

The Collapse of the Schwarzschild Radius: The End of Black Holes A Revised Escape Velocity Is Valid Under Strong Gravitational Fields with Potential Significant Implications for Cosmology

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Abstract

In this paper we introduce an exact escape velocity that also holds under very strong gravitational fields, even below the Schwarzschild radius. The standard escape velocity known from modern physics is only valid under weak gravitational fields. This paper strongly indicates that an extensive series of interpretations around the Schwarzschild radius are wrong and were developed as a result of using an approximate escape velocity that not is accurate when we approach strong gravitational fields. Einstein's general relativity escape velocity as well as the gravitational time dilation and gravitational redshift that are derived from the Schwarzschild metric need to be modified; in reality, they are simply approximations that only give good predictions in low gravitational fields. This paper could potentially have major implications for gravitational physics as well as a long series of interpretations in cosmology.

Key words: Escape velocity, strong gravitational field, Schwarzschild radius, gravitational time dilation, gravitational redshift, special relativity, general relativity, Planck quantization, Newton, Einstein, Schwarzschild.

1 Short Background on the Derivation on the Standard Escape Velocity

Derivation of the standard classical escape velocity is accomplished by solving the following equation with respect to v

$$\begin{aligned} E_k - \frac{GmM}{r} &= 0 \\ \frac{1}{2}Mv_e^2 - \frac{GmM}{r} &= 0 \\ v_e^2 &= \frac{\frac{GmM}{r}}{\frac{1}{2}m} \\ v_e^2 &= \frac{2GM}{r} \\ v_e &= \sqrt{\frac{2GM}{r}} \end{aligned} \tag{1}$$

which is the well known escape velocity, with important applications in rocket science and cosmology. Exactly the same escape velocity formula can be derived directly from Einstein's general relativity using the Schwarzschild metric. However, as pointed out by Augousti and Radosz (2006), for example, the formula derived from the Schwarzschild metric under general relativity theory only holds for a weak gravitational field. That is when we are considerably far away from the Schwarzschild radius of the mass in question. Both the standard way of deriving the classical escape velocity and the same escape velocity formula derived from general relativity theory using the Schwarzschild metric are only approximations

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that are very inaccurate in strong gravitational fields. At and below the Schwarzschild radius, the standard escape velocity formula has no logic and has led to a series of likely incorrect speculative conclusions that have had a significant affect on our view of gravitational physics and cosmology.

2 An Exact Escape Velocity That Also Holds Under Strong Gravitational Fields

In this section I will derive the exact escape velocity based on the exact kinetic energy formula. The kinetic energy formula typically used to demonstrate the derivation of the escape velocity is $E_k \approx \frac{1}{2}Mc^2$, see above. It is (or should be) understood that this is only an approximation formula for the exact kinetic energy formula that is given by:

$$E_k = \frac{Mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - Mc^2 \quad (2)$$

where M is the rest mass. By performing a Taylor series expansion of Einstein's "moving mass" formula we get:

$$\begin{aligned} \frac{Mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} &= Mc^2 + \frac{1}{2}Mv^2 + \frac{3}{8}M\frac{v^4}{c^2} + \frac{5}{16}M\frac{v^6}{c^4} + \frac{35}{128}M\frac{v^8}{c^6} + \frac{63}{256}M\frac{v^{10}}{c^8} \\ &+ \frac{231}{1024}M\frac{v^{12}}{c^{10}} + \frac{429}{2048}M\frac{v^{14}}{c^{12}} + \frac{6435}{32768}M\frac{v^{16}}{c^{14}} + \frac{12155}{65536}M\frac{v^{18}}{c^{16}} \\ &+ \frac{46189}{262144}M\frac{v^{20}}{c^{18}} + \frac{88179}{524288}M\frac{v^{22}}{c^{20}} + \frac{676039}{4194304}M\frac{v^{24}}{c^{22}} + \dots \end{aligned}$$

