Improvements to
The Theory of Ultralogics - I.

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1 JUN 2016. Revised 10 AUG 2018

Abstract: The major purpose for this article is to reestablish Theorem 9.3.1, with the modified Robinson approach, and make other improvements in Section 9 of The Theory of Ultralogics (Herrmann, (1978-93, 1999). Additionally, what constitutes a saturated enlargement is now fixed as of this date.

1. Improvements to Section 9 in Herrmann (1978-93) and notational conventions.

[Note: After presenting 27 articles at the arXiv.org, they are apparently discriminating against me and my research findings. Thus, I will not even attempt to make any further revisions to my articles that appear there. Any revisions will appear elsewhere.]

[[Certain notational conventions have been employed throughout all of my recent presentations in nonstandard analysis. You have certain notation that indicates objects within the standard superstructure and the same notion appears relative to the superstructure as it is embedded into the nonstandard model. The content indicates that the result symbolically expressed is a member of one or the other superstructure. My notation is NOT consistent with that used by other authors. Due to the construction of the model used, the embedded standard objects correspond to the equivalence classes that contain the constant sequences.

For example, let A be a standard member. Some authors denote ${}^\sigma A$ as the set of all such equivalence classes determined by members of A. However, in my writings ${}^\sigma A = \{{}^*a \mid a \in A\}$. Prior to introducing the ultrapower style construction, some authors for certain sets A, notationally identify such classes with the same symbol as A. I have taken the same approach but it is relative to the equivalence classes of constant sequences. That is, in my writings you will find statements such as $\mathbf{D} \subset {}^*\mathbf{D}$. The complete notation indicates that this is a statement about the "nonstandard" superstructure. This $\mathbf{D}$ is the set of all constant sequence equivalence classes determined by each standard d $\in$ D. I could, but have not, employed a specific notation, such as ${}^oA$, to indicate this.

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Thus, in general, one sees statements such as $\mathbf{D} \subset {}^*\mathbf{D}$ and ${}^\sigma\mathbf{D} \subset {}^*\mathbf{D}$. These, generally state different properties about different subsets of ${}^*\mathbf{D}$. Further, for certain finite sets, $\mathbf{B}$, if follows that under this symbolic identification that ${}^*\mathbf{B} = \mathbf{B}$.]]

The coding i as originally defined might need to be extended, due to the possible increase in cardinality of the $\mathcal{W}'$. It may need to be a bijection onto various if not all of the real numbers as indicated in Herrmann (1978-93, p. 88). However, incorporating an entire real number alphabet is obviously not necessary since we are using, at least, a two language approach. The actual language being investigated can vary greatly. On the other hand, the modified Robinson approach can now be applied. This simply amounts to **removing the i notation**. This may seem to have been done in various cases. However, the code was merely notationally suppressed.

There has actually been a controversy since Gödel first coded the symbols of a first-order language as to how the symbols and the informal natural numbers are employed. In Mendelson (1987, p. 149) is a Gödel coding. The individual constant symbols are denoted as $a_1, a_2, \ldots, a_n, \ldots$. Thus the usual symbols for the informal natural numbers are employed. Usually these are taken only as symbolic forms. But the Gödel coding for these symbols is $g(a_k) = 7 + 8k,\ k = 1, 2, 3, \ldots$. There has been a group of mathematicians who do not allow this dual meaning for the symbols. That is they do not accept dissecting the symbolic form in this manner. Kleene suggested using his "tick" notation and the intuitive rule for counting to avoid this problem. That is the actual constant symbol is, say, $a_{||||}$, where the $||||$ corresponds to the informal natural number 4. Since such a symbolic change is possible, I see no need to actually do it. Having a pure dissectible symbolic form on one hand then bijectively corresponding it to something else is the mathematicians way of doing such things via a multi-language approach. This is the "object-language" and "meta-language" (observer-language) differences.

There is the developmental paradigm, and for nondetailed descriptions the *general developmental paradigm.* But now we have something totally new — the *general paradigm.* It is important to note that the general paradigm is considered to be distinct from developmental paradigms, although certain results that hold for general paradigms will hold for developmental paradigms and conversely. For example, associated with each general paradigm $\mathrm{G_A}$ is an ultraword w_g such that the set $\mathbf{G_A} \subset {}^*\mathbf{S}(\{w_g\})$ and all other theorems relative to such ultrawords hold for general paradigms. The general paradigm is a collection of words that discuss, in general, the behavior of entities and other constituents of a natural system. They, usually, do not contain a time statement W_i as it appears in section 7.1 for developmental paradigm descriptions. Our interest in this section is relative to only two such general paradigms. The reader can easily generate many other general paradigms.

