Chapter 10

LAWS, RULES AND OTHER THINGS

10.1 More About Ultrawords.

Previously, we slightly investigated the composition of an ultraword $w \in {}^*\mathbf{M_d} - \mathbf{d}$. Using the idea of the minimum informal language $\mathrm{P}_0 \subset \mathrm{P}$, where d is denumerable and P is a propositional language, our interest now lies in completely determining the composition of ${}^*\mathbf{S}(\{w\})$. [Note: since our language is informal axiom (3) and (4) are redundant in that superfluous parentheses have been removed.] First, two defined sets.

$$\mathrm{A} = \{\mathrm{x} \mid \mathrm{x} \in \mathrm{P}_0 \text{ is an instance of an axiom for S}\} \tag{10.1.1}$$

$$\mathrm{C} = \{\mathrm{x} \mid \mathrm{x} \in \mathrm{P}_0 \text{ is a finite } (\geq 1) \text{ conjunction of members of d}\} \tag{10.1.2}$$

Notice that it is also possible to refine the set C by considering C to be an ordered conjunction with respect to the ordering of the indexing set used to index members of d. Further, as usual, we have that A, C, d are mutually disjoint.

Theorem 10.1.1 *Let $w \in {}^*\mathbf{M_d} - {}^*\mathbf{d}$ be an ultraword for infinite $\mathbf{d} \subset {}^*\mathbf{S}(\{w\})$. Then ${}^*\mathbf{S}(\{w\}) = {}^*\mathbf{A} \cup Q_1 \cup d_1'$, where for internal *-finite d_1', $\mathbf{d} \subset d_1' \subset {}^*\mathbf{d}$ and internal $Q_1 \subset {}^*\mathbf{C}$ is composed of *-finite (≥ 1) conjunctions (i.e. $i(|||\mathrm{and}|||)$) of distinct members of d_1' and $w \in Q_1$. Further, each member of d_1' and no other *-proposition is used to form the *-finite conjunctions in Q_1, the only *-propositions in ${}^*\mathbf{S}(\{w\})$ are those in w, and ${}^*\mathbf{A}$, Q_1 and d_1' are mutually disjoint.*

Proof. The intent is to show that if $\mathrm{w} \in \mathrm{M_d} - \mathrm{d}$, then $\mathrm{S}(\{\mathrm{w}\}) = \mathrm{A} \cup \mathrm{Q} \cup \mathrm{d}'$, where $\mathrm{Q} \subset \mathrm{C}$, finite $\mathrm{d}' \subset \mathrm{d}$ and Q is composed of finite (≥ 1) conjunctions of members of d', each member of d' is used to form these conjunctions and no other propositions.

Let J be the set of propositional atoms in the composite w. (**0**) Then $\mathrm{J} \subset \mathrm{S}(\{\mathrm{w}\})$. If K is the set of all propositional atoms in $\mathrm{S}(\{\mathrm{w}\})$, then $\mathrm{J} \subset \mathrm{K}$. Let $\mathrm{b} \in \mathrm{K} - \mathrm{J}$. It is obvious that $\mathrm{b} \notin \mathrm{S}(\{\mathrm{w}\})$ since otherwise $\{\mathrm{w}, \mathrm{b}\} \subset \mathrm{S}_0(\{\mathrm{w}\})$ but $\not\models_{\mathrm{S}_0} \mathrm{w} \to \mathrm{b}$. Thus, $\mathrm{J} = \mathrm{K}$. Consequently, $\mathrm{J} \subset \mathrm{S}(\{\mathrm{w}\})$, $\mathrm{J} \subset \mathrm{d}$ and there does not exists an $\mathrm{F} \in \mathrm{d} - \mathrm{J}$, such that $\mathrm{F} \in \mathrm{S}(\{\mathrm{w}\})$. (**1**) *Let* $\mathrm{J} = \mathrm{d}'$. *The only propositional atoms in* $\mathrm{S}(\{\mathrm{w}\})$ *are those in w.* Obviously $\mathrm{A} \subset \mathrm{S}(\{\emptyset\})$.

Assume the language P_0 is inductively defined from the set of atoms d. Recall that for our axioms $\mathcal{X} = \mathcal{D} \to \mathcal{F}$, the strongest connective in $\mathcal{X}$ is $\to$. While in $\mathcal{D}$, or $\mathcal{F}$ when applicable, the strongest connective is $\wedge$. Since $\emptyset \subset \{\mathrm{w}\}$, it follows that $\mathrm{S}(\{\mathrm{w}\}) = \mathrm{S}(\emptyset) \cup \mathrm{S}(\{\mathrm{w}\})$.

Let $\mathrm{b} \in \mathrm{S}(\emptyset)$. The only steps in the formal proof for b contain axioms or follows from modus ponens. Suppose that step $\mathrm{B_k} = \mathrm{b}$ is the first modus ponens step obtained from steps $\mathrm{B_i},\ \mathrm{B_j},\ \mathrm{i}, \mathrm{j} < \mathrm{k}$, where $\mathrm{B_i} = \mathrm{A} \to \mathrm{b},\ \mathrm{B_j} = \mathrm{A}$. The strongest connective for each axiom is $\to$. However, since $A \to b$ is an axiom, the strongest connective in A is $\land$. This contradicts the requirement that A must also be an axiom with strongest connect $\to$. Thus no modus ponens step can occur in a formal proof for b. Hence, (**2**) $\mathrm{A} = \mathrm{S}(\emptyset)$. (No modus ponens step can occur using two axioms.)

Let $\mathrm{B_k} = \mathrm{b_1} \in \mathrm{P_0}$ and suppose (a) that $\mathrm{b_1} = \mathrm{w}$, or (b) $\mathrm{b_1} \neq \mathrm{w}$ and is the first nonaxiom step that appears in a formal demonstration from the hypothesis w. Assume (b). Then all steps $\mathrm{B_i} \in \{\mathrm{w}\} \cup \mathrm{A},\ 0 \leq \mathrm{i} < \mathrm{k}$. Then the only way that $\mathrm{b_1}$ can be obtained is by means of modus ponens. However, all other steps, not including that which is w, are axioms. No modus ponens step can occur using two axioms. Thus one of the steps used for modus ponens must not be an axiom. The only nonaxiom that occurs prior to the step $\mathrm{B_k}$ is the step $\mathrm{B_m} = \mathrm{w}$. Hence, one of the steps required for $\mathrm{B_k}$ must be $\mathrm{B_m} = \mathrm{w}$. The other step must be an axiom of the form $\mathrm{w} \to \mathrm{b_1}$ and $\mathrm{b_1} \neq \mathrm{w}$. Thus, from the definition of the axioms (**3**) $\mathrm{b_1}$ is either a finite (≥ 1) conjunction of atoms in d', or a single member of d'. Assume strong induction. Hence, for $\mathrm{n} > 1$, statement (**3**) holds for all r, $1 \leq \mathrm{r} \leq \mathrm{n}$. A similar argument shows that (**3**) holds for the $\mathrm{b_{n+1}}$ nonaxiom step. Thus by induction, (**3**) holds for all nonaxiom steps.

Hence, there exists a $\mathrm{Q} \subset \mathrm{C}$ such that each member of Q is composed of finitely many (≥ 1) distinct members of d' and the set G(Q) of all the proposition atoms that appear in any member of $\mathrm{Q} = \mathrm{d}' = \mathrm{J}$ since $\mathrm{w} \in \mathrm{Q}$. Moreover, (**4**) $\mathrm{S}(\{\mathrm{w}\}) = \mathrm{A} \cup \mathrm{d}' \cup \mathrm{Q}$ and (**5**) $\mathrm{A},\ \mathrm{d}',\ \mathrm{Q}$ are mutually disjoint.

$$\forall x(x \in \mathbf{M_d} - \mathbf{d} \to \exists y \exists z((y \in F(\mathbf{d})) \land (z \subset \mathbf{C}) \land (\mathbf{S}(\{x\}) =$$
$$\mathbf{A} \cup y \cup z) \land (\mathbf{A} \cap y = \emptyset) \land (\mathbf{A} \cap z = \emptyset) \land (x \in z) \land$$
$$(y \cap z = \emptyset) \land \mathbf{G}(z) = y)). \tag{10.1.3}$$

holds in $\mathcal{M}$, hence also in ${}^*\mathcal{M}$. So, let w be an ultraword. Then there exists internal $Q_1 \subset {}^*\mathbf{C},\ w \in Q_1$ and *- finite $d_1' \subset {}^*\mathbf{d}$ such that

$\mathbf{d} \subset {}^*\mathbf{S}(\{w\}) = {}^*\mathbf{A} \cup d_1' \cup Q_1$; ${}^*\mathbf{A}$, d_1', Q_1 are mutually disjoint and ${}^*G(Q_1) = d_1' = {}^*\mathbf{J}$. Hence, $\mathbf{d} \subset d_1'$.

