# The Division of One by Zero

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## Abstract

**Unfortunately, however, the relation between a finite and an infinite is not always so straightforward. The infinite and the finite mutually related as sheer others are inseparable. A related point is that while the infinite is determined in its own self by the other of itself, the finite, the finite itself is determined by its own infinite. Each of both is thus far the unity of its own other and itself. The inseparability of the infinite and the finite does not mean that a transition of the finite into the infinite and vice versa is not possible. In the finite, as this negation of the infinite, we have the satisfaction that determinateness, alteration, limitation et cetera are not vanished, are not sublated. The finite is a finite only in its relation to its own infinite, and the infinite is only infinite in its relation to its own finite. As will become apparent, the infinite as the empty beyond the finite is burdened by the fact that determinateness, alteration, limitation et cetera are vanished. The relation between the finite and the infinite finds its mathematical formulation in the division of one by zero. As we will see, it is $+1/+0=+\infty$.**

## Keywords

**Number theory, Quantum theory, Relativity theory, Unified field theory, Causality**

## 1. Introduction

Time determined as opposed to an other is negatively determined not only against an other but as an infinite it opposed to a finite. Even when time is represented as unlimited and infinite, a 'point in time' still constitutes its element. *Is infinity itself free from any opposition or contradiction*? The infinite, a purely self-related, is something relative, it is related to its own other, it is the negation of its own other, of its own finite. The infinite even if determined by its own other, by the finite, is determined as the non-being of an other, while the finite itself stands as opposed to its own infinite. Both are equally others to each other. The infinite, in separation from the other, separated from the finite, above or beyond the finite, the finite as being here and the infinite being there, is related to the finite. What could justify the assumption that an infinite does stand as something complete and finished and superior or above to the finite? Even as separated, the infinite and the finite are connected by that which separates both. As therefore each is in its other and through its own other determining its own self, the finite and the infinite, even if separated are inseparable, which is equally the inner unity. Consequently, is there a point where a finite changes into an infinite, where a finite negate its negation and to become infinite? In other words, where does infinity begins? How can something pass beyond the finite into the infinite and vice versa? The changing or the necessary transition of the finite into the infinite, and vice versa. In mathematics, several types of indeterminate forms are distinguished, in principle. Some typical indeterminate forms considered in the literature are denoted by 0/0 or by 1/0 or by ∞/∞ or by 0×∞ or by ∞−∞ or by $0^0$ or by $1^\infty$ and by $\infty^0$ et cetera. Our today's contemporary mathematical viewpoint of infinity is attributed to the English mathematician John *Wallis*. In 1655, John Wallis (1616-1703) for the first time introduced the symbol ∞ for infinity. John Wallis, pointed out: "esto enim ∞ nota numeri infiniti" [1]. Translated into English: 'let the symbol ∞ denote infinity'. In particular, Wallis himself claimed in 1656 without a proof that "1/∞ ... habenda erit pro nihilo" [2] or

$$\frac{+1}{+\infty} = +0 \tag{1}$$

Thus far, John Wallis [3] is demanding without a mathematical proof too, that

$$\frac{+1}{+\infty} \times +\infty = +1 \tag{2}$$

Isaac *Newton* himself followed Wallis [4] in his position. In his own book *Opuscla,* Isaac Newton claimed without a proof that "*1 / 0 = Infinitae*" [5]. We may ask ourselves, can Einstein's theory of special relativity tell us anything about the division of one by zero?

## 2. Material and methods

### 2.1. Definitions

#### Definition. The time of a stationary observer $_{R}t$ and the time of a co-moving observer $_{O}t$

Time is dependent on the observer's reference frame. Especially, clocks moving at close to the speed of light c will slow down with respect to a stationary observer R (observer at rest). Thus far, let $_{R}t$ denote the time as measured by a stationary observer, i. e. the relativistic time. Let $_{O}t$ denote the time as measured by a moving observer O. The relationship between the time $_{O}t$ as measured by a clock moving at constant velocity v in relation to the time $_{R}t$ as measured by a clock of a stationary observer R is determined by Einstein's relativistic time dilation [6] as

$$_{O}t = {}_{R}t \times \sqrt[2]{1 - \frac{v^2}{c^2}} \tag{3}$$

where $_{O}t$ denotes time as measured by a moving observer O, $_{R}t$ denotes the time as measured by a stationary observer R, v denotes the relative velocity between both observers and c denotes the speed of light in vacuum. Equally, it is

$$\frac{_{O}t}{_{R}t} = \sqrt[2]{1 - \frac{v^2}{c^2}} \tag{4}$$

or

$$\frac{_{O}t}{c^2} \times \frac{c^2}{_{R}t} = \sqrt[2]{1 - \frac{v^2}{c^2}}$$

(5)