The formal language is the usual first-order set-theoretic language with variables and constants. And, as used throughout, $\mathcal{W}'$ is a set of words formed by an alphabet, where if $w \in \mathcal{W}'$, then there is no set a in our structure such that $a \in w$. Obviously, if infinite $\mathcal{W}'$ is not denumerable, then the modified Robinson approach is the most appropriate, relative to the nonstandard language. Depending upon the application, the alphabet is assumed to have symbols for informal mathematical entities. Thus, there are mostly two mathematically styled languages, the symbolic language N, which is part of the "object language" that denotes the informal natural numbers considered as constants and the formal natural number $\mathbb{N}$ used to analyze the language. In the form of constants, members of N, only have, as previously defined, intuitive meaning. This allows one, as done by Robinson, to consider formal relations that tend to characterize the intuitive meanings.

Consider the symbol c' and let $\mathrm{n}' \in \mathrm{N}$. These symbols form a denumerable subset of $\mathcal{W}'$. The symbol $0' \notin \mathrm{N}$. These symbols are considered as alphabet members and correspond to constants that further correspond to the nonzero natural numbers. Hence, as set-theoretic entities $\mathrm{N} \subset \mathcal{W}'$. In what follows, the intended alphabet symbols are employed as constants of the formal first-order language. The formal mathematical structure also has the usual array of constants that denote the members. [Note: Within some of my papers on this subject you may find the notation $\underline{\mathcal{W}}$ or $\underline{\mathcal{W}'}$. These sets are the same as equivalence class representations $\mathcal{E}$ and $\mathcal{E}'$, respectively.]

Now consider the following informally defined set of words. Of course, in the extended case, it can be assumed that the cardinality of $\mathcal{W}'$ is no greater than that of $\mathbb{R}$. It should be noted that the members of $\mathrm{G_A}$ are but linguistic forms that do, at least, partially have meaning when interpreted physically. Due to the possible non-countable cardinality of $\mathcal{W}'$, the modified Robinson approach is employed in what follows.

$$\begin{gathered}\mathrm{G_A} = \{\mathrm{An|||elementary|||particle|||}\alpha(\mathrm{n}')\mathrm{|||with|||} \\ \mathrm{kinetic|||energy|||c'}{+}1/(\mathrm{n}').\mid n' \in \mathrm{N}\}\end{gathered} \tag{9.3.1}$$

Of particular interest is the composition of members of ${}^*\mathbf{G_A} - \mathbf{G_A}$.

Theorem 9.3.1 *A set* $[g] \in {}^*\mathbf{G_A} - \mathbf{G_A}$ *if and only if there exists a* $f \in {}^*(P_{55})$ *and a nonstandard* $\nu \in {}^*\mathrm{N} - \mathrm{N}$ *such that* $f \in [g]$, *and* $f(55) = \mathrm{A},\ f(54) = \mathrm{n},\ f(53) = |||, \cdots, f(30) = f(2), \cdots, f(3) = (, f(2) = \nu,\ f(1) =),\ f(0) = .$

Proof. From the definition of $\mathrm{G_A}$ the sentences

$$\forall z((z \in \mathbf{G_A}) \to \exists x \exists w((w \in \mathrm{N}) \wedge (x \in P_{55}) \wedge (x \in z) \wedge$$
$$((55, \mathrm{A}) \in x) \wedge ((54, \mathrm{n}) \in x) \ \wedge \cdots \wedge (x(30) = x(2)) \wedge \cdots \wedge$$
$$((3, () \in x) \wedge (x(2) = w) \wedge$$
$$((1,)) \in x) \wedge ((0, .) \in x))). \tag{9.3.2}$$
$$\forall x \forall w((x \in P_{55}) \wedge (w \in \mathrm{N}) \wedge$$
$$((55, \mathrm{A}) \in x) \wedge ((54, \mathrm{n}) \in x) \ \wedge \cdots \wedge (x(30) = x(2)) \wedge \cdots \wedge$$
$$((3, () \in x) \wedge (x(2) = w) \wedge ((1,)) \in x) \wedge ((0, .) \in x) \to$$
$$\exists z((z \in \mathbf{G_A}) \wedge (x \in z))).$$

hold in $\mathcal{M}$, hence in ${}^*\mathcal{M}$. There is in the standard structure bijection $j[\mathrm{N}] = \mathbb{N}'$. Hence, bijection ${}^*j[{}^*\mathrm{N}] = {}^*\mathbb{N}'$. Consequently ${}^*j[{}^*\mathrm{N} - \mathrm{N}] = {}^*\mathbb{N} - \mathbb{N}$. Since ${}^*j[\mathrm{N}] = j[\mathrm{N}]$ under our notational convention, where, for atoms a, ${}^*a = a$, then there is a nonstandard $\nu \in {}^*\mathrm{N} - \mathrm{N}$ that satisfies the *-transformed statements 9.3.2 for a $[g] \in {}^*\mathbf{G_A} - \mathbf{G_A}$, where internal partial sequence $f \in [g]$ is the member that characterizes the alphabet members and, thus, also varies over members of ${}^*\mathrm{N} - \mathrm{N}$ for $f(2)$ and $f(2) = f(30)$. ▮