Now to analyze the objects in Q_1. Let $\mathrm{d} = \{\mathrm{F_i} \mid \mathrm{i} \in \mathbb{N}\}$. Consider a bijection $h\colon \mathbb{N} \to \mathbf{d}$ defined by $h(n) = \mathbf{F_n} = [f]$, where $f \in T^0$ is the special member of $\mathbf{F_n}$ such that $f = \{(0, f(0))\}$, $f(0) = i(\mathrm{F_n}) = q_n \in i[\mathrm{d}]$. From the above analysis, (A) $[g] \in \mathbf{S}(\{w\}) - \mathbf{A} - \mathbf{d}$, $(w \in \mathbf{M_d} - \mathbf{d})$, iff there exist $k,\ j \in \mathbb{N}$ such that $k < j$ and $f_1' \in i[\mathrm{P_0}]^{2(j-k)}$ such that $[f_1'] = [g]$, and this leads to (B) that for each even $2p$, $0 \leq 2p \leq 2(j-k)$; $f_1'(2p) = q_{k+p} \in i[\mathrm{P_0}] \subset \mathcal{W}$, $[(0, q_{k+p})] \in \mathbf{d}'$, all such q_{k+p} being distinct. For each odd $2p+1$ such that $0 \leq 2p+1 \leq 2(j-k)$, $f_1'(2p+1) = i(|||\mathrm{and}|||)$. Also (C) $h(p) \in h[[k,j]]$ iff there exists an even $2p$ such that $0 \leq 2p \leq 2(j-k)$ and $f_1'(2p) = h(p) = q_{k+p} \in i[\mathrm{P_0}]$. [Note that 0 is considered to be an even number.]

By *-transfer of the above statements (A), (B) and (C), $[g] \in Q_1$ iff there exists some $j,\ k \in {}^*\mathbb{N}$, $k < j$, and $f' \in {}^*(i[\mathrm{P_0}])^{2(j-k)}$ such that $[f'] = [g]$ and ${}^*h[[k,j]] \subset {}^*\mathbf{d}$. Moreover, each ${}^*h(r)$, $r \in [k,j]$ is a distinct member of ${}^*\mathbf{d}$. The conjunction "codes" for $i(|||\mathrm{and}|||) \in \mathcal{W}$ that are generated by each odd $2p+1$ are all the same and there are *-finitely many of them. Hence, Q_1 is the *-finite (≥ 1) conjunctions of distinct members of d_1', no other *-propositions are utilized and since ${}^*\mathbf{G}(Q_1) = d_1'$, all members of d_1' are employed for these conjunctions. This completes the proof. ▮

Corollary 10.1.1.1 *Let* $w \in {}^*\mathbf{M_d} - {}^*\mathbf{d}$ *be an ultraword for denumerable* d *such that* $\mathbf{d} \subset {}^*\mathbf{S}(\{w\})$. *Then* ${}^*\mathbf{S}(\{w\}) \cap \mathbf{P_0} = \mathbf{A} \cup \mathbf{Q} \cup \mathbf{d}$ *and* A, Q, d *are mutually disjoint. The set* Q *is composed of finite* ≥ 1 *conjunctions of members of* d *and all of the members of* d *are employed to obtain these conjunctions.*

Proof. Recall that due to the finitary character of our standard objects ${}^\sigma\mathbf{A} = \mathbf{A} = {}^*\mathbf{A} \cap \mathbf{P_0}$. In like manner, since $\mathbf{d} \subset d_1'$, $d_1' \cap \mathbf{P_0} = \mathbf{d}$. Now $\mathbf{P_0} \cap Q_1$ are all of the standard members of Q_1. For each $k \in {}^*\mathbb{N}$, ${}^*h(k) = \mathbf{F_k} \in {}^*\mathbf{d}$ and conversely. Further, $\mathbf{F_k} \in \mathbf{d}$ iff $k \in \mathbb{N}$. Restricting $k,\ j \in \mathbb{N}$ in the above theorem yields standard finite ≥ 1 conjunctions of standard members of d_1'; hence, members of $\mathbf{d}$. Since ultraword $w \in Q_1$, we know that there exists some $\eta \in {}^*\mathbb{N} - \mathbb{N}$ and $f_1' \in {}^*(i[\mathrm{P_0}])^{2\eta}$, where f_1' satisfies the *-transfer of the properties listed in the above theorem . Since finite conjunctions of standard members of d_1' are *-finite conjunctions of members of d_1' and $\mathbf{d} = \mathbf{d} \cap d_1'$, it follows that all possible finite conjunctions of members of $\mathbf{d}$ that are characterized by the function $f_1' \in i[\mathrm{P_0}]^{2(j-k)}$ are members of Q_1 for each such $j,\ k < \eta$. Also for such $j,\ k$ the values of f_1' are standard. On the other hand, any value of f_1' is nonstandard iff it corresponds to a member of $d'_1 - \mathbf{d}$. Thus $Q_1 \cap \mathbf{P_0} = \mathbf{Q}$ and this completes the proof. ∎

If it is assumed that each member of d describes a Natural event (i.e. N-event) at times indicated by X_i, dropping the X_i may still yield a denumerable developmental paradigm without specifically generated symbols such as the "i." Noting that d'_1 is *-finite and internal leads to the conclusion that we can have little or no knowledge about the word-like construction of each member of $d'_1 - \mathbf{d}$. These pure nonstandard objects can be considered as describing pure NSP-world events, as will soon be demonstrated. Therefore, it is important to understand the following interpretation scheme, where descriptions are corresponded to events.

> *Standard or internal NSP-world events or sets of events are interpreted as directly or indirectly influencing N-world events. Certain external objects, such as the standard part operator, among others, are also interpreted as directly or indirectly influencing N-world events.*

Notice that standard events can directly or indirectly affect standard events. In the micro-world, the term *indirect evidence* or verification is a different idea than indirect influences. You can have direct or indirect evidence of direct or indirect influences when considered within the N-world. An indirect influence occurs when there exists, or there is assumed to exist, a mediating "something" between two events. Of course, indirect evidence refers to behavior that can be observed by normally accepted human sensors as such behavior is assumed to be caused by unobserved events. However, the evidence for pure NSP-world events that directly or indirectly influence N-world events must be indirect evidence under the above interpretation.

In order to formally consider NSP-world events for the formation of objective standard reality, proceed as follows: let $\mathcal{O}$ be the subset of $\mathcal{W}$ that describes those Natural events that are used to obtain developmental or general paradigms and the like. Let $\mathrm{E_j} \in \mathcal{O}$. Linguistically, assume that each $\mathrm{E_j}$ has the spacing symbol ||| immediately to the right. Thus within each T_i, there is a finite symbol string $\mathrm{F_i} = \mathrm{E_i} \in \mathcal{O}$ that can be joined by the justaposition (i.e. join) operation to other event descriptions. Assume that $\mathcal{W}_1$ is the set of nonempty symbol strings (with repetitions) formed from members of $\mathcal{O}$ by the join operation. These finite strings of symbols generate the basic elements for our partial sequences.

Obviously, $\mathcal{W}_1 \subset \mathcal{W}$. Consider $\mathrm{T}'_i = \{\mathrm{XW}_i \mid \mathrm{X} \in \mathcal{W}_1\}$ and note that in many applications the time indicator W_i need not be of significance for a given $\mathrm{E_j}$ in some of the strings. Obviously, $\mathrm{T}'_i \subset \mathrm{T}_i$ for each i. For our isomorphism i, the following hold.

$$\forall y(y \in \mathcal{E} \to (y \in \mathbf{T}'_i \leftrightarrow \exists x \exists f \exists w((\emptyset \neq w \in F(i[\mathcal{O}])) \wedge (x \in \mathbb{N}) \wedge \\ (f(0) = i[W_i]) \wedge (f \in P) \wedge \forall z((z \in \mathbb{N}) \wedge (0 < z \leq x) \to \\ f(z) \in w) \wedge (f \in y)))). \tag{10.1.4}$$

$$\forall x(x \in \mathbb{N} \to \exists f \exists w((\emptyset \neq w \in F(i[\mathcal{O}])) \wedge (f \in P) \wedge \\ \forall z(z \in \mathbb{N} \to (0 < z \leq x \leftrightarrow f(z) \in w)))). \tag{10.1.5}$$

$$\forall w(\emptyset \neq w \in F(i[\mathcal{O}]) \to \exists x \exists f \exists y((f \in P) \wedge (x \in \mathbb{N}) \wedge (y \in \mathbf{T}'_i) \wedge \\ (f \in y) \wedge \forall z(z \in \mathbb{N} \to (0 < z \leq x \leftrightarrow f(z) \in w)))). \tag{10.1.6}$$

Since each finite segment of a developmental pardigm corresponds to a member of $\mathbf{T}'_i$, each nonfinite hyperfinite segment should correspond to a member of ${}^*(\mathbf{T}'_i) - \mathbf{T}'_i$ and it should be certain individual segments of such members of ${}^*(\mathbf{T}'_i) - \mathbf{T}'_i$ that correspond to the ultranatural events produced by an ultraword; UN-events that cannot be eliminated from an NSP-world developmental paradigms. [Note: for a scientific language, 10.1.4 - 10.1.6 and other such statements would correspond to a $\mathcal{W}'$ as generated by, at the least, a denumerable alphabet such as used in 9.2, 9.3.]

10.2 Laws and Rules.

One of the basic requirements of human mental activity is the ability to recognize the symbolic differences between finitely long strings of symbols as necessitated by our reading ability and to apply linguistic rules finitely many times. Gödel numberings specifically utilize such recognitions and the rules for the generation of recursive functions must be comprehended with respect to finitely many applications. Observe that Gödel number recognition is an “ordered” process while some fixed intuitive order is not necessary for the application of the rules that generate recursive functions.

In general, the simplest “rule” for ordered or unordered finite human choice, a rule that is assumed to be humanly comprehensible by finite recognition, is to simply *list* the results of our choice (assuming that they are symbolically representable in some fashion) as a partial finite sequence for ordered choice or as a finite set of finitely long symbol strings for an unordered choice. Hence, the end result for a finite choice can itself be considered as an algorithm “for that choice only.” The next application of such a *finite choice rule* would yield the exact same partial sequence or choice set. Another more general rule would be a statement which would say that you should “choose a specific number of objects" from a fixed set (of statements). Yet, a more general rule would be that you simply are required to "choose a finite set of all such objects," where the term "finite" is intuitively known. Of course, there are numerous specifically described algorithms that will also yield finite choice sets.