**Scholium.**
Coordinate systems can be chosen freely, deepening upon circumstances. In many coordinate systems, an event can be specified by one time coordinate and three spatial coordinates. The time as specified by the time coordinate is denoted as coordinate time. Coordinate time is distinguished from proper time. The concept of proper time, introduced by Hermann Minkowski in 1908 and denoted as $_{O}t$, incorporates Einstein's time dilation effect. In principle, Einstein is defining time exclusively for every place where a watch measuring this time is located.

"... Definition ... der ... Zeit ... für den Ort, an welchem sich die Uhr … befindet ..." [7]

In general, a watch is treated as being at rest relative to the place where the same watch is located.

"Es werde ferner mittels der im ruhenden System befindlichen ruhenden Uhren die Zeit t [$_{R}t$, author] des ruhenden Systems ... bestimmt, ebenso werde die Zeit τ [$_{O}t$, *author*] des bewegten Systems, in welchen sich relativ zu letzterem ruhende Uhren befinden, bestimmt..." [8]

Only, the place where a watch at rest is located can move together with the watch itself. Therefore, due to Einstein, it is necessary to distinguish between clocks as such which are qualified to mark the time $_{R}t$ when at rest relatively to the stationary system R, and the time $_{O}t$ when at rest relatively to the moving system O.

"Wir denken uns ferner eine der Uhren, welche relativ zum ruhenden System ruhend die Zeit t [$_{R}t$, author], relativ zum bewegten System ruhend die Zeit τ [$_{O}t$, *author*] anzugeben befähigt sind ..." [9]

In English:
<Further, we imagine one of the clocks which are qualified to mark the time t [$_{R}t$, author] when at rest relatively to the stationary system, and the time τ [$_{O}t$, *author*] when at rest relatively to the moving system …. >

### Definition. The normalized relativistic time dilation

As defined above, due to Einstein's special relativity, it is

$$\frac{_{O}t}{_{R}t} = \sqrt[2]{1 - \frac{v^2}{c^2}} \tag{6}$$

The normalized relativistic time dilation relation [10] follows as

$$\frac{_{O}t^2}{_{R}t^2} + \frac{v^2}{c^2} = 1 \tag{7}$$

## 2.2. Axioms

The following theory is based on the next axiom.

### Axiom I. (Lex identitatis)

$$+1 = +1 \qquad \text{(Axiom I)}$$

## 3. Results

Isaac Newton [11] created his own world view. Centuries later, Albert Einstein's (1879-1955) published his theory of special relativity. Einstein's theory of special relativity which has passed a lot of observational and experimental investigations could be of use to solve the problem of the division of 1 by 0.

### 3.1. Theorem. The square root of zero

**Claim.**
In general, it is

$$\sqrt[2]{+1-1} = \sqrt[2]{+i^2-i^2} = \sqrt[2]{0} = |0| \tag{8}$$

**Direct proof.**
Due to our Axiom I it is

$$+1 = +1 \tag{9}$$

According to our today's rules of algebra, we obtain

$$+1-1 = +1-1 = +i^2-i^2 = +0 \tag{10}$$

where the imaginary unit i is per defintionem $i^2=-1$. The square root operation is equivalent with

$$\sqrt[2]{+1-1} = \sqrt[2]{+i^2-i^2} = \sqrt[2]{+0} \tag{11}$$

At this point, we don't know the result of the square root of zero. Consequently, we assume that

$$\sqrt[2]{+1-1} = \sqrt[2]{+i^2-i^2} = \sqrt[2]{+0} \equiv X \tag{12}$$

Following the rules of algebra, it is equally valid that

$$+1-1 = +i^2-i^2 = +0 = X^2 \tag{13}$$

This equation is equivalent with

$$+1-1 = +i^2-i^2 = +0 = X \times X \tag{14}$$

In last consequence it is

$$+0 = X \times X \tag{15}$$

or

$$\sqrt[2]{+0} = |0| \tag{16}$$

**Quod erat demonstrandum.**

### 3.2. Theorem. The division of one by zero I

Let us perform a thought experiment under extreme conditions of inertial frames of reference where $_{R}t = +\infty$, $_{0}t=+1$ and v=c.