Using Theorem 9.3.1, each member of ${}^*\mathbf{G_A} - \mathbf{G_A}$, when interpreted, has only two positions with a single missing standard object since positions 30 and 2 do not correspond to any symbol string in our language $\mathcal{W}'$. This interpretation still retains a vast amount of content, however. The members of ${}^*\mathbb{N} - \mathbb{N} = \mathbb{N}_\infty$ correspond to the infinite Robinson numbers. Thus, considering "new" constant symbols not used in our language, such as $\wr$, and let them denote infinite numbers, we have symbolic forms such as

$$\mathrm{G}'_\mathrm{A} = \mathrm{An}|||\mathrm{elementary}|||\mathrm{particle}|||\alpha(\wr)|||\mathrm{with}|||$$
$$\mathrm{kinetic}|||\mathrm{energy}|||\mathrm{c}'{+}1/(\wr). \tag{9.3.3}$$

9.4 Interpretations

Recall that the Natural world portion of the NSP-world model may contain *undetectable* objects, where "undetectable" means that there does not appear to exist human, or humanly constructible machine sensors that directly detect the objects or directly measure any of the objects physical properties. The rules of the scientific method utilized within the micro-world of subatomic physics allow all such undetectable Natural objects to be accepted as existing in reality.[1] The properties of such objects are indirectly deduced from the observed properties of gross matter. In order to have indirect evidence of the objectively real existence of such objects, such indirectly obtained behavior will usually satisfy a specifically accepted model.

Although the numerical quantities associated with these undetectable Natural (i.e. standard) world objects, if they really do exist, cannot be directly and exactly measured via any known instrumentation, these quantities are still represented by standard mathematical entities. By the rules of correspondence for interpreting pure NSP-world entities, such entities with a property being described by G'_{A} must be considered as undetectable pure NSP-world objects, assuming any of them exist in this background world. On the other hand, physical entities could satisfy this behavior, when viewed from the substratum. The G'_{A} type statements are actually being **predicted by the mathematical method employed**. Consequently, some such measures *may be* assumed to have an indirect affect within the Natural world. The predicted measure $1/\wr$ is that of an infinitesimal. From a substratum viewpoint, when c' is interpreted as the $0 \in \mathbb{N}$, it rationally verifies a stance original held by Newton that such measures are "real" as well as a remark by Robinson that such measures may be of significance in the world of particle physics.

The concept of *realism* often dictates that all interpreted members of a mathematical model be considered as existing in reality. The philosophy of science that accepts only *partial realism* allows for the following technique. One can stop at any point within a mathematically generated physical interpretation. Then proceed from that point to deduce an intuitive physical theory, but only using other not interpreted mathematical formalism as auxiliary constructs or as catalysts. Entities having such infinitesimal measures could be restricted to the substratum. Or, as mentioned, they could be physical entities that exhibit such behavior only when viewed from the substratum. With respect to the NSP-world, another aspect of this interpretation enters the picture. Assuming realism, then the question remains which, if any, entity with infinitesimal behavior actually indirectly influences Natural world processes? Partial realism allows for the possibility that none of these pure NSP-world measures has any affect upon the standard world. These ideas should always be kept in mind.

If you accept that such particle measures as described by G_{A} can exist in reality, then the philosophy of realism leads to the next interpretation.

> (1) *If there exists an elementary particle with Natural system behavior described by* G_{A}, *then there exists an entity that displays the behavior described by statement* G'_{A}.

The concept of absolute realism would require that the acceptance of entities with behavior described by G_{A} is indirect evidence for the existence of the G'_{A} described behavior. I caution the reader that the interpretation we apply to such sets of sentences as $\mathrm{G_A}$ are only to be applied to such sets of sentences.

The these rsults may, of course, be interpreted in infinitely many different ways. Indeed, the NSP-world model with its physical-type language can also be applied in infinitely many ways to infinitely many scenarios. I have applied it to such models as the MA, GD, GID and GGU-models among others. In this section, I consider another possible interpretation relative to those Big Bang cosmologies that postulate real objects at or near infinite temperature, energy or pressure. These theories incorporate the concept of the *initial singularity(ies).*

One of the great difficulties with many Big Bang cosmologies is that no meaningful physical interpretation for formation of the initial singularity is forthcoming from the theory itself. The fact that a proper and acceptable theory for creation of the universe requires that consideration not only be given to the moment of zero cosmic time but to what might have occurred "prior" to that moment in the nontime period is what partially influenced Wheeler to consider the concept of a *pregeometry.*[3], [4] It is totally unsatisfactory to dismiss such questions as "unmeaningful" simply because they cannot be discussed in your favorite theory. Scientists must search for a broader theory to include not only the question but a possible answer.