From the symbol string viewpoint, there are trivial machine programmable algorithms that allow for the comparison of finitely long symbols with each member of a finite set of symbol strings B that will determine whether or not a specific symbol string is a member of B. These programs duplicate the results of human symbol recognition. As is well-known, there has not been an algorithm described that allows us to determine whether or not a given finite symbol string is a member of the set of all theorems of such theories as formal Peano Arithmetic. If one accepts Church's Thesis, then no such algorithm will ever be described.

Define the general finite human choice relation on a set A as $H_0(A) = \{(A, x) \mid x \in F_0(A)\}$, where F_0 is the finite power set operator (including the empty set = no choice is made). Obviously, the inverse H_0^{-1} is a function from $F(A)$ onto $\{A\}$. There are choice operators that produce sets with a specific number of elements that can be easily defined. Let $F_1(A)$ be the set of all singleton subsets of A. The axioms of set theory state that such a set of singleton sets exists. Define $H_1(A) = \{(A, x) \mid x \in F_1(A)\}$, etc. Considering such functions as defined on sets X that are members of a superstructure, then these relations are subsets of $\mathcal{P}(X) \times \mathcal{P}(X)$ and as such are also members of the superstructure.

Let $\mathrm{A} = \mathrm{P}_0$. Observe that ${}^\sigma\mathbf{H_0}(\mathbf{A}) = \{({}^*\mathbf{A}, x) \mid x \in F_0(\mathbf{A})\}$ and ${}^*\mathbf{H_i}(\mathbf{A}) = \{({}^*\mathbf{A}, x) \mid x \in {}^*(F_i(\mathbf{A}))\}$ $(i \geq 0)$. Now ${}^*(F_0(\mathbf{A})) = {}^*F_0({}^*\mathbf{A})$ is the set of all *-finite subsets of ${}^*\mathbf{A}$. On the other hand, for the $i > 0$ cardinal subsets, ${}^*(F_i(\mathbf{A})) = F_i({}^*\mathbf{A})$ for each $i \geq 1$. With respect to an ultraword w that generates the general and developmental paradigms, we know that $w \in {}^*\mathbf{P_0} - \mathbf{P_0}$ and that $({}^*\mathbf{P_0}, \{x\}) \in {}^*\mathbf{H_1}(\mathbf{P_0})$. The actual finite choice operators are characterized by th set-theoretic second projector operator P_2 as it is defined on $H_i(\mathrm{A})$. This operator embedded by the injection θ is the same as P_2 as it is defined on $\mathbf{H_i}(\mathbf{A})$. Thus, when $h = (\mathbf{A}, x) \in \mathbf{H_i}(\mathbf{A})$, then we can define $x = P_2(h) = C_i(h) = \mathbf{C_i}(h)$. The maps C_i and $\mathbf{C_i}$, formally defined below, are the specific finite choice operators. For consistency, we let C_i and $\mathbf{C_i}$ denote the appropriate finite choice operators for $H_i(\mathrm{A})$ and $\mathbf{H_i}(\mathbf{A})$, respectively.

Since the ${}^*P_2$ defined on say $\mathbf{H_i}(\mathbf{A})$ is the same as the set-theoretic second projection operator P_2, it would be possible to denote ${}^*\mathbf{C_i}$ as $\mathbf{C_i}$ on internal objects. For consistency, the notation ${}^*\mathbf{C_i}$ for these special finite choice operators is retained. Formally, let $\mathbf{C_i}\colon \mathbf{H_i}(\mathbf{A}) \to F_i(\mathbf{A})$. Observe that ${}^\sigma\mathbf{C_i} = \{{}^*(a,b) \mid (a,b) \in \mathbf{C_i}\} = \{(({}^*\mathbf{A}, b), b) \mid b \in F_i(\mathbf{A})\} \subset {}^*\mathbf{C_i}$; and, for $b \in F_i(\mathbf{A})$, $\mathbf{C_i}((\mathbf{A}, b)) = b$ implies that ${}^\sigma(\mathbf{C_i}((\mathbf{A}, b))) = \{{}^*a \mid a \in b\} = b$ from the construction of $\mathcal{E}$. Thus in contradistinction to the consequence operator, for each $({}^*\mathbf{A}, b) \in {}^\sigma\mathbf{H_i}$, the image $({}^\sigma\mathbf{C_i})(({}^*\mathbf{A}, b)) = {}^\sigma(\mathbf{C_i}((\mathbf{A}, b))) = ({}^*\mathbf{C_i})(({}^*\mathbf{A}, b)) = b$. Consequently, the set map ${}^\sigma\mathbf{C_i}\colon {}^\sigma\mathbf{H_i} \to F_i(\mathbf{A}) = {}^\sigma(F_i(\mathbf{A}))$ and ${}^*\mathbf{C_i} \mid {}^\sigma\mathbf{H_i} = {}^\sigma\mathbf{C_i}$. Finally, it is not difficult to extend these finite choice results to general internal sets.

In the proofs of such theorems as 7.2.1, finite and other choice sets are selected due to their set-theoretic existence. The finite choice operators C_i are not specifically applied since these operators are only intended as a mathematical model for apparently effective human processes — procedures that generate acceptable algorithms. As is well-known, there are other describable rules that also lead to finite or infinite collections of statements. Of course, with respect to a Gödel encoding i for the set of all words $\mathcal{W}$ the finite choice of readable sentences in $\mathcal{E}$ is one-to-one and effectively related to a finite and, hence, recursive subset of $\mathbb{N}$.

From this discussion, the descriptions of the finite choice operators would determine a subset of the set of all algorithms ("rules" written in the language $\mathcal{W}$) that allow for the selection of readable sentences. Notice that before algorithms are applied there may be yet another set of readable sentences that yields conditions that must exist prior to an application of such an algorithm and that these application rules can be modeled by members of $\mathcal{E}$.

In order to be as unbiased as possible, it has been required for N-world applications that the set of all frozen segments be infinite. Thus, within the proof of Theorem 7.2.1, every N-world developmental, as well as a general paradigm, is a proper subset of a *-finite NSP-world paradigm, and the *-finite paradigm is obtained by application of the *-finite choice operator ${}^*\mathbf{C_0}$. As has been shown, such *-finite paradigms contain pure unreadable (subtle) sentences that may be interpreted for developmental paradigms as pure refined NSP-world behavior and for general paradigms as specific pure NSP-world ultranatural events or objects.

Letting Γ correspond to the formal theory of Peano Arithmetic, then assuming Church's Thesis, there would not exist a N-world algorithm (in any human language) that allows for the determination of whether or not a statement F in the formal language used to express Γ is a member of Γ. By application of the *-finite choice operator

${}^*\mathbf{C_0}$, however, there does exist a *-finite Γ' such that ${}^\sigma\Gamma = \Gamma \subset \Gamma'$ and, hence, within the NSP-world a "rule" that allows the determination of whether or not $\mathrm{F} \in \Gamma'$. If such internal processes mirror the only allowable procedures in the NSP-world for such a "rule," then it might be argued that we do not have an effective NSP-world process that determines whether or not F is a member of Γ for Γ is external.

As previously alluded to at the beginning of this section, when a Gödel encoding i is utilized with the N-world, the injection i is not a surjection. When such Gödel encodings are studied, it is usually *assumed*, without any further discussion, that there is some human mental process that allows us to recognize that one natural number representation (whether in prime factored form or not) is or is not distinct from another such representation. It is not an unreasonable assumption to assume that the same *effective* (but external) process exists within the NSP-world. Thus within the NSP- world there is a "process" that determines whether or not an object is a member of ${}^*\mathbb{N} - \mathbb{N} = \mathbb{N}_\infty$ or $\mathbb{N}$. Indeed, from the ultraproduct construction of our nonstandard model, a few differences can be detected by the human mathematician. Consequently, this assumed NSP-world effective process would allow a determination of whether or not $\mathbf{F} = [f_m]$ is a member of Γ by recalling that $f_m \in P_m$ signifies that $[f_m] \in {}^*\Gamma - \Gamma$ implies $m \in \mathbb{N}_\infty \simeq {}^*(i[\mathcal{W}]) - i[\mathcal{W}]$.

The above NSP-world recognition process is equivalent, as defined in Theorem 7.2.1, to various applications of a single (external) set-theoretic intersection. Therefore, there are internal processes, such as ${}^*\mathbf{C_0}$, that yield pure NSP-world developmental paradigms and a second (external) but acceptable NSP-world effective process that produces specific N-world objects. Relative to our modeling procedures, it can be concluded that both of these processes are intrinsic ultranatural processes.

With respect to Theorem 10.1.1, the NSP-world developmental or general paradigm generated by an ultraword is *-finite and, hence, specifically NSP- world obtainable prior to application of ${}^*\mathbf{S}$ through application of ${}^*\mathbf{C_0}$ to ${}^*\mathbf{d}$. However, this composition can be reversed. The NSP-world (IUN) process ${}^*\mathbf{C_1}$ can be applied to the appropriate ${}^*\mathbf{M_d}$ type set and an appropriate ultraword $w \in {}^*\mathbf{M_d}$ obtained. Composing ${}^*\mathbf{C_1}$ with ${}^*\mathbf{S}$ would yield d'_1 in a slightly less conspicuous manner. Obviously, different ultrawords generate different standard and nonstandard developmental or general paradigms.