**Claim.**
Under conditions of special relativity (inertial frames of reference) where $_{R}t = +\infty$, $_{0}t=+1$ and v=c the division of one by zero is possible and allowed. In particular, it is

$$\frac{+1}{|0|} = +\infty \tag{17}$$

**Direct proof.**
Due to our Axiom I it is

$$+1 = +1 \tag{18}$$

Multiplying this equation with $_{O}t$, the "proper" time, we obtain

$$_{O}t \times 1 = _{O}t \times 1 \tag{19}$$

In general, due to Einstein's special relativity it is equally

$$_{O}t = _{R}t \times \sqrt[2]{1 - \frac{v^2}{c^2}} \tag{20}$$

In general, Einstein's special relativity demands that

$$\frac{_{O}t}{\sqrt[2]{1 - \frac{v^2}{c^2}}} = _{R}t \tag{21}$$

Under conditions of inertial frames of reference, Einstein's relativistic time-dilation relation is generally valid. Theoretically, a stationary observer is allowed to measure a time $_{R}t$ which is infinite. Thus far even under circumstances where $_{R}t = +\infty$, Einstein's relativistic time-dilation relation is valid. Thus far, there is no theoretical evidence that Einstein's relativistic time-dilation relation is not valid under conditions where $_{R}t=+\infty$. Rearranging equation before, we obtain

$$\frac{_{O}t}{\sqrt[2]{1 - \frac{v^2}{c^2}}} = +\infty \tag{22}$$

The validity of Einstein's relativistic time-dilation relation is not limited and will not break down if the co-moving observer measures the time $_{0}t=+1$. While the stationary observer measures an infinite time $_{R}t$, the stationary observer can measure the amount of $_{0}t=+1$. The equation before changes to

$$\frac{+1}{\sqrt[2]{1 - \frac{v^2}{c^2}}} = +\infty \tag{23}$$

There are circumstances where *the relative velocity v* between the stationary observer R and the co-moving observer 0 is equality to *v=c*. Even under these conditions, Einstein's relativistic time-dilation relation is valid. We obtain

$$\frac{+1}{\sqrt[2]{1-\frac{c^2}{c^2}}}=\frac{+1}{\sqrt[2]{1-\frac{1^2}{1^2}}}=\frac{+1}{\sqrt[2]{1-1}}=\frac{+1}{\sqrt[2]{0}}=+\infty \tag{24}$$

Finally, even under extreme conditions, where ${}_{R}t = +\infty$, ${}_{0}t=+1$ and $v=c$, Einstein's relativistic time-dilation is valid. Based on Einstein's theory of special relativity and the theorem 3.1, we obtain

$$\frac{+1}{|0|}=+\infty \tag{25}$$

**Quod erat demonstrandum.**

*Scholium*.
In other words, all of the above equations incorporate the instance that $+1/(-0)=-\infty$ while $+1/(+0)=+\infty$.

## 3.3. Theorem. The division of one by zero II

Einstein's theory of special relativity is valid even under conditions where the relative velocity v = 0. Under conditions where the relative velocity v = 0 the wave energy (of a quantum mechanical object) as such is not destroyed but converted completely into pure energy of a 'particle'.