The Taylor series consists of an infinite array of terms, but when $v \ll c$ then only the first two terms in the Taylor series are needed for a good approximation:

$$\frac{Mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \approx Mc^2 + \frac{1}{2}Mv^2.$$

By subtracting the rest mass energy Mc^2 from the formula above, we get the classical kinetic energy formula $E_k \approx \frac{1}{2}Mv^2$. But again this approximate kinetic energy formula only holds when the velocity is much smaller than c . It is not an exact kinetic energy formula and it is very inaccurate for velocities approaching the speed of light. If we are using this approximate kinetic energy formula in deriving the escape velocity then the escape velocity will also be approximation that only is valid for $v \ll c$.

Here we will derive the escape formula from the exact kinetic energy formula; the escape velocity should then be exact. We are basically combining Newton's gravitational potential with Einstein's special relativity theory to solve for the escape velocity:

$$\begin{aligned} 0 &= E_k - \frac{GmM}{r} \\ 0 &= \frac{mc^2}{\sqrt{1 - \frac{\bar{v}_e^2}{c^2}}} - mv^2 - \frac{GmM}{r} \end{aligned} \quad (3)$$

we note the escape velocity here for \bar{v}_e rather than v or v_e to distinguish the notation for the exact escape velocity from the standard (approximate) escape velocity v_e . We get

$$\begin{aligned} \bar{v}_e &= \frac{c\sqrt{GM}\sqrt{GM + 2c^2r}}{GM + c^2r} \\ \bar{v}_e &= \frac{c\sqrt{G^2M^2 + GM2c^2r}}{GM\left(1 + \frac{c^2r}{GM}\right)} \\ \bar{v}_e &= c\frac{\sqrt{1 + \frac{2c^2r}{GM}}}{1 + \frac{c^2r}{GM}} \end{aligned} \quad (4)$$

This is the exact escape velocity formula. We can go further and obtain a quantized version of formula 4 based on the principle of Haug (2016a,b). We will set the Newton gravitational constant to

$$G_p = \frac{\aleph^2 c^3}{\hbar} \quad (5)$$

where \hbar is the reduced Planck's constant and c is the well tested round-trip speed of light. We could call this Planck's form of the gravitational constant. The parameter \aleph is unknown constant that can be set equal to the Planck length if this is know, or alternatively it can be calibrated to the measured gravitational constant and then we have found the Planck length indirectly. From this the Planck length is given by

$$l_p = \sqrt{\frac{\hbar G_p}{c^3}} = \sqrt{\frac{\hbar \frac{\aleph^2 c^3}{\hbar}}{c^3}} = \aleph \quad (6)$$

and the Planck mass is given by

$$m_p = \sqrt{\frac{\hbar c}{G_p}} = \sqrt{\frac{\hbar c}{\frac{\aleph^2 c^3}{\hbar}}} = \frac{\hbar}{\aleph c} \quad (7)$$

Using the gravitational constant in the Planck form, as well as the rewritten Planck units, we are easily able to rewrite the exact escape velocity in a quantized form as well

$$\begin{aligned} \bar{v}_e &= c \frac{\sqrt{1 + \frac{2c^2 r}{G_p M}}}{1 + \frac{c^2 r}{G_p M}} \\ \bar{v}_e &= c \frac{\sqrt{1 + \frac{2c^2 r}{G_p \aleph m_p}}}{1 + \frac{c^2 r}{G_p \aleph m_p}} \\ \bar{v}_e &= \frac{c \sqrt{1 + \frac{2c^2 r}{\frac{\aleph^2 c^3}{\hbar} \aleph \frac{\hbar}{\aleph} \frac{1}{c}}}}{\left(1 + \frac{c^2 r}{\frac{\aleph^2 c^3}{\hbar} \aleph \frac{\hbar}{\aleph} \frac{1}{c}}\right)} \\ \bar{v}_e &= c \frac{\sqrt{1 + \frac{2r}{\aleph \aleph}}}{1 + \frac{r}{\aleph \aleph}} \end{aligned} \quad (8)$$