To complete the actual mental-type processes that lead to the proper ordered event sequences, the above discussion for the finite choice operators is extended to the human mental ability of ordering a finite set in terms of rational number subscripts. New choice operators are defined that model not just the selection of a specific set of elements that is of a fixed finite cardinality but also choosing the elements in the required rational number ordering. The ultrawords w that exist are *-finite in length. By application of the inverses of the f and τ functions of section 7.1, where they may be considered as extended standard functions $^*f$ and $^*\tau$, there would be from analysis of extended theorem 7.3.2 a hyperfinite set composed of standard or nonstandard frozen segments contained in an ultraword. Further, in theorem 7.3.2, the chosen function f does not specifically differentiate each standard or nonstandard frozen segment with respect to its "time" stamp subscript. There does exist, however, another function in the *-equivalence class $[g] = w$ that will make this differentiation. It should not be difficult to establish that after application of the ultralogic $^*\mathbf{S}$, there is applied an appropriate mental-like hyperfinite ordered choice operator (an IUN-selection process) and that this would yield that various types of event sequences. Please note that each event sequence has a beginning point of observation. This point of observation need not indicate the actual moment when a specific Natural system began its development.

Various subdevelopmental (or subgeneral) paradigms $\mathrm{d_i}$ are obtained by considering the actual descriptive content (i.e. events) of specific theories Γ_i that are deduced from hypotheses η_i, usually, by finitary consequence operators $\mathrm{S_i}$ (the inner logics) that are compatible with S. In this case, $\mathrm{d_i} \subset \mathrm{S_i}(\eta_\mathrm{i})$. It is also possible to include within $\{\mathrm{d_i}\}$ and $\{\eta_\mathrm{i}\}$ the assumed descriptive chaotic behavior that seems to have no apparent set of hypotheses except for that particular developmental paradigm itself and no apparent deductive process except for the identity consequence operator. In this way, such scientific nontheories can still be considered as a formal theory produced by a finitary consequence operator applied to an hypothesis. Many of these hypotheses η_i contain the so-called natural laws (or first-principles) peculiar to the formal theories Γ_i and the theories language, where it is assume that such languages are at least closed under the informal conjunction and conditional.

Consider each η_i to be a general paradigm. For the appropriate M type set constructed from the denumerable set $\mathrm{B} = \{\bigcup\{\mathrm{d_i}\} \cup (\bigcup\{\eta_\mathrm{i}\}),$ redefine $\mathrm{M_B}$ to be the smallest subset of $\mathrm{P_0}$ containing B and closed under finite (≥ 0) conjunction. (The usual type of inductively defined $\mathrm{M_B}$.) Then there exist ultrawords $w_i \in {}^*\mathbf{M_B} - {}^*\mathbf{B}$ such that $\eta_\mathbf{i} \subset {}^*\mathbf{S}(\{w_i\})$ (where due to parameters usually *ultranatural laws* exist in ${}^*\mathbf{S}(\{w_i\}) - \eta_\mathbf{i}$) and $\mathbf{d_i} \subset {}^*\mathbf{S}(\{w_i\})$. Using methods such as those in

Theorem 7.3.4, it follows that there exists some $w" \in {}^*\mathbf{M_B} - \mathbf{M_B}$ such that $w_i \in {}^*\mathbf{S}(\{w"\})$ and, consequently, $\eta_\mathbf{i} \cup \mathbf{d_i} \subset {}^*\mathbf{S}(\{w"\})$. Linguistically, it is hard to describe the ultraword $w"$. Such a $w"$ might be called an *ultimate ultranatural hypothesis* or *the ultimate building plain.*

Remark. It is not required that the so-called Natural laws that appear in some of the η_i be either cosmic time or universally applicable. They could refer only to local first-principles. It is not assumed that those first-principles that display themselves in our local environment are universally space-time valid.

Since the consequence operator S is compatible with each $\mathrm{S_i}$, it is useful to proceed in the following manner. First, apply the IUN-process ${}^*\mathbf{S}$ to $\{w"\}$. Then $\mathbf{d_i} \cup \eta_\mathbf{i} \subset {}^*\mathbf{S}(\{w"\})$. It now follows that $\mathbf{d_i} \cup \eta_\mathbf{i} \subset {}^*\mathbf{S}(\{w_i\}) \subset {}^*\mathbf{S_i}({}^*\mathbf{S}(\{w_i\})) \subset {}^*\mathbf{S_i}({}^*\mathbf{S_i}(\{w_i\})) = {}^*\mathbf{S_i}(\{w_i\})$. Observe that for each $\mathrm{a} \in \Gamma_\mathrm{i}$ there exists some finite $\mathrm{F_i} \subset \eta_\mathrm{i}$ such that $\mathbf{a} \in \mathbf{S_i}(\mathbf{F_i})$. However, $\mathbf{F_i} \subset \eta_\mathbf{i}$ for each member of $F(\eta_\mathrm{i})$ implies that $\mathbf{a} \in {}^*\mathbf{S_i}(\mathbf{F_i}) \subset {}^*\mathbf{S_i}({}^*\mathbf{S}(\{w_i\}))$. Consequently, $\mathbf{\Gamma_i} \subset {}^*\mathbf{S_i}({}^*\mathbf{S}(\{w_i\}))$. The ultimate ultraword suffices for the descriptive content and inner logics associated with each theory Γ_i.

We now make the following observations relative to "rules" and deductive logic. It has been said that science is a combination of empirical data, induction and deduction, and that you can have the first two without the last. That this belief is totally false should be self-evident since the philosophy of science requires its own general rules for observation, induction, data collection, proper experimentation and the like. All of these general rules require logical deduction for their application to specific cases — the metalogic. Further, there are specific rules for linguistics that also must be properly applied prior to scientific communication. Indeed, we cannot even open the laboratory door — or at least describe the process — without application of deductive logic. The concept of deductive logic as being the patterns our "minds" follow and its use exterior to the inner logic of some theory should not be dismissed for even the (assumed?) mental methods of human choice that occur prior to communicating various scientific statements and descriptions.

Finally, with respect to the hypothesis rule in [9], it might be argued that we can easily analyze the specific composition of all significant ultrawords, as has been previously done, and the composition of the nonstandard extension of the general paradigm. Using this assumed analysis and an additional alphabet, one *might* obtain specific information about pure NSP-world ultranatural laws or refined behavior. Such an argument would seem to invalidate the cautious hypothesis rule and lead to appropriate speculation. However, such an argument would itself be invalid.

Let $\mathcal{W}_1$ be an infinite set of meaningful readable sentences for some description and assume that $\mathcal{W}_1$ does not contain any infinite subset of readable sentences each one of which contains a mathematically interpreted entry such as a real number or the like. Since $\mathcal{W}_1 \subset \mathcal{W}$ and the totality $\mathrm{T}_i = \{\mathrm{XW}_i \mid \mathrm{X} \in \mathcal{W}\}$ is denumerable, the subtotality $\mathrm{T}_i' = \{\mathrm{XW}_i \mid \mathrm{X} \in \mathcal{W}_1\}$ is also denumerable. Hence, the external cardinality of ${}^*\mathbf{T_i'} \geq 2^{|\mathcal{M}|}$.

Consider the following sentence

$$\forall z(z \in i[\mathcal{W}_1] \to \exists y \exists x((y \in \mathcal{W}^{[0,1]}) \wedge (x \in \mathbf{T_i'}) \wedge (y \in x) \wedge ((0, i(\mathrm{W}_i)) \in y) \wedge ((1, z) \in y)))). \tag{10.2.1}$$

By *-transfer and letting "z" be an element in ${}^*(i[\mathcal{W}_1]) - i[\mathcal{W}_1]$ it follows that we can have little knowledge about the remaining and what must be unreadable portions that take the "X" position. If one assumes that members of $\mathcal{W}_1$ are possible descriptions for possible NSP-world behavior at the time t_i, then it may be assumed that at the time t_i the members of ${}^*\mathbf{T_i'} - \mathbf{T_i}$ describe NSP-world behavior at NSP- world (and N-world) time t_i. Now as i varies over ${}^*\mathbb{N}$, pure nonstandard subdevelopmental paradigms (with or without the time index statement W_i) exist with members in ${}^*\mathcal{T}$ and may be considered as descriptions for time refined NSP-world behavior, especially for a NSP-world time index $i \in \mathbb{N}_\infty$.

CHAPTER 10 REFERENCES

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Chapter 11

Propertons (Subparticles)

11.1 Propertons.

What is a propertons? Or, what is an infant, or subparticle, I first used the name infant for these strange objects. I then coined the term subparticle. Since these names lead to incorrect images, the name properton is employed. This gives a more intuitive meaning in that it carries, in coded form, physical or physical-like properties. As stated in [9], these objects are not to be described in terms of any geometric configuration. These multifaceted things, these propertons, are not to be construed as either particles nor waves nor quanta nor anything that can be represented by some fixed imagery. Propertons are to be viewed only operationally. Propertons are only to be considered as represented by a *-finite sequence $\{a_i\}_{i=1}^n,\ n \in {}^*\mathbb{N}$, of hyperreal numbers. Indeed, the idea of the n-tuple $(a_1, a_2, \ldots, a_i, \ldots)$ notation is useful and we assume that n is a fixed member of $\mathbb{N}_\infty$. The language of coordinates for this notation is used, where the i'th coordinate means the i'th value of the sequence. Obviously, 0 is not a domain member for our sequential representation.

