 + \left(\frac{n_2}{l_2}\right)^2 + \left(\frac{n_3}{l_3}\right)\right] \tag{4}$$

and the corresponding wave function is

$$\psi_{n_1,n_2,n_3} = \sqrt{\frac{8}{l_1 l_2 l_3}} \sin\left(\pi n_1 \frac{x}{l_1}\right) \sin\left(\pi n_2 \frac{x}{l_2}\right) \sin\left(\pi n_3 \frac{x}{l_3}\right). \tag{5}$$

The corresponding pressures are

$$p_{23} = -\frac{1}{l_2 l_3}\frac{\partial E_{n_1,n_2,n_3}}{\partial l_1} \tag{6}$$

$$p_{13} = -\frac{1}{l_1 l_3}\frac{\partial E_{n_1,n_2,n_3}}{\partial l_2} \tag{7}$$

$$p_{12} = -\frac{1}{l_1 l_2}\frac{\partial E_{n_1,n_2,n_3}}{\partial l_3}. \tag{8}$$

The physical interpretation of eqs. (6–8) is, that if some pressure p is at wall l_{ij}, then the same pressure is instantaneously without retardation at all opposite walls (the quantum Pascal law).

Let us only remark that the quantum pressure derived here is the perfect proof that the wave function in quantum mechanics is physical reality and not only mathematical object. The wave function is in such a way the objective form of matter, where matter is continuum which forms Universe. The similar quantum model is the Casimir effect, which is base on the quantum field theory.

The Casimir effect and the Casimir-Polder force are physical forces arising from a quantized field. They are named after the Dutch physicist Hendrik Casimir who predicted it in 1948.

The Casimir effect is an interaction between disjoint neutral bodies caused by the fluctuations of the electrodynamic vacuum. It can be explained by considering the normal modes of electromagnetic fields, which explicitly depend on the boundary (or matching) conditions on the interacting bodies surfaces. Since electromagnetic field interaction is strong for a one-atom-thick material, the Casimir effect is of interest for graphene too.

At the most basic level, the field at each point in space is a simple quantum harmonic oscillator. Excitations of the field (oscillator) correspond to the elementary particles of particle physics. However, even the vacuum has a complex structure, all calculations must be made in relation to such model of the vacuum.

The Casimir effect at zero temperature

In order to understand the Casimir effect, we follow Holstein (1992) and imagine two capacitor plates with a separation a. The field modes permitted by the boundary condition have the electrical intensity vanishing on the surface on the plates. If the normal to the surface defines the z-direction, then for the propagation in this direction wavelength varies from zero to a. If the zero point energy of the oscillators representing the quantum field is $\hbar\omega_k/2$ (Berestetskii et al., 1999), then then the total energy between the plates is given by the formula

$$U(a) = \sum_k \frac{1}{2}\hbar\omega_k. \tag{9}$$

When the plate separation is increased, more modes are permitted so the energy is increasing function of separation a. In case that the separation a is lowered, then the energy is also lowered which means that the change of energy is force of the form:

$$F = -\frac{\partial U(a)}{\partial a}. \tag{10}$$

The force has been detected for instance by Sparnay (1958) and represents the macroscopic manifestation of the validity of quantum field theory.

The quantitative evaluation of the Casimir force is as follows. Let be wave numbers k_x, k_z in the x, y direction. Then the density of states is given by the formula

$$A \int \frac{d^2k}{(2\pi)^2}, \tag{11}$$

where A is the area of the plates.

In the z-direction, on the other hand, the boundary conditions $\mathbf{E}(0) = \mathbf{E}(a) = 0$ requires

$$E \sim \sin(k_z z) \tag{12}$$

with

$$k_z = \frac{n\pi}{a} \; n = 1, 2, ... \tag{13}$$

The frequencies are

$$\omega_k = \sqrt{k_x^2 + k_y^2 + \left(\frac{n\pi}{a}\right)^2}. \tag{14}$$

The total vacuum energy of photons (with two polarizations) between plates is evidently as follows:

$$U(a) = 2\sum_{n=1}^{\infty} A \int \frac{d^2k}{(2\pi)^2} \frac{1}{2}\omega_k. \tag{15}$$

Defining

$$k = \sqrt{k_x^2 + k_y^2}, \tag{16}$$

we have from eq. (14)

$$kdk = \omega d\omega \tag{17}$$

and the new mathematical form of the total intermediate vacuum energy is

$$U(a) = A\sum_{n=1}^{\infty}\frac{1}{2\pi}\int_{\frac{n\pi}{a}}^{\infty} d\omega\omega^2. \tag{18}$$

Using the cutoff operation with $\exp(-\varepsilon\omega)$, we get the following formulas:

$$U(a) = \frac{A}{2\pi}\sum_{n=1}^{\infty}\int_{\frac{n\pi}{a}}^{\infty} d\omega\omega^2 e^{-\varepsilon\omega} = \frac{A}{2\pi}\frac{d^2}{d\varepsilon^2}\sum_{n=1}^{\infty}\int_{\frac{n\pi}{a}}^{\infty} d\omega e^{-\varepsilon\omega} =$$

$$\frac{A}{2\pi}\frac{d^2}{d\varepsilon^2}\sum_{n=1}^{\infty}\frac{1}{\varepsilon}e^{-\frac{n\pi\varepsilon}{a}} = \frac{A}{2\pi}\frac{d^2}{d\varepsilon^2}\frac{1}{\varepsilon}\left(\frac{1}{1-e^{\frac{\varepsilon\pi}{a}}} - 1\right). \tag{19}$$

After application the formula with the Bernoulli numbers B_n (Prudnikov et al., 1984)

$$\frac{1}{1-e^{-t}} = -\sum_{n=1}^{\infty} B_n \frac{t^{n-1}}{n!}, \tag{20}$$

we get for $\varepsilon \to 0$ the final formula for the attraction of two plates immersed in the quantum vacuum (Holstein, 1992):

$$\frac{1}{A}F = -\frac{\partial}{\partial a}\frac{1}{A}U(a) = -\frac{\pi^2}{240a^4}. \tag{21}$$

Let us remark in conclusion on the perspective of our physical meditation. Quantum mechanics in the box does not involve the retarded wave functions. Retardation is possible only in electromagnetism and acoustics (Pardy, 2013a ; ibid., 2013b). It means that the quantum pressure is instantaneous at the wall of the box. The instantaneous process is equal to the action at a distance, or the existence of the superluminal signals inside the quantum box. The similar situation is in case of the Casimir effect between two capacitor plates. It is not excluded that such process is preamble to the superluminal communication between astronauts. Similarly, for the colonization of the Universe, superluminal communication is the absolute necessity (Welsh et al., 1985).

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