**Claim.**
Under conditions of special relativity (inertial frames of reference) there are circumstances, where the relative velocity v = 0. Under conditions where the relative velocity v = 0 we must accept that

$$\frac{+1}{+0}=+\infty \tag{26}$$

**Direct proof.**
Due to our Axiom I it is

$$+1=+1 \tag{27}$$

Multiplying this equation with ${}_{O}t$, the "proper" time, we obtain

$${}_{O}t\times 1={}_{O}t\times 1 \tag{28}$$

In general, due to Einstein's special relativity it is equally

$${}_{O}t={}_{R}t\times\sqrt[2]{1-\frac{v^2}{c^2}} \tag{29}$$

In general, Einstein's special relativity demands that

$$\frac{{}_{O}t \times {}_{O}t}{{}_{R}t \times {}_{R}t} + \frac{v \times v}{c \times c} = 1 \tag{30}$$

Under experimental conditions of special relativity where **v=c** we obtain

$$\frac{{}_{O}t \times {}_{O}t}{{}_{R}t \times {}_{R}t} + \frac{c \times c}{c \times c} = 1 \tag{31}$$

or

$$\frac{{}_{O}t \times {}_{O}t}{{}_{R}t \times {}_{R}t} + 1 = 1 \tag{32}$$

or

$$\frac{{}_{O}t \times {}_{O}t}{{}_{R}t \times {}_{R}t} = +0 \tag{33}$$

In particular, even under extreme conditions, where ${}_{R}t = +\infty$, ${}_{0}t=+1$ and v=c, Einstein's normalized relativistic time-dilation is valid. Based on these assumptions, we obtain

$$\frac{+1 \times +1}{+\infty \times +\infty} = +0 \tag{34}$$

Due to our theorem before, it is $+1/+\infty=+0$. We obtain

$$+0 \times +0 = +0 \tag{35}$$

or at the end

$$+0 = +0 \tag{36}$$

**Quod erat demonstrandum.**

### 3.4. Theorem. The normalization of the relationship between a finite and an infinite

**Claim.**
The normalization of the finite and the infinite follows as

$$\frac{+1}{+\infty} + \frac{+\infty - 1}{+\infty} = +1 \tag{37}$$

**Direct proof.**
Due to our Axiom I it is

$$+1 = +1 \tag{38}$$

Rearranging equation, we obtain

$$+1-1=+0 \tag{39}$$

Adding $+\infty$ it is

$$+1+\infty-1=+\infty+0 \tag{40}$$

In general, it is $+\infty+0=+\infty$ and it follows that

$$+1+\infty-1=+\infty \tag{41}$$

Dividing by $+\infty$, we obtain

$$\frac{+1}{+\infty}+\frac{+\infty-1}{+\infty}=\frac{+\infty}{+\infty} \tag{42}$$

Our goal is to normalize the relationship between the finite +1 and the infinite $+\infty$. Therefore, time even if infinite is something real. Thus far, infinity divided by infinity is equal to one. We obtain

$$\frac{+1}{+\infty}+\frac{+\infty-1}{+\infty}=+1 \tag{43}$$

**Quod erat demonstrandum.**

Scholium.
Due to the theorem before, it is $+1/+\infty=+0$. Consequently, it is

$$+0+\frac{+\infty-1}{+\infty}=+1 \tag{44}$$

or

$$\frac{+\infty}{+\infty}-\frac{+1}{+\infty}=+1 \tag{45}$$

or

$$\frac{+\infty}{+\infty}-0=+1 \tag{46}$$

or

$$\frac{+\infty}{+\infty}=+1 \tag{47}$$

which was assumed by us.

## 4. Discussion

In general, we were able to provide evidence that $+1/+0=+\infty$. Even if the proof itself is self-consistent, there are still some questions about the validity of such an approach. Why should it be allowed that a stationary observer measured a stationary time of $_{R}t = +\infty$ while the co-moving observer measures the time $_{0}t=+1$ and all this while the relative velocity between the stationary observer and the co-moving observer is equal to v=c. Are there such circumstances at all? Equally, under conditions where the relative velocity between the stationary observer R and the co-moving observer 0 is equal to v=c, the rest-mass is equal to zero. In particular, another question arises. We found that 0/0=1. Multiplying by 0 we obtain $0\times0/0 =0\times1=0$. Since $0\times0 = 0$ we obtain that 0/0=0 which is a contradiction to our finding which was that 0/0=1. Yet it still remains a mystery to divide 0 by 0. Thus, to perhaps clarify the issue here once again, it is necessary to work out the rules of *operation precedence* of (arithmetic and algebraic) operations to assure logical consistency or at least to use of *brackets*. In this context it is necessary to consider the following. We found that $(0/0)=(0\times(1/0))=1$. Due to our theorem above this is equivalent with $0\times(\infty)=1$. Multiplying by 0 we obtain $(0\times(\infty))\times0=1\times0=0$. Since $(0\times(\infty)) = 1$we obtain that $1\times0=1\times0=0$ which is a correct result.