where N is the total number of Planck masses, m_p , in the mass, M , we are trying to escape from.¹ Formula 4 and 8 will give the exact same output values, they differ in that the formula 4 requires the gravitational constant as input and the mass in kg, while the formula 8 requires the number of Planck masses the mass makes up, the Planck length, and the reduced Planck constant. From the exact escape velocity formula we can see there is no radius where the escape velocity is larger than c . In other words, the formula predicts that light can always escape an object no matter how massive it is or how strong the gravitational field is. In other words, the notion of a black hole is a mathematical illusion from an approximate escape velocity formula that is not valid in the presence of strong gravitational fields.

An interesting case is what the exact escape velocity is at the Schwarzschild radius $r_s = \frac{GM}{c^2} = 2N\aleph$. This gives

$$\begin{aligned} \bar{v}_e &= c \frac{\sqrt{1 + \frac{2r_s}{\aleph \aleph}}}{1 + \frac{r_s}{\aleph \aleph}} \\ \bar{v}_e &= c \frac{\sqrt{1 + \frac{2 \times 2N\aleph}{\aleph \aleph}}}{1 + \frac{2N\aleph}{\aleph \aleph}} \\ \bar{v}_e &= c \frac{\sqrt{5}}{3} \approx 0.745355992c \end{aligned} \quad (9)$$

First, at a radius considerably below the Schwarzschild radius, the escape velocity is approaching c . This is in sharp contrast to the standard approximate escape velocity of modern physics that predicts that

¹This formula reminds me of relativistic Doppler shift. If it not was for the number 2 inside the formula it could have been written as $\bar{v}_e = c \sqrt{\frac{1 - \frac{r}{\aleph \aleph}}{1 + \frac{r}{\aleph \aleph}}}$.

the escape velocity at the Schwarzschild radius is c and that the escape velocity inside the Schwarzschild radius is $> c$. In this scenario, the standard escape velocity predicts that not even a photon can escape if it passes inside the Schwarzschild radius. Such interpretations are the result of the misuse of approximations and artifact coordinates in the Schwarzschild metric. Table 1 shows the exact escape velocity and the approximate standard escape velocity. At the surface of earth we must go out to the 6 decimals to see differences between the exact and the approximate escape velocities. This is probably the reason that the standard escape velocity has been used so successfully and that no one has focused on the fact that it is only an approximation. Nevertheless, reliance on the standard escape velocity, an approximation that is not valid in strong gravitational fields, may very well have produced a series of deep misinterpretations in cosmology.

Table 1: The table shows the exact escape velocity from an Earth-sized mass at different radiuses compared to the standard escape velocity. Assumed mass: $M = 5.972 \times 10^{24}$ kg.

Multiples of the Schwarzschild radius :	Radius meter :	Exact escape velocity meters per second :	Standard escape velocity meters per second :
Surface earth: 718, 306, 435 r_s	6,371,000	11,185.768431	11,185.768436
100 r_s	0.886947366	29,867,359.67	29,979,245.80
10 r_s	0.088694737	91,409,921.62	94,802,699.26
5 r_s	0.044347368	124,892,875.60	134,071,263.05
r_s :	0.008869474	$c\frac{\sqrt{5}}{3} \approx$ 223,452,105	299,792,458.00=c
0.5 r_s	0.004434737	259,627,884.49	$> c$ Impossible to escape=Black hole
0.1 r_s	0.000886947	295,599,349.98	$> c$ Impossible to escape=Black hole
0.001 r_s	8.86947E-06	299,791,860.81	$> c$ Impossible to escape=Black hole
0.0001 r_s	8.86947E-07	299,792,452.01	$> c$ Impossible to escape=Black hole
0.00001 r_s	8.86947E-08	299,792,457.94	$> c$ Impossible to escape=Black hole
0 r_s	0	299,792,458.00	Equation breaks= BH singularity

I will claim that the Schwarzschild radius is nothing special in the physical world. The Schwarzschild radius and its misinterpretations arise from the use of an approximate escape velocity and likely are also from coordinate artifacts in the Schwarzschild solution of the Einstein field equation when $r = r_s$, see Eddington (1922). It is well known that some of the Schwarzschild metric components blow up at $r = r_s$ and $r = 0$. The misinterpretation of the Schwarzschild radius should become even clearer when we move on to gravitational time dilation and gravitational redshift.