The first coordinate a_1 is a "naming" coordinate. The remaining coordinates are used to represent various real numbers, complex numbers, vectors, and the like physical qualities needed for different physical theories. For example, $a_2 = 1$ might be a counting coordinate. Then $a_i,\ 3 \leq i \leq 6$ are hyperreal numbers that represent NSP-world coordinate locations of the properton named by a_1 — $a_7,\ a_8$ represent the positive or negative charges that can be assigned to every properton — $a_9,\ a_{11},\ a_{13}, \ldots$ hyperreal representations for the inertial, gravitational and intrinsic (rest) mass, etc. For vector quantities, continue this coordinate assignment and assign specific coordinate locations for the vector components. So as not to be biased, include as other coordinates hyperreal measures for qualities such as energy, apparent momentum, and all other physical qualities required within theories that must be combined in order to produce a reasonable description for N-world behavior. For the same reason, we do not assume that such N-world properties as the uncertainty principle hold for the NSP-world. (See note (2) on page 128.)

It is purposely assumed that the qualities represented by the coordinate $a_i,\ i \geq 3$ are not inner-related, in their basic construction, by any mathematical relation since it is such inner-relations that are assumed to mirror the N-world laws that govern the development of not only our present universe but previous as well as future developmental alterations. The same remarks apply to any possible and distinctly different universes that may or not occur. Thus, for these reasons, we view the properton as being totally characterized by such a sequence $\{a_i\}$ and always proceed cautiously when any attempt is made to describe all but the most general properton behavior. Why have we chosen to presuppose that propertons are characterized by sequences, where the coordinates are hyperreal numbers?

Let r be a positive real number. The number r can be represented by a decimal-styled number, where for uniqueness, the repeated 0s case is used for all terminating decimals. From this, it is seen that there is a sequence S_i of natural numbers such that $S_i/10^i \to r$. Consequently, for any $\omega \in \mathbb{N}_\infty = {}^*\mathbb{N} - \mathbb{N}$, it follows that $\pm {}^*S_\omega/10^\omega \in \mu(\pm r)$, where ${}^*S_\omega \in {}^*\mathbb{N}$ and $\mu(\pm r)$ is the monad about $\pm r$. In [9], it is assumed that each coordinate $a_i,\ i \geq 3$ is characterized by the numerical quantity $\pm 10^{-\omega},\ \omega \in \mathbb{N}_\infty$. Obviously, we need not confine ourselves to the number $10^{-\omega}$.

Theorem 11.1.1 *For each $0 < i \in \mathbb{N}$, let $0 < m_i \in \mathbb{N}$ and $m_i \to \infty$. Let any $\omega,\ \lambda \in \mathbb{N}_\infty$. Then, for each $r \in \mathbb{R}$, there exists a $b/{}^*m_\omega \in \{x/{}^*m_\omega \mid (x \in {}^*\mathbf{Z}) \wedge (|x| < \lambda\, {}^*m_\omega\}$, where ${}^*m_\omega \in \mathbb{N}_\infty$, and $b/{}^*m_\omega \approx r$ (i.e. $b/{}^*m_\omega \in \mu(r)$). If $r \neq 0$, then $|b| \in \mathbb{N}_\infty$.*

Proof. For $r \in \mathbb{R}$, there exists a unique integer $n \in \mathbf{Z}$ such that $n \leq r < n+1$. Partition $[n, n+1)$ as follows: for each $0 < i \in \mathbb{N}$, and $0 < m_i \in \mathbb{N}$, consider $[n, n+1/m_i), \ldots, [n+(m_i-1)/m_i, n+1)$. Then there exists a unique $c_i \in \{0, 1, \ldots, m_i - 1\}$ such that $r \in [n + c_i/m_i, n + (c_i+1)/m_i)$. Let $S_i = (m_i n + c_i)/m_i = f_i/m_i$. Since $0 \leq r - S_i < 1/m_i$ and $m_i \to \infty$, then $S_i \to r$. This yields two sequences $S\colon \mathbb{N} \to \mathbf{Q}$ and $f\colon \mathbb{N} \to \mathbf{Z}$, where, for each $\omega \in \mathbb{N}_\infty$, ${}^*S_\omega = {}^*f_\omega/{}^*m_\omega \approx r$ and ${}^*f_\omega \in {}^*\mathbf{Z}$. Observe that ${}^*f_\omega/{}^*m_\omega$ is a finite (i.e. limited) number and ${}^*m_\omega \in \mathbb{N}_\infty$. Hence, $|{}^*f_\omega/{}^*m_\omega| < \lambda$ entails that $|{}^*f_\omega| < \lambda\, {}^*m_\omega$. Therefore, ${}^*f_\omega/{}^*m_\omega \in \{x/{}^*m_\omega \mid (x \in {}^*\mathbf{Z}) \wedge (|x| < \lambda\, {}^*m_\omega\}$. If ${}^*f_\omega \in \mathbf{Z}$, then ${}^*f_\omega/{}^*m_\omega \approx 0$. ▮

Corollary 11.1.1.1 *For each $0 < i \in \mathbb{N}$, let $0 < m_i \in \mathbb{N}$ and $m_i \to \infty$. Let any $\omega,\ \lambda \in \mathbb{N}_\infty$. Then, for each $r \in \mathbb{R}$, there is a sequence $f\colon \mathbb{N} \to \mathbf{Z}$ such that ${}^*f_\omega/{}^*m_\omega \in \mu(r)$. There are unique $n \in \mathbf{Z}$, $c_\omega = 0$ or $c_\omega \in \mathbb{N}_\infty$ such that $c_\omega \leq {}^*m_\omega - 1$ and ${}^*f_\omega = {}^*m_\omega n + c_\omega$.*

For the *ultra-properton*, each coordinate $a_i = 1/10^\omega\ i \geq 3$ and odd, $a_i = -1/10^\omega\ i \geq 4$ and even, $\omega \in \mathbb{N}_\infty$. From the above theorem, the choice of $10^{-\omega}$ as the basic numerical quantity is for convenience only and is not unique accept in its infinitesimal character. Of course, the sequences chosen to represent the ultra-properton are pure internal objects and as such are considered to directly or indirectly affect the N-world. Why might the *-finite "length" of such propertons (here is where we have replaced the NSP-world entity by its corresponding sequence) be of significance?

First, since our N-world languages are formed from a finite set of alphabets, it is not unreasonable to assume that NSP-world "languages" are composed from a *-finite set of alphabets. Indeed, since it should not be presupposed that there is an upper limit to the N-world alphabets, it would follow that the basic NSP-world set of alphabets is an infinite *-finite set. Although the interpretation method that has been chosen does not require such a restriction to be placed upon NSP-world alphabets, it is useful, for consistency, to assume that descriptions for substratum processes that affect, in either a directly or indirectly detectable manner, N-world events be so restricted. For the external NSP-world viewpoint, all such infinite *-finite objects have a very significant common property. Note: in what follows $\mathcal{M}$ is the standard superstructure constructed on page 76 and not the object defined on page 57.

Theorem 11.1.2 *All infinite *-finite members of our (ultralimit) model ${}^*\mathcal{M}$ have the same external cardinality which is $\geq |\mathcal{M}|$.*

Proof. Hanson [8] and Zakon [16] have done all of the difficult work for this result to hold. First, one of the results shown by Henson is that all infinite *-finite members of our ultralimit model have the same external cardinality. Since our model is a comprehensive enlargement, Zakon's theorem 3.8 in [16] applies. Zakon shows that there exists a *-finite set, A, such that $|A| \geq |\mathcal{M}| = |\mathcal{R}|$. Since A is infinite, Hanson's result now implies that all infinite *-finite members of our model satisfy this inequality.∎

For an infinite standard set A, it is well-known that $|{}^*A| \geq |\mathcal{M}|$. One may use these various results and establish easily that there exist more than enough propertons to obtain all of the cardinality statements relative to the three substratum levels that appear in [9] even if we assume that there are a continuum of finitely many properton qualities that are needed to create all of the N-world.

Consider the following infinite set of statements expressed in an extended alphabet.

$$\begin{aligned}\mathrm{G_A} = \{&\mathrm{An|||elementary|||particle|||k'(i',j')|||with|||}\\ &\mathrm{total|||energy|||c'{+}1/(n')}.\mid ((i,j,n)\in\\ &\mathbb{N}^+\times\mathbb{N}^+\times\mathbb{N}^+)\wedge(1\leq k\leq m)\},\end{aligned} \tag{11.1.1}$$

where $\mathbb{N}^+$ is the set of all nonzero natural numbers and $m \in \mathbb{N}^+$. Applying the same procedure that appears in the proof of Theorem 9.3.1 and with a NSP-world alphabet $\mathcal{W}'$, we obtain

$$\begin{aligned} \mathrm{G}'_{\mathrm{A}} = \{&\mathrm{An|||elementary|||particle|||k'(i',j')|||with|||} \\ &\mathrm{total|||energy|||c'{+}1/(n').} \mid ((i,j,n) \in \\ &{}^*\mathbb{N}^+ \times {}^*\mathbb{N}^+ \times {}^*\mathbb{N}^+) \wedge (1 \le k \le m)\}, \end{aligned} \tag{11.1.2}$$

Assume that there is at least one type of elementary particle with the properties stated in the set $\mathrm{G_A}$. It will be shown in the next section that within the NSP-world there may be simple properties that lead to N-world energy being a manifestation of mass. For $c = 0$, we have another internal set of descriptions that forms a subset of $\mathrm{G'_A}$.

$$\begin{aligned} \{&\mathrm{An|||elementary|||particle|||k'(i',j')|||with|||} \\ &\mathrm{total|||energy|||c'{+}1/(10^{\gamma'}).} \mid ((i,j,\gamma) \in \\ &{}^*\mathbb{N}^+ \times {}^*\mathbb{N}^+ \times {}^*\mathbb{N}^+) \wedge (1 \le k \le m)\}, \end{aligned} \tag{11.1.3}$$

For our purposes, (11.1.3) leads immediately to the not ad hoc concept of propertons with infinitesimal proper mass. As will be shown, such infinitesimal proper mass can be assumed to characterize any possible zero proper mass N-world entity. The set $\mathrm{G'_A}$ has meaning if there exists at least one natural entity that can possess the energy expressed by $\mathrm{G_A}$, where this energy is measured in some private unit of measure.