Although the initial singularity for a Big Bang type of state of affairs apparently cannot be discussed in a meaningful manner by many standard physical theories, unless one adjoins to the theory an ad hoc quantum field, it can be discussed by application of our NSP-world language. Let c' be a symbol that represents any fixed real number. Define

$$\begin{aligned}\mathrm{G_B} = \{&\mathrm{An}|||\mathrm{elementary}|||\mathrm{particle}|||\alpha(\mathrm{n}')|||\mathrm{with}|||\\ &\mathrm{total}|||\mathrm{energy}|||\mathrm{c}'+\mathrm{n}'.\mid n\in \mathbb{N}\},\end{aligned}\tag{9.4.1}$$

Application of Theorem 9.3.1 to $\mathrm{G_B}$ yields the form

$$\begin{aligned}\mathrm{G'_B} = &\mathrm{An}|||\mathrm{elementary}|||\mathrm{particle}|||\alpha(\zeta')|||\mathrm{with}|||\\ &\mathrm{total}|||\mathrm{energy}|||\mathrm{c}'+\wr\end{aligned}\tag{9.4.2}$$

(2) *If there exist an elementary particle with Natural system behavior described by* $\mathrm{G_B}$*, then there exist an entity that displays behavior described by* $\mathrm{G'_B}$.

The entities being described by $\mathrm{G'_B}$ have infinite energy. This infinite energy **does not** behave in the same manner as would the real number energy measures discussed in $\mathrm{G_B}$. As is usual when a metalanguage physical theory is generated from a formalism, we can further extend and investigate the properties of $\mathrm{G'_B}$ described entities by imposing upon them the corresponding behavior of the positive infinite hyperreal numbers. This produces some interesting propositions. Hence, we are able to use a nonstandard physical world language in order to give further insight into the state of affairs at or near a cosmic initial singularity. This gives *one* solution to a portion of the pregeometry problem. I point out that there are other NSP-world models for the beginnings of our universe, if there was such a beginning. Of course, the statement G'_B need not be related at all to any Natural world physical scenario, but could refer only to the behavior of pure NSP-world entities.

Notice that Theorems such as 7.3.1 and 7.3.4 relative to the generation of developmental paradigms by ultrawords, also apply to general paradigms, where $\mathrm{M}, \mathrm{M}_\mathrm{B}, \mathrm{P}_0$ are defined appropriately. The following is a slight extension of Theorem 7.3.2 for general paradigms. Theorem 9.4.1 will also hold for developmental paradigms.

Theorem 9.4.1 *Let* G_C *be any denumerable general paradigm. Then there exists an ultraword* $w \in {}^*\mathbf{P}_0$ *such that for each* $\mathbf{F} \in \mathbf{G_C}$, $\mathbf{F} \in {}^*\mathbf{S}(\{w\})$ *and there exist infinitely many* $[g] \in {}^*\mathbf{G_C} - \mathbf{G_C}$ *such that* $[g] \in {}^*\mathbf{S}(\{w\})$.

Proof. In the proof of Theorem 7.3.2, it is shown that there exists some $\nu \in {}^*\mathbb{N} - \mathbb{N}$ and a bijection h such that ${}^*h[[0,\nu]] \subset {}^*\mathbf{S}(\{w\})$ and ${}^*h[[0,\nu]] \subset {}^*\mathbf{G_C}$. Since $|{}^*h[[0,\nu]]| \geq |\mathcal{M}_1|^+$, then $|{}^*h[[0,\nu]] - h[\mathbb{N}]| \geq |\mathcal{M}_1|^+$. This completes the proof. ∎

Corollary 9.4.1.1 *Theorem 9.4.1 holds, where* G_C *is replaced by a developmental paradigm.*

> (3) *Let* G_C *be a denumerable general paradigm. There exists an intrinsic ultranatural process,* ${}^*\mathbf{S}$, *such that objects described by members of* G_C *are produced by* ${}^*\mathbf{S}$. *During this production, numerously many pure NSP-objects as described by statements in* ${}^*\mathbf{G_C} - \mathbf{G_C}$ *are produced.*

2. On my use of saturated models.

It is to be understood that any saturated enlargement that I employ is obtained only by application of the following theorem.

> **8.2 Theorem.** Given any superstructure $V(X)$ and cardinal κ there is a κ-saturated superstructure $V({}^*X)$ and monomorphism $*\colon V(X) \to V({}^*X)$. (Hurd and Loeb, (1985, p. 105)).

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