The identity of the inertial mass with the gravity mass

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April 23, 2016

Abstract

The motion of the mathematical extra-terrestrial pendulum is considered in the spherical gravitational field. The potential energy of the pendulum bob is approximated to give the nonlinear equation of motion. After solving it by the Landau-Migdal method we obtain the frequency of motion and the swing amplitude. The crucial point of our experiment is, that if the inertial mass m depends on its distance from the reference body M, then the calculated frequuency of our pendulum is not identical with the measured frequency. This crucial knowledge of the pendulum project can have substantional influence on the future development of general relativity and physics of elementary particles.

Key words. Mathematical pendulum, Newton gravity potential, nonlinear equation, Landau-Migdal iteration method.

We suggest experiment for the test if the inertial mass m of a body depends on its distance from the reference body with mass M. It can be tested by the large mathematical pendulum, which we denoted as the cosmical extra-terrestrial pendulum.

We specify such cosmical pendulum by the definition that it is the extra-terrestrial mathematical pendulum, the pendant of which moves along a geostationary orbit, where the geostationary Earth orbit or geosynchronous equatorial orbit (GEO) is a circular orbit 35,786 kilometres above the Earth equator and following the direction of the Earth rotation. An object in such an orbit has an orbital period equal to the Earth rotational period.

The length of the (carbon tube) fibre of the pendulum is considered sufficiently long to detect the sphericity of the globe gravity.

The mathematics of pendulums are in general quite complicated problem. However, there are some simplifying assumptions, which allows the equations of motion to be solved analytically for small-angle oscillations. We consider here the pendulum bob motion in the nonuniform gravity field expressed by the Newton gravitational formula. The parameters of pendulum is as follows: the swing fibre is massless, inextensible and always remains of the constant length, the moving bob with mass m is a point mass, motion occurs only in two dimensions, i.e. the bob trajectory is an arc, the motion does not lose energy to friction or air resistance, and finally, the pendant point does not move (the pendant point is on the geo-stationary satellite).

In order to get the differential equation of motion, we first derive the gravitational potential energy generated by the point with mass M and with the mass of the bob m.

The force acting on the point mass in such gravity field is

$$\mathbf{F} = -\kappa \frac{Mm}{r^3}\mathbf{r}. \tag{1}$$

The performed work by this force is defined by the following formula

$$W = -\int_{\mathbf{r_1}}^{\mathbf{r_2}} \mathbf{F} \cdot d\mathbf{r} = \kappa Mm \int_{\mathbf{r_1}}^{\mathbf{r_2}} \frac{\mathbf{r} \cdot d\mathbf{r}}{r^3}, \tag{2}$$

where the sign (-) in front of integral formula denotes the negative work performed by the gravitational field. Using $\mathbf{r} \cdot d\mathbf{r} = |\mathbf{r}||d\mathbf{r}| \cos\alpha = rdr$, we get

$$W = \kappa Mm \int_{r_1}^{r_2} \frac{rdr}{r^3} = -\kappa Mm \left(\frac{1}{r_2} - \frac{1}{r_1}\right). \tag{3}$$

Now, let us introduce the general point r by $r_2 \to r$ and reference point R by , $r_1 \to R$ then the potential energy is of the form

$$E_p = -\kappa Mm \left(\frac{1}{r} - \frac{1}{R}\right). \tag{4}$$

The potential energy of a point m at the vicinity of the reference point R, i.e. at point $R+h$, where $h \ll R$ is then

$$E_p(R+h) = -\frac{\kappa Mm}{R}\left(\frac{1}{1+h/R} - 1\right) \approx \frac{\kappa Mmh}{R^2}\left(1 - h/R\right). \qquad (5)$$

With regard to the fact that the local acceleration at point R is

$$g = \frac{\kappa M}{R^2}, \qquad (6)$$

we write the potential energy in the simple form:

$$E_p = mgh\left(1 - \frac{h}{R}\right) = mgh - \frac{mg}{R}h^2, \qquad (7)$$

which is the suitable formula of energy with spherical gravity correction in order to construct the differential equation for motion of pendulum.