Human beings combine together finitely many sentences to produce comprehensible descriptions. Moreover, all N-world human construction requires the composition of objectively real N-world objects. We model the idea of *finite composition* or *finite combination* by an N-world process. This produces a corresponding NSP-world intrinsic ultranatural process *ultrafinite composition* or *ultrafinite combination* that can either directly or indirectly affect the N-world, where its effect is indirectly inferred.

Let the index j vary over a hyperfinite interval and fix the other indices. Then the set of sentences

$$\begin{aligned} \mathrm{G''_A} = \{&\mathrm{An|||elementary|||particle|||k'(i',j')|||with|||} \\ &\mathrm{total|||energy|||c'{+}1/(n').} \mid (j \in {}^*\mathbb{N}^+) \\ &\wedge (1 \le j \le \lambda)\}, \end{aligned} \tag{11.1.4}$$

where $\lambda \in {}^*\mathbb{N}^+$, $3 \le i \in {}^*\mathbb{N}$, $n \in \mathbb{N}_\infty$ and $1 \le k \le m$, forms an internal linguistic object that can be assumed to describe a hyperfinite collection of ultranatural entities. Each member of $\mathrm{G''_A}$ has the i'th coordinate that measures the proper mass and is infinitesimal (with respect to NSP-world private units of measure). In the N-world, finite combinations yield an event. Thus, with respect to such sets as $\mathrm{G}''_{\mathrm{A}}$, one can say that there are such N-world events iff there are ultrafinite combinations of NSP-world entities. And such ultrafinite combinations yield a NSP-world event that is an ultranatural entity.

Associated with such ultrafinite combinations for the entities described in $\mathrm{G}''_{\mathrm{A}}$ there is a very significant procedure that yields the i'th coordinate value for the entity obtained by such ultrafinite combinations. Such entities are called *intermediate propertons*. Let $m_0 \geq 0$ be the N-world proper mass for an assumed elementary particle denoted by k'. If $m_0 = 0$, then let $\lambda = 1$. Otherwise, from Theorem 11.1.1, we know that there is a $\lambda \in {}^*\mathbb{N}$ such that $\lambda/(10^\omega) \in \mu(m_0)$, where $\omega \in \mathbb{N}_\infty$ and since $m_0 \neq 0$, $\lambda \in \mathbb{N}_\infty$. Consequently, for $b_n = 10^{-\omega}$, the *-finite sum

$$\sum_{n=1}^{\lambda} b_n = \sum_{n=1}^{\lambda} \frac{1}{10^\omega} = \frac{\lambda}{10^\omega} \tag{11.1.5}$$

has the property that $\mathtt{st}(\sum_{n=1}^{\lambda} 1/(10^\omega)) = m_0$. (Note the special summation notation for a constant summand.) The standard part operator st is an important external operator that is a continuous [11] NSP-world process that yields N-world effects. The appropriate interpretation is that

> *ultrafinite combinations of ultra-propertons yield an intermediate properton that, after application of the standard part operator, has the same effect as an elementary particle with proper mass m_0.*

An additional relevant idea deals with the interpretation that the *-finite set $\mathrm{G}''_{\mathrm{A}}$ exists at, say, nonstandard time, and that such a set is manifested at standard time when the operator st is applied. The standard part operator is one of those external operators that can be indirectly detected by the presence of elementary particles with proper mass m_0.

The above discussion of the creation of intermediate propertons yields a possible manner in which ultra-propertons are combined within the NSP-world to yield appropriate energy or mass coordinates for the multifaceted propertons. But is there an indication that all standard world physical qualities that are denoted by qualitative measures begin as infinitesimals?

Consider the infinitesimal methods used to obtain such things as the charge on a sphere, charge density and the like. In all such cases, it is assumed that charge can be infinitesimalized. In 1972, it was shown how a classical theory for the electron, when infinitesimalized, leads to the point charge concept of quantum field theory and then how the *-finite many body problem produced the quasi-particle. [15] Although this method is not the same as the more general and less ad hoc properton approach, it does present a procedure that leads to an infinitesimal charge density and then, in a very ad hoc manner, it is assumed that there are objects that when *-finitely combined together entail a real charge and charge density. Further, it is the highly successful use of the modeling methods of infinitesimal calculus over hundreds of years that has lead to our additional presumption that all coordinates of the basic sequential properton representation are a $\pm$ fixed infinitesimal.

In order to retain the general independence of the coordinate representation, *independent *-finite coordinate summation* is allowed, recalling that such objects are to be utilized to construct many possible universes. [This is the same idea as *-finitely repeated simple affine or linear transformations.] Thus, distinct from coordinatewise addition, *-finitely many such sequences can be added together by means of a fixed coordinate operation in the following sense. Let $\{a_i\}$ represent an ultra-properton. Fix the coordinate j, then the sequence $\{c_i\},\ c_i = a_i,\ i \neq j$ and $c_j = 2a_j$ forms an intermediate properton. As will be shown, it is only after the formation of such intermediate propertons that the customary coordinatewise addition is allowed and this yields, after the standard part operator is applied, representations for elementary particles. Hence, from our previous example, we have that ultrafinite combinations of ultra-propertons yield propertons with "proper mass" $\lambda/(10^\omega) \approx m_0$ while all other coordinates remain as $\pm 10^\omega$. This physical-like process is not a speculative ad hoc construct, but, rather, it is modeled after what occurs in our observable natural world. Intuitively, this type of summation is modeled after the process of inserting finitely many pieces of information (mail) into a single "postal box," where these boxes are found in rectangular arrays in post offices throughout the world.

Now other ultra-propertons are ultrafinitely combined and yield for a specific coordinate the $\pm$ unite charge or, if quarks exist, other N-world charges, while all other coordinates remain fixed as $\pm 1/(10^\omega)$, etc. Rationally, how can one conceive of a combination of these intermediate propertons, a combination that will produce entities that can be characterized in a standard particle or wave language?

Recall that a finite summation is a *-finite summation within the NSP-world. Therefore, a finite combination of intermediate propertons is an allowed internal process. [Note that external processes are always allowed but with respect to our interpretation procedures we always have direct or indirect knowledge relative to application of internal processes. Only for very special and reasonable external processes do we have direct or indirect knowledge that they have been applied.] Let $\gamma_i \in \mu(0),\ i = 1, \ldots, n$. Then $\gamma_1 + \cdots \gamma_n \in \mu(0)$. The final stage in properton formation for our universe — the final stage in particle or wave substratum formation — would be *finite coordinatewise* summation of finitely many intermediate propertons. This presupposes that the N-world environment is characterized by but finitely many qualities that can be numerically characterized. This produces the following type of coordinate representation for a specific coordinate j after n summations with n other intermediate propertons that have only infinitesimals in the j coordinate position.

$$\sum_{i=1}^{\lambda}(1/(10^{\omega}) + \sum_{i=1}^{n}\gamma_i. \tag{11.1.6}$$

Assuming λ is one of those members of $\mathbb{N}_\infty$ or equal to 1 as used in (11.1.5), then the standard part operator can now be applied to (11.1.6) and the result is the same as $\mathtt{st}(\sum_{i=1}^{\lambda}(1/(10^{\omega}))$.

The process outlined in (11.1.6) is then applied to finitely many distinct intermediate propertons — those that characterize an elementary particle. The result is a properton each coordinate of which is infinitely close to the value of a numerical characterization or an infinitesimal. When the standard part operator is applied under the usual coordinatewise procedure, the coordinates are either the specific real coordinatewise characterizations or zero. Therefore, N-world formation of particles, the dense substratum field, or even gross matter may be accomplished by a ultrafinite combination of ultra-propertons that leads to the intermediate properton; followed by finite combinations of intermediate propertons that produce the N-world objects. Please note, however, that prior to application of the standard part operator such propertons retain infinitesimal nonzero coordinate characterizations in other noncharacterizing positions. (See note (1) on 128.)

We must always keep in mind the hypothesis law [9] and avoid unwarranted speculation. We do not speculate whether or not the formed particles have point-like or "spread out" properties within our space-time environment. These additional concepts may be pure catalyst type statements within some standard N-world theory and could have no significance for either the N-world or NSP-world.

With respect to field effects, the cardinality of the set of all ultra-propertons clearly implies that there can be ultrafinite combinations of ultra-propertons "located" at every "point" of any finite dimensional continuum. Thus the field effects yielded by propertons may present a completely dense continuum type of pattern within the N-world environment although from the monadic viewpoint this is not necessarily how they "appear" within the NSP-world.

There are many scenarios for quantum transitions if such occur in objective reality. The simplest is a re-ultrafinite combination of the ultra-propertons present within the different objects. However, it is also possible that this is not the case and, depending upon the preparation or scenario, the so-called "conservation" laws do not hold in the N-world.

