Planck Dimensional Analysis of Big G

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Abstract

This is a short note to show how big G can be derived from dimensional analysis by assuming that the Planck length [1] is much more fundamental than Newton's gravitational constant.

Key words: Big G, Newton, Planck units, Dimensional analysis, Fundamental constants, Atomism.

1 Dimensional Analysis

Haug [2, 3, 4] has suggested that Newton's gravitational constant (Big G) [5] can be written as

$$G = \frac{l_p^2 c^3}{\hbar}$$

Writing the gravitational constant in this way helps us to simplify and quantify a long series of equations in gravitational theory without changing the value of G, see [3]. This also enables us simplify the Planck units. We can find this G by solving for the Planck mass or the Planck length with respect to G, as has already been done by Haug.

We will claim that G is not anything physical or tangible and that the Planck length may be much more fundamental than the Newton's gravitational constant. If we assume the Planck length is a more fundamental constant than G, then we can also find G through "traditional" dimensional analysis. Here we will assume that the speed of light c, the Planck length l_p, and the reduced Planck constant $\hbar$ are the three fundamental constants. The dimensions of G and the three fundamental constants are

$$[G] = \frac{L^3}{MT^2}$$

$$[\hbar] = M\frac{L^2}{T}$$

$$[c] = \frac{L}{T}$$

$$[l_p] = L$$

Based on this, we have

$$
\begin{aligned}
G &= l_p^\alpha c^\beta \hbar^\gamma \\
\frac{L^3}{MT^2} &= L^\alpha \left(\frac{L}{T}\right)^\beta \left(M\frac{L^2}{T}\right)^\gamma
\end{aligned}
\tag{1}
$$

Based on this, we obtain the following three equations

$$
\begin{aligned}
\text{Lenght}: \quad & 3 = \alpha + \beta + 2\gamma & (2)\\
\text{Mass}: \quad & -1 = \gamma & (3)\\
\text{Time}: \quad & -2 = -\beta - \gamma & (4)
\end{aligned}
$$

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*e-mail espenhaug@mac.com. Thanks to Victoria Terces for helping me edit this manuscript. In version v1 I had unfortunately mistakenly swapped the dimensions for G with $\hbar$, the end results where correct, but not the derivation.

This gives us

$$\alpha = 2$$

$$\beta = 3$$

$$\gamma = -1$$

which means

$$G = \frac{l_p^2 c^3}{\hbar} \tag{5}$$

The Planck length l_p is also the reduced Compton wavelength of a Planck mass. We should also consider this in a historical perspective. The Newton gravitational constant was discovered long before the Planck length was even thought about in 1906. The Planck length was derived from the gravitational constant, the speed of light, and the Planck constant. However, it could have been done the other way around if the Planck length first had been introduced as a "hypothetical" fundamental entity. That the Newton gravitational constant was discovered before the Planck length does not necessarily make it more fundamental than the Planck length. The Newton gravitational constant was likely discovered first because it was much easier to measure; this is true even if Big G is hard to measure accurately, see [6, 7, 8, 9, 10]. Still, Big G is actually much easier to measure (indirectly) than the Planck length.

We will likely never be able to measure the Planck length directly, but only indirectly through G as well as some other measuremnts. That we can measure G and not l_p does not necessarily mean that l_p is less fundamental than the gravitational constant.

The current view that subatomic particles do not have a spatial dimension is a rather modern view. Newton assumed the most fundamental particles had spatial dimensions (a diameter) and that this particle not could be broken down further. We think it is in high time to reintroduce the Newtonian view, and we will claim that the Planck length is the diameter of the only truly fundamental particle making up all other particles and energy. The corspscular particles introduced by Newton were very similar to the indivisible particle that the Greek atomists had suggested around 2500 B.C. We think that one of the biggest mistakes in modern physics has been to totally abandon Newton's view. Haug [11] has recently shown that everything in special relativity theory can be derived from Atomism (the Newton particle), including: $E = mc^2$ and $E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$, relativistic Doppler shift and more.

Even if the reader does not support this view on the interpretation of the Planck length, the approach to writing the gravitational constant shown here has important benefits for physics because it can be used to simplify Planck units and to quantize many gravitational formulas.

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