Currently, dividing by zero on a computer causes problems. Consequently, computer hardware manufacturers are using the concept of "not a number" to define circumstances that a meaningful result/number can't be returned. Anderson [16] et al. are defining a new arithmetic which has no arithmetical exceptions. Following Anderson et al. the transreal numbers include all of the real numbers, plus three other mathematical constructs: infinity ($\infty$), negative infinity ($-\infty$) and "nullity" ($\phi$). In other words, Anderson et al. are treating infinity as something trans (non-) real. Anderson [16] et al. invented a new number, which their called "nullity". Formally, Anderson [16] et al. concept of "nullity" is not completely identical with *the concept of "not a number"* as already known in floating point arithmetic on computers but effectively, it is. What are the properties of "nullity"? In last consequence, "nullity" is nothing else but "not a number" and equally a kind of a reformulation of the basic mathematical concept of "undefined". Altogether, even a breathlessly repeated proclamation that a deep and important problem (i. e. $1/0=\infty$) is solved, does not solve such a problem. Anderson [16] et al. are reaching the result that $1/0=\infty$ (theorem 8). An axiom is something which is already well-established or at least so self-evident that it is accepted without any question or controversy by the scientific community. In particular, $1/0=\infty$ is neither well-established nor self-evident and not accepted at all as solved. Thus far, using Axiom 16 as the starting point or premise for further theorems, reasoning or arguments as done in theorem 8 is useless. Anderson et al. approach to the problem of $1/0=\infty$ is based on a premise (Axiom 16), which itself requires a proof. In principle, Anderson et al. reasoning is not suitable for solving the problem of 1/0 or 0/0.

Bergstra [17] et al. are trying to solve the problem of the division by 0 by an axiomatic concept of a field. In last consequence, due to Bergstra et al. the inverse of zero is zero or in other words 0/0=0. Following Bergstra [17] et al., fields itself cannot be defined by equations. Consequently, a field itself, described by a kind of a mathematical framework, cannot be equivalent to itself. In contrast to Bergstra et al., in physics and especially Einstein's field theory of gravitation, fields are determined through field equations. Thus far, Bergstra [17] et al. starting point is wothless. Further, if we accept that 0/0=0, we must give up Einstein's special theory of relativity. Due to Einstein's special theory of relativity, it is 0/0=1 [15]. Consequently, Bergstra [17] et al. approach to the problem of the division by 0is completely without any sense.

Hiroshi Michiwaki et al. [18] are claiming that Z/0=0. Equally, it is possible that Z=0. The issues that arise here are subtle and delicate. Altogether, Hiroshi Michiwaki et al. [18] and Bergstra et al. [17] contributions in this context are not solving any problems claimed. Following Hiroshi Michiwaki et al. [18] and Bergstra et al. [17] we must accept that 0/0=0 and abandon Einstein's special theory of relativity completely because Einstein's special relativity itself demands [15] that 0/0=1. In contrast to Bergstra et al. [17] and Hiroshi Michiwaki et al. [18] positions, special relativity is generally accepted and experimentally well confirmed. And much more than this. Einstein's special relativity theory and especially the equations defining the metric are nothing but the Pythagorean theorem [19] applied to the differentials of the co-ordinates.

Currently, the Pythagorean theorem itself is proofed as correct since thousands of years by many proofs while Einstein's theory of relativity is confirmed by a lot of experiments. Still, anybody is free to ask an extremely shocking and most fundamental question which itself raises subtle problems. In particular, are we allowed to rely on the Pythagorean theorem and Einstein's theory of special relativity at least in principle?

## 5. Conclusions

In general, **under conditions of the validity of Pythagoreans theorem and the theory of special relativity**, it is $(+1/+0) = +\infty$. The problem of the division of one by zero is solved.

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