As an example, the neutrino could be a complete fiction, only endorsed as a type of catalyst to force certain laws to hold under a particular scenario. Consider the set of sentences

$$\begin{aligned}\mathrm{G_B} = \{&\mathrm{An|||elementary|||particle|||k'(i',j')|||with|||}\\ &\mathrm{total|||energy|||c'{+}n'.} \mid ((i,j,n) \in\\ &\mathbb{N}^+ \times \mathbb{N}^+ \times \mathbb{N}^+) \land (1 \leq k \leq m)\}.\end{aligned} \tag{11.1.7}$$

It is claimed by many individuals that such objects as being described in $\mathrm{G_B}$ exist in objective reality. Indeed, certain well-known scenarios for a possible cosmology require, at least, one "particle" to be characterized by such a collection $\mathrm{G_B}$. By the usual method, these statements are *- transferred to

$$\begin{aligned}\mathrm{G'_B} = \{&\mathrm{An|||elementary|||particle|||k'(i',j')|||with|||}\\ &\mathrm{total|||energy|||c'{+}n'.} \mid ((i,j,n) \in\\ &{}^*\mathbb{N}^+ \times {}^*\mathbb{N}^+ \times {}^*\mathbb{N}^+) \land (1 \leq k \leq m)\}.\end{aligned} \tag{11.1.8}$$

Hence, letting $n \in \mathbb{N}_\infty$ then various "infinite" NSP-world energie emerge from our procedures. With respect to the total energy coordinate(s), ultra-propertons may also be ultrafinitely combined to produce such possibilities. Let $\lambda = 10^{2\omega}$ [resp. $\lambda = \omega^2$] and $\omega \in \mathbb{N}_\infty$. Then

$$\sum_{n=1}^{\lambda} \frac{1}{10^\omega} = 10^\omega \ [\text{resp. } \sum_{n=1}^{\lambda} \frac{1}{\omega} = \omega] \in \mathbb{N}_\infty. \tag{11.1.9}$$

Of course, these numerical characterizations are external to the N-world. Various distinct "infinite" qualities can exist rationally in the NSP-world without altering our interpretation techniques. The behavior of the infinite hypernatural numbers is very interesting when considered as a model for NSP-world behavior. A transfer of finite energy, momentum and, indeed, all other N-world characterizing quantities, back and forth, between these two worlds is clearly possible without destroying NSP-world infinite conservation concepts.

Further, observe that various intermediate propertons carrying nearstandard coordinate values could be present at nearstandard space-time coordinates, and application of the continuous and external standard part operator would produce an apparent not conserved N-world effect. These concepts will be considered anew when we discuss the Bell inequality.

Previously, ultrawords were obtained by application of certain concurrent relations. Actually, basic ultrawords exist in any elementary nonstandard superstructure model, as will now be established for the general paradigm.

Referring back to $\mathrm{G_A}$ equation (11.1.1), for some fixed k, $1 \leq k \leq m$, let $\mathrm{h}_k\colon \mathbb{N}^+ \times \mathbb{N}^+ \times \mathbb{N}^+ \to \mathrm{G_A}$ be defined as follows: $\mathrm{h}_k(i,j,n) =$ $\mathrm{An|||elementary|||particle|||k'(i',j')|||with|||total|||energy|||c'+}$ $\mathrm{1/(n')}$. Since the set $F(\mathbb{N}^+ \times \mathbb{N}^+ \times \mathbb{N}^+)$ is denumerable, there exists a bijection $\mathrm{H}\colon \mathbb{N} \to F(\mathbb{N}^+ \times \mathbb{N}^+ \times \mathbb{N}^+)$. For each $1 \leq \lambda \in \mathbb{N}$ and fixed $i,\ n \in \mathbb{N}^+$, let $\mathrm{G_A}(\lambda) = \{\mathrm{An|||elementary|||particle|||k'(i',j')}$ $\mathrm{|||with|||total|||energy|||c'+1/(n').}\ \mid\ (1 \leq \mathrm{j} \leq \lambda) \wedge (\mathrm{j} \in \mathbb{N}^+)\}$. Let $p \in \mathbb{N}$. If $|\mathrm{H}(p)| \geq 2$, define finite $\mathrm{M}(\mathrm{h}_k[\mathrm{H}(p)]) =$ $\{\mathrm{A_1|||and|||A_2|||and|||}\cdots\mathrm{|||and|||A}_m\}$, where $\mathrm{A}_j \in \mathrm{h}_k[\mathrm{H}(p)], m =$ $|\mathrm{H}(p)|$. If $|\mathrm{H}(p)| \leq 1$, then define $\mathrm{M}(\mathrm{h}_k[\mathrm{H}(p)]) = \emptyset$. Let $\mathrm{M}^0 =$ $\bigcup\{\mathrm{M}(\mathrm{h}_k[\mathrm{H}(p)]) \mid p \in \mathbb{N}\}$. Please note that the $\mathrm{k'}$ represents the "type" or name of the elementary particle, assuming that only finitely many different types exist, $\mathrm{i'}$ is reserved for other purposes, and the $\mathrm{j'}$ the number of such elementary particles of type $\mathrm{k'}$.

Theorem 11.1.3 *For any $i,\ n,\ \lambda \in {}^*\mathbb{N}^+$, such that $2 \leq \lambda$, there exists $w \in {}^*\mathbf{M^0} - \mathbf{G_A},\ {}^*\mathbf{G_A}(\lambda) \subset {}^*\mathbf{S}(\{w\})$ and if $A \in {}^*\mathbf{G_A} -$ ${}^*\mathbf{G_A}(\lambda)$, then $A \notin {}^*\mathbf{S}(\{w\})$.*

Proof. Let $i,\ j,\ \lambda \in \mathbb{N}^+$ and $2 \leq \lambda$. Then there exists some $r \in \mathbb{N}$ such that $\mathrm{h}_k[\mathrm{H}(r)] = \mathrm{G_A}(\lambda)$. From the construction of M^0, there exists some $r' \in \mathbb{N}$ such that $\mathrm{w}(r') =$ $\mathrm{An|||elementary|||particle|||k'(i',1')|||with|||total|||energy|||c'+1/(n').}$ $\mathrm{|||and|||An|||elementary|||particle|||k'(i',2')|||with|||total|||energy|||}$ $\mathrm{c'+1/(n').|||and|||}\cdots\mathrm{|||and|||An|||elementary|||particle|||k'(i',\lambda')|||}$ $\mathrm{with|||total|||energy|||c'+1/(n').}\ \in \mathrm{M}[\mathrm{h}_k[\mathrm{H}(r')]$. Note that $\mathrm{w}(r') \notin$ $\mathrm{G_A},\ \mathrm{h}_k[\mathrm{H}(r')] \subset \mathrm{S}(\{\mathrm{w}(r')\})$ and if $\mathrm{A} \in \mathrm{G_A} - \mathrm{h}_k[\mathrm{H}(r)]$, then $\mathrm{A} \notin$ $\mathrm{S}(\{\mathrm{w}(r')\})$. The result follow by our embedding and *-transfer. ▮

The ultrawords utilized to describe various propertons, whether obtained as in Theorem 11.1.3 or by concurrent relations, are called *ultramixtures* due to their applications. The ultrafinite choice operator $\mathbf{C_1}$ can select them, prior to application of ${}^*\mathbf{S}$. Moreover, application of the ultrafinite combination operator entails a specific intermediate properton with the appropriate nearstandard coordinate characterizations. Please notice that the same type of sentence collections may be employed to infinitesimalize all other quantities, although the sentences need not have meaning for certain popular N-world theories. Simply because substitution of the word "charge" for "energy" in the above sentences $\mathrm{G_A}$ does not yield a particular modern theory description, it does yield the infinitesimal charge concept prevalent in many older classical theories.

Using such altered $\mathrm{G_A}$ statements, one shows that there does exist ultramixtures w_i for each intermediate properton and, thus, a single ultimate ultramixture w such that ${}^*\mathbf{S}(\{w_i\}) \subset {}^*\mathbf{S}(\{w\})$. Each elementary particle may, thus, be assumed to originate from w through application of the ultralogic ${}^*\mathbf{S}$.

Recall that if a standard $A \subset \mathbb{R}$ is infinite, then it is external, and if B is internal, $A \subset B$, then $B \neq A$. Therefore, there exists some $\eta \in B$ such that $\eta \notin A$. This simple fact yields many significant nonstandard results. For example, as the next theorem shows, if $\eta \in \mathbb{N}_\infty$, then there exists some $\lambda \in \mathbb{N}_\infty$ such that $10^{2\lambda} < \eta$.

Theorem 11.1.4 *Let $f\colon \mathbb{N} \to \mathbb{N}$ and $f[\mathbb{N}]$ be infinite. If $\eta \in \mathbb{N}_\infty$, then there exists some $\lambda \in \mathbb{N}_\infty$ such that ${}^*f(\lambda) < \eta$.*

Proof. For $\eta \in \mathbb{N}_\infty$, consider the nonempty internal set $B = \{{}^*f(x) \mid ({}^*f(x) < \eta) \land (x \in {}^*\mathbb{N})\} \subset {}^*f[{}^*\mathbb{N}]$. Let $n \in \mathbb{N}$. Then $f(n) \in \mathbb{N}$ and $f(n) < \eta$ imply that $({}^\sigma f)[\mathbb{N}] = {}^\sigma(f[\mathbb{N}]) = f[\mathbb{N}] \subset B$. Since $f[\mathbb{N}]$ is infinite, it is external. Thus ${}^*f\colon {}^*\mathbb{N} \to {}^*\mathbb{N}$ implies that there exists some $\lambda \in {}^*\mathbb{N}$ such that ${}^*f(\lambda) \in B - f[\mathbb{N}]$. However, ${}^*f$ is a function. Hence, $\lambda \in \mathbb{N}_\infty$ and ${}^*f(\lambda) < \eta$. ∎

Theorem 11.1.4 has many applications and can be extended to other functions not just those with domain and codomain $\mathbb{N}$, and other B type relations.