Let us consider the pendulum with the equilibrium z-coordinate at point $z = R$ and the support coordinate (the pendant point) is at $z = R + l$, where l is the length of our pendulum. Then, the standard expression for the h-coordinate is

$$h = 2l\sin^2(\varphi/2) \approx \frac{l}{2}\varphi^2; \varphi \to 0, \qquad (8)$$

where φ is the standard deflection angle of the swing with regard to the z-coordinate.

The total energy of the pendulum is

$$E = \frac{1}{2}mv^2 + mgh - \frac{mg}{R}h^2. \qquad (9)$$

We obtain for small φ that $\lim_{\varphi\to 0}(dh/dt) \approx l\varphi(d\varphi/dt)$ and from equation $dE/dt = 0$, we get following equation

$$\ddot{\varphi} + \frac{g}{l}\varphi - \frac{gl}{R}\varphi^3 = 0, \qquad (10)$$

or,

$$\ddot{\varphi} + \omega_0^2\varphi = \lambda\varphi^3; \quad \omega_0^2 = g/l; \quad \lambda = gl/R. \qquad (11)$$

The next step is to solve the last differential equation (11) by the appropriate approximate method.

We will solve the eq. (11) by iteration. In order to avoid the resonance solution, we use the method described for instance in the Migdal special book on special mathematical methods in quantum mechanics (Migdal, 1975). This method was also used by author at solwing the Gross-Pitaevskii equation for the superfluid medium (Pardy, 1989) with the goal to detect the gravity waves by the superfluid system (instead of LIGO and eLISA).

The first step is that we rewrite eq. (11) as follows:

$$\ddot{\varphi} + \omega^2\varphi = \lambda\varphi^3 + (\omega^2 - \omega_0^2)\varphi; \quad \omega_0^2 = g/l; \quad \lambda = gl/R, \tag{12}$$

where the fundamental solution of the left side is $\varphi_0 = C\sin\omega t$, where constant C must be determined from the initial conditions. Then, the equation for the first iteration is as follows

$$\ddot{\varphi}_1 + \omega^2\varphi_1 = \frac{1}{4}\lambda C^3(3\sin\omega t - \sin 3\omega t) + C(\omega^2 - \omega_0^2)\sin\omega t. \tag{13}$$

The mathematical consistency demands the coefficient with $\sin\omega t$, must be zero from which follows that

$$\omega = \left(\omega_0^2 - \frac{3}{4}\lambda C^2\right)^{1/2}; \quad \omega_0^2 = g/l; \quad \lambda = gl/R. \tag{14}$$

Now, we must solve the equation

$$\ddot{\varphi}_1 + \omega^2\varphi_1 = -\frac{\lambda}{4}C^3(\sin 3\omega t). \tag{15}$$

The partial solution of eq. (15) is $\varphi_p = A\sin 3\omega t$, which gives after insertion this function in eq. (15), that

$$A = \frac{\lambda}{32}\frac{1}{\omega^2}C^3 \tag{16}$$

and it means that the first iteration solution of eq. (12) is

$$\varphi_1 = C\sin\omega t + \left(\frac{\lambda}{32}\frac{1}{\omega^2}C^3\right)\sin 3\omega t; \quad \omega = \left(\omega_0^2 - \frac{3}{4}\lambda C^2\right)^{1/2}. \tag{17}$$

We have seen how to solve, in approximation, the mathematical pendulum moving in the spherical gravity field. We considered here the large extra-terrestrial pendulum with the long fibre. The pendant point is considered on the geo-stationary satellite. While the bob of the Foucalt pendulum moves over the Earth surface, our pendulum bob moves in ionosphere.

We know from history, that Isaac Newton $\sim$ 1680 measured the period of pendulums of different mass but identical length difference with the result being less than 1 part in 1000. Friedrich Wilhelm Bessel in 1832 measured the period of pendulums of different mass but identical length with no measurable difference. So, we here follow the great scientists of pendulum experiments.

The crucial point of our experiment is, that if the inertial mass m depends on its distance from the reference body M, then the calculated frequuency of our pendulum is not identical with the measured frequency. This crucial knowledge can have substantional influence on the future development of general relativity and physics of elementary particles.

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