3 Gravitational Time Dilation

Einstein's gravitational time dilation is given by

$$t_0 = t_f \sqrt{1 - \frac{2GM}{r}} = \sqrt{1 - \frac{v_e^2}{c^2}} \quad (10)$$

where $v_e = \sqrt{\frac{2GM}{r}}$ the standard escape velocity and r is the radius out from the center of the mass, and t_f is the time gone by for a clock so far from the gravitational center that it is basically unaffected by the gravitational field. To calculate the Einstein gravitational time dilation we need to know the escape velocity. Einstein's standard gravitational time dilation formula uses the approximate escape velocity formula that is only valid under weak gravitational fields. The approximate escape velocity formula gives extremely accurate values when we are at the surface of Earth or at radiuses similar to that of the GPS satellites, for example.

However, when we approach strong gravitational fields in the range of the Schwarzschild radius, then the standard Einstein approximate gravitational time dilation formula is likely to give incorrect values. At the Schwarzschild radius, the formula above gives highly inaccurate predictions and below the Schwarzschild radius the formula simply breaks down. It is also worth mentioning that Haug (2016b) has recently quantized the standard Einstein gravitational time dilation, see Appendix B. The quantization is not important for the conclusions in this paper, but we mention it here, as we will quantize the exact gravitational time dilation that holds at extremely strong gravitational fields.

Exact gravitational time dilation

The exact gravitational time dilation is obtained simply by replacing the approximate escape velocity used in modern physics with the exact escape velocity derived above

$$\begin{aligned}
 t_0 &= t_f \sqrt{1 - \frac{\bar{v}_e^2}{c^2}} \\
 t_0 &= t_f \sqrt{1 - \frac{\left(c \frac{\sqrt{1 + \frac{2c^2 r}{GM}}}{1 + \frac{c^2 r}{GM}} \right)^2}{c^2}} \\
 t_0 &= t_f \sqrt{1 - \frac{\left(1 + \frac{2c^2 r}{GM} \right)}{\left(1 + \frac{c^2 r}{GM} \right)^2}} \tag{11}
 \end{aligned}$$

Alternatively we could write this in a more informative and elegant quantized form

$$\begin{aligned}
 t_0 &= t_f \sqrt{1 - \frac{\bar{v}_e^2}{c^2}} \\
 t_0 &= t_f \sqrt{1 - \frac{\left(c \frac{\sqrt{1 + \frac{2r}{N\aleph}}}{1 + \frac{r}{N\aleph}} \right)^2}{c^2}} \\
 t_0 &= t_f \sqrt{1 - \frac{1 + \frac{2r}{N\aleph}}{\left(1 + \frac{r}{N\aleph} \right)^2}} \tag{12}
 \end{aligned}$$

The quantized and nonquantized forms give exactly the same output values, except that the quantized form comes in quantized steps. An interesting case is when we set the radius to the Schwarzschild radius, $r_s = \frac{2GM}{c^2} = 2N\aleph$. Then we get

$$\begin{aligned}
 t_0 &= t_f \sqrt{1 - \frac{1 + \frac{2r}{N\aleph}}{\left(1 + \frac{r}{N\aleph} \right)^2}} \\
 t_0 &= t_f \sqrt{1 - \frac{1 + \frac{2 \times 2N\aleph}{N\aleph}}{\left(1 + \frac{2N\aleph}{N\aleph} \right)^2}} \\
 t_0 &= t_f \sqrt{1 - \frac{5}{9}} \approx t_f 0.666666667 \tag{13}
 \end{aligned}$$

In other words, time does not stand still at the Schwarzschild radius. For the exact time dilation, the Schwarzschild radius is not unique and nothing special happens at this radius. The standard interpretation that time stands still at the Schwarzschild radius is very likely to be an incorrect interpretation rooted in the use of the approximate escape velocity.