11.2 Ultraenergetic Propertons.

There is a possibility that propertons can have additional and unusual properties when they are generated by statements such as $\mathrm{G_B}$. With respect to the translated $\mathrm{G'_B}$ statements, we have coined the term *ultraenergetic* to discuss propertons that have various infinite energies. I again note that such ultraenergetic propertons may be considered as under the control of our previously discussed ultralogics and *ultranatural choice operators* (i.e. hyperfinite choice). Further, it is possible to place these ultraenergetic propertons into pools of infinite energy that are mathematically termed as "galaxies" and that have interesting mathematical properties. However, these properties will not be discussed in this present book.

If our universe or any portion of it began or exists at this present epoch in a state of "infinite" energy, then the ultraenergetic propertons could play a critical role. [I point out that general developmental paradigms indicate, as will be shown, that the actual state of affairs for any beginnings of our universe cannot be known by the present methods of the scientific method.] Now, any such singularity that might exist cosmologically, even in our local environment, may owe its existence to various ultralogically generated ultraenergetic propertons. Of course, this is pure speculation, but these NSP-world alternative explanations for *assumed* quantum physical phenomena yield indirect evidence for the acceptance of the NSP-world model.

Quantum mechanics has now become highly positivistic in character although certain previous states of affairs have been partially accepted. This important possibility was stated by Bernard d'Espagnet with respect to one of our preliminary investigations — the experimental disproof of the Bell inequality and the local variable concept [17] — that "seems to imply that in some sense all of the objects [particles or aggregates] constitute an indivisible whole" [2]. One aspect of the original MA-model, (11.4.5) of section 11.4, can be used as an aid to model this statement.

The ideas developed within our theory of developmental paradigms do not contradict d'Espagnet's (weak) definition of realism. He simply requires that if we can describe a relation between physical entities produced by some experimental process, a relation that is not observed and, thus, not described prior to the experiment, then their must be a cause that has produced this new relation. I don't believe that it is necessary, under his definition, that this cause be describable.

In May 1984, this author became aware of the Bell inequality and d'Espagnat's discussion of local realism [3]. In particular, we discovered that d'Espagnat may, to some degree, embraced our the statements that appear in (11.4.5) section 11.4 as an explanation for this experimental disproof. "Perhaps in such a world the concept of an independent existing reality can retain some meaning, but it will be altered and one remote from everyday experience" [4]. But is there a NSP-world cause, indeed, a mechanism that for such behavior?

There are many scenarios as to how "instantaneous" informational signals may be transmitted within the NSP-world without violating Einstein separability in the N-world (i.e. no influence of any kind within the N-world can propagate faster than the speed of light [5].) The basic N-world interpretation for any verified effect of the Special Theory would imply that the only reason for Einstein separability is a relation between those propertons that create the N-world and the NSEM field propertons [20]. But ultraenergetic propertons are not of either of these types and need not interact with the NSEM field for many reasons. The most obvious is that the NSEM field is not dense from the NSP-world viewpoint but is scattered. Obviously ultraenergetic propertons may be used for this purpose. Recall that we should be very careful when speculating about the NSP-world due to the difficulty of describing refined behavior. However, this should not completely restrain us, especially when the general paradigm method states that such things as these exist logically.

It is possible to describe a mechanism and a possible new type of properton that can send N-world instantaneous informational signals between all standard material particles, field objects or aggregates and not violate N-world Einstein separability. One possibility is that these influences would be imparted by means of independent coordinate summation to the propertons that comprise these objects and yet in doing so these new entities could not be humanly detected, not detectable except as far as the instantaneous state change indicates, since the total energy (in this case classical kinetic) utilized by this NSP-world mechanism would be infinitesimal. If it is an instantaneous energy change, then, as will be shown, only that specific energy change would appear in the N-world.

From the methods employed to construct propertons, it is immediately clear that there exists a very "large" quantity of propertons that are not used for standard particle and field effect construction. This can be seen by allowing the i'th symbol is such statements as G'_{A} to vary from 1 to some value in $\mathbb{N}_\infty$. The cardinality of such collections of statements would be great than or equal to $2^{|\mathcal{M}|}$. We simply pass this external cardinality statement to the propertons being described.

It is a basic tenet of infinitesimal reasoning that without further justification the only properties that we should associate with such unutilized objects are of the simplest classical type. The logic of particle physics allows us to logically accept the existence of such propertons without any additional justification. Let $\lambda \in \mathbb{N}_\infty$. Then $(1/\lambda)^4 \in \mu(0)$. Let a pure NSP-world properton, not one used to construct a universe, have mass coordinate of the value $m = (1/\lambda)^4$. Call this properton P and have P increase its velocity over a finite NSP-time interval from zero to λ. Propertons that attain such velocities are called *ultrafast propertons.*

Extending the classical idea of kinetic energy to the NSP-world it follows that the (kinetic) energy attained by this properton, when it reaches its final velocity, is $(1/2)(1/\lambda)^2 \in \mu(0)$. Suppose that there is a continuum or less of positions within our universe. Since $c < \lambda$, the kinetic energy used to accelerate enough of these propertons to the λ velocity so that they could effect every position in our universe would be less than $(1/2)(1/\lambda) \in \mu(0)$, assuming the energy is additive in the NSP-world.

Each of these ultrafast propertons besides altering some other specific coordinate would also add its total kinetic energy to the intermediate properton since the new intermediate properton simply includes this new one. But then suppose that our universe has existed for less than or equal to a continuum of time. Then since $2c < \lambda$, once again the amount of energy that would be added to our universe over such a time period, if each of these informational propertons combined with one member of an intermediate, would be infinitesimal. All the state alterations give the N-world appearance of being instantaneously obtained although the existence of the G function of Theorem 7.5.1 clearly states that in the NSP-world such alterations are actually hypercontinuous and hypersmooth.

What if the state change itself depends upon the velocity of such an ultrafast properton? We use kinetic energy as an example. Say the change is in the kinetic energy coordinate in the standard amount of h. Then all one needs to consider is an ultrafast properton with infinitesimal mass $m = 2h(1/\lambda)^2$. Moving with a velocity of λ, such an ultrafast properton has the requisite kinetic energy. Things can clearly be arranged so that all other coordinates of such ultrafast propertons are infinitesimal. Independent coordinate summation for any finite number of alterations will leave all other nonaltered coordinates of the intermediate properton infinitely close to the original values for the alteration is but obtained by the addition of a finite number of new propertons to the collection.

We acknowledge that the N-world inner coordinate relations have been used to obtain these alterations. This need not be the way it could be done. Can we describe the method of capture and other sorts of behavior? Probably too much has already been described in the language of this book. One should not forget that descriptions may exist for such NSP-world behavior but not in a readable language.

The state of affairs described above lends credence to d'Espagnate's explanation of why the Bell inequality is violated and gives further evidence for the acceptance of the NSP-world model. "The basic law that signals cannot travel faster than light is demoted from a property of external [N-world] reality to a feature of mere communicatable human experience. ....the concept of an independent or external reality can still be retained as a possible explanation of *observed* regularities in experiments. It is necessary, however, that the violation of Einstein separability be included as a property, albeit a well-hidden and counterintuitive property...."[6]

11.3 More on Propertons.

See the material on properton generation in http://vixra.org/abs/1308.0125

11.4 MA-Model. (1993)

In this section, we look back and gather together various observations relative to formal theorems that yield the concept I have described as the Metaphoric-Anamorphosis (i.e. MA) model. The MA-model is a specific application of the General Grand Unification Model as described by a hyperfinite GID-design. The different types of developmental paradigms that can be selected by ultrafinite (i.e. ultranatural) choice and a few of our previous results leads immediately to the following logically acceptable possibilities. [Of course, as is the case with all mathematical modeling, simply because a possibility exists it need not be utilized to describe an actual scenario and if it is used, then it need not be an objectively real description.]

> (11.4.1) *Entire microscopic, macroscopic or large scale natural systems can apparently appear or disappear or be physically altered suddenly.*
>
> (11.4.2) *All such alterations can be designed in an ultracontinuous manner.*
>
> (11.4.3) *None of these NSP-world concepts are related to the notion of hidden variables.*
>
> (11.4.4) *The GGU-model produced GID-designs that are relative to any numerical quantity associated with any elementary particle, field effect or aggregate is related to every numerical quantity associated with every other elementary particle or aggregate via restrictions of hypercontinuous, hyperuniform and hypersmooth pure NSP-world functions. These functions*

may be interpreted as representing the IUN-altering process of utilizing ultrafinite composition (i.e. ultranatural composition) in order to "change" any elementary particle, field effect or aggregate into any type of elementary particle, field effect or aggregate.

Statement (11.4.4) is particular significant in that it may be coupled with ultralogics and ultrafinite choice operators and entails an additional manifestation for the possibility that there is no N-world independent existing objective reality. **Further, notice that depending upon the space-time neighborhood, statements such as (11.4.1) need not be humanly verifiable (i.e. they may be undetectable).**

CHAPTER 11 REFERENCES

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(1) For propertons, only two possible intrinsic properties for elementary particle formation are here considered. Assuming that there are such things as elementary particles, then they would be differentiated one from the other by their intrinsic properties that are encoded within properton coordinates. When there are particle interactions, these intrinsic properties can be altered or even changed to extrinsic properties. How the alteration from intrinsic to extrinsic occurs probable cannot be known since it most likely is an ultranatural event. For further results on this subject, see http://arxiv.org/abs/quant-ph/9909078

(2) To conceive of propertons properly, quantum theory is viewed as an approximation. Moreover, in terms of physically determined units, the numerical characteristics produced by applications of the standard part operator are considered as exact.