In Table 2 one can study the differences in predictions between Einstein's gravitational time dilation rooted in the Schwarzschild metric and the modified exact solution presented here. The gravitational time dilation works all the way down to radius zero, which is far below the Schwarzschild radius.

4 Gravitational Redshift

The Einstein gravitational redshift derived from the Schwarzschild metric is given by

$$\begin{aligned}
 \lim_{r \rightarrow +\infty} z(r) &= \frac{1}{\sqrt{1 - \frac{2GM}{R_e c^2}}} - 1 \\
 \lim_{r \rightarrow +\infty} z(r) &= \frac{1}{\sqrt{1 - \frac{v_e^2}{c^2}}} - 1 \tag{14}
 \end{aligned}$$

Table 2: The table shows the exact time dilation for a Earth-sized mass at different radiuses compared to the standard gravitational time dilation. Assumed mass: $M = 5.972 \times 10^{24}$ kg..

Multiples of the Schwarzschild radius :	Radius meters :	Time dilation from exact escape velocity :	Einstein time dilation factor :
Mountain top radius $718,419,181r_s$	6,372,000	0.999999999304027	0.999999999304027
Earth surface $718,306,435r_s$	6,371,000	0.999999999303918	0.999999999303918
$100r_s$	0.886947	0.995024875621891	0.994987437106620
$10r_s$	0.088695	0.952380952380952	0.948683298050514
$5r_s$	0.044347	0.909090909090909	0.894427190999916
r_s	0.008869	$\sqrt{1 - \frac{5}{9}} \approx \mathbf{0.666667}$	0 (time stands still)
$0.5r_s$	0.004435	0.500000000000000	equation collapse=Black hole
$0.1r_s$	0.000887	0.166666666666667	equation collapse=Black hole
$0.001r_s$	8.8695E-06	0.001996007983993	equation collapse=Black hole
$0.0001r_s$	8.8695E-07	0.000199960007936	equation collapse=Black hole
$0.00001r_s$	8.8695E-08	0.000019999595592	equation collapse=Black hole
$0r_s$	0	0.000000000000000	equation collapse=Black hole

where R_e is the distance between the center of the mass of the gravitating body and the point at which the photon is emitted and v_e is the well known standard escape velocity.² The escape velocity used in the Einstein gravitational redshift formula is an approximate escape velocity that dose not work well when we approaches the Schwarzschild radius. In other words Einsteins gravitational redshift almost for sure gives very wrong redshift predictions for photons emitted from strong gravitational field.

The gravitational redshift based on the exact escape velocity formula that holds under strong gravitational fields must be

$$z(r) = \frac{1}{\sqrt{1 - \frac{v_e^2}{c^2}}} - 1 \quad (15)$$

We can rewrite this as

$$z(r) = \frac{1}{\sqrt{1 - \frac{\left(c \frac{\sqrt{1 + \frac{2c^2 R_e}{GM}}}{1 + \frac{c^2 r}{GM}} \right)^2}{c^2}}} - 1$$

$$z(r) = \frac{1}{\sqrt{1 - \frac{1 + \frac{2c^2 R_e}{GM}}{\left(1 + \frac{c^2 R_e}{GM} \right)^2}}} - 1 \quad (16)$$

Or alternatively in the quantized form

$$z(r) = \frac{1}{\sqrt{1 - \frac{\left(c \frac{\sqrt{1 + \frac{2R_e}{N\hbar}}}{1 + \frac{R_e}{N\hbar}} \right)^2}{c^2}}} - 1$$

$$z(r) = \frac{1}{\sqrt{1 - \frac{1 + \frac{2R_e}{N\hbar}}{\left(1 + \frac{R_e}{N\hbar} \right)^2}}} - 1 \quad (17)$$

In Table 3 we have calculated predicted gravitational redshifts for a mass containing 10 solar masses with the standard Einstein gravitational redshift formula and our modified gravitational redshift formula that also holds down and even below the Schwarzschild radius. It is clear from the table that massive dense objects can have a very high gravitational redshift. The standard model is not able to give predictions for photons emitted from an area below the Schwarzschild radius. As we have discussed, the predictions from the standard theory in this regard break down at the Schwarzschild radius and are thus interpreted as black holes.

²See Haug (2016b) for quantization of this formula, even if that not is important in the context here.

Table 3: The table shows the exact red-shift for a 10 Solar-sized mass at different radiuses compared to the standard gravitational red shift. Assumed mass of object, 10 solar masses : $M = 9.134 \times 10^{38}$ kg.

Multiples of Schwarzschild radius :	Radius photons emitted from (meters) :	Haug exact redshift z(r) :	Einstein/Schwarzschild redshift z(r) :
$216r_s$	Radius Sun : 6,371,000	0.00231716062796	0.00232524570812
$100r_s$	2,952,526.072	0.005000000000000	0.00503781525921
$10r_s$	295,252.607	0.050000000000000	0.05409255338946
$5r_s$	147,626.304	0.100000000000000	0.11803398874989
r_s	29,525.261	0.500000000000000	Equation break down
$0.5r_s$	14,762.63	1.000000000000000	Equation break down
$0.2r_s$	5,905.052	2.500000000000000	Equation break down
$0.1r_s$	2,952.526	5.000000000000002	Equation break down
$0.01r_s$	295.253	50.00000000002820	Equation break down
$0.001r_s$	29.525	499.9999997491300	Equation break down
$0.0001r_s$	2.953	4999.99998073019000	Equation break down
$0.00001r_s$	0.295	50000.03185862340000	Equation break down
$0r_s$	0	Equation break down	Equation break down

Possibly Cosmological Implications

Hawkins (2010) has done an impressive empirical job in observing and studying redshift in quasars. Surprisingly he did not find excess time dilation in the high Z quasars as expected by predictions from standard cosmology. However, instead of claiming the data were right and the current cosmology theories were incomplete he introduced new ideas such as the existence of growing black holes that would offset the lacking excess gravitational time dilation exactly. This rather than claiming the standard models had to be incomplete (fair enough if one wants to get published inside established academic journals that are extremely negative to anyone criticizing the standard framework one could get tempted to come up with explain to save the epicycles.). We suggest that the correct explanation of the high Z quasar studies may be based in the fact that the standard gravitational theory does not have a gravitational redshift theory that works well close to and below the Schwarzschild radius. As a result, many of the high Z redshift interpretations in cosmology are possibly wrong. Very dense and massive objects like quasars will, in our modified theory, be predicted to have much higher gravitational redshifts than those predicted by standard cosmology. Based on our theory, it is also not surprising if one should find high Z objects in front of lower Z objects, as those claimed to be observed by Arp (1998).

Further, in light of this theory many other predictions in cosmology, including the theory of the Big Bang interpretation of the universe could also be misguided.

5 Conclusion

We have derived an exact escape velocity based on Newton and special relativity theory that holds also for very strong gravitational fields. The standard escape velocity used in modern physics is only an approximate escape velocity that not is valid in strong gravitational fields. This paper suggests that the interpretations of the Schwarzschild radius in modern physics are incorrect. There are likely no black holes, the escape velocity at the Schwarzschild radius is not c , and time does not stand still at the Schwarzschild radius. There is “nothing” special about the Schwarzschild radius except perhaps a set of mathematical artifacts that are the result of mathematical approximations, indeed approximations that are not valid in strong gravitational fields. In addition to the central discussion, we have also quantified the escape velocity, the gravitational time dilation and the gravitational redshift. Some people may claim that the solutions given in this paper must be incomplete since they do not use GR to be derived. We look forward to a debate on these topics. The new escape velocity and time dilation introduced this paper can hopefully be a small piece in helping to bring physics and cosmology back on the right track.

Appendix A

Derivation of the standard escape velocity from Planck scale as first shown by Haug (2016b)

$$\begin{aligned}
E &\approx \frac{1}{2}mv^2 - \frac{GmM}{r} \\
E &\approx \frac{1}{2}N_1m_p v^2 - \frac{GN_1m_p N_2m_p}{r} \\
E &\approx \frac{1}{2}N_1 \frac{\hbar}{N} \frac{1}{c} v^2 - \frac{N_1 \frac{\hbar^2 c^3}{N} \frac{1}{N} N_2 \frac{\hbar}{N} \frac{1}{c}}{r} \\
E &\approx \frac{1}{2}N_1 \frac{\hbar}{N} \frac{1}{c} v^2 - N_1 N_2 \frac{\hbar}{r} c
\end{aligned} \tag{18}$$

where N_1 is the number of Planck masses in the smaller mass m (for example a rocket) and N_2 is the number of Planck masses in the other mass. This we have to set to 0 and solve with respect to v to find the escape velocity:

$$\begin{aligned}
\frac{1}{2}N_1 \frac{\hbar}{N} \frac{1}{c} v^2 - N_1 N_2 \frac{\hbar}{r} c &= 0 \\
v^2 &= 2 \frac{N_1 N_2 \frac{\hbar}{r} c}{N_1 \frac{\hbar}{N} \frac{1}{c}} \\
v^2 &= 2N_2 \frac{\hbar c^2}{r} \\
v &= c \sqrt{N_2 \frac{2N}{r}}
\end{aligned} \tag{19}$$

This is a quantized escape velocity. Bear in mind that the kinetic energy of $\frac{1}{2}mv^2$ is only a good approximation for $v \ll c$. Still, for all planets in our solar system and even for the massive Sun itself, the escape velocity from the surface of these “objects” will be so small that $v \ll c$. Only when we approach the escape velocity at the Schwarzschild radius are the approximations in this appendix inaccurate. Since N_1 cancels out, we can simply call N_2 for N and write the escape velocity as

$$v = c \sqrt{N \frac{2N}{r}} \tag{20}$$

where N is the number of Planck masses in the mass we are trying to escape from.

Appendix B: Gravitational Time Dilation at Planck Scale

We can rewrite the standard Einstein gravitational time dilation in the form of quantized escape velocity (derived above).

$$\begin{aligned}
t_o &= t_f \sqrt{1 - \frac{v_e^2}{c^2}} \\
t_o &= t_f \sqrt{1 - \frac{\left(c \sqrt{2N \frac{N}{r}}\right)^2}{c^2}} \\
t_o &= t_f \sqrt{1 - \frac{2NN}{r}}
\end{aligned} \tag{21}$$

Let’s see if we can calculate the time dilation at, for example, the surface of the Earth from Planck scale gravitational time dilation. The Earth’s mass is 5.972×10^{24} kg. And again, the Earth’s mass in terms of the Planck mass must be $\frac{5.972 \times 10^{24}}{2.17651 \times 10^{-8}} \approx 2.74388 \times 10^{32}$. Further, the radius of the Earth is $r \approx 6\,371\,000$ meters. We can now just plug this into the quantized gravitational time dilation

$$\begin{aligned}
t_o &= t_f \sqrt{1 - \frac{2NN}{r}} \\
t_o &= t_f \sqrt{1 - \frac{2 \times 2.74388 \times 10^{32} \times 1.61622837 \times 10^{-35}}{6\,371\,000}} \approx t_f \times 0.999999999303915
\end{aligned}$$

That is for every second that goes by in outer space (a clock far away from the massive object), 0.99999999930391500 seconds goes by on the surface of the Earth. That is, for every year in outer space (very far from the Earth), there are about 22 milliseconds left to reach an Earth year. This is naturally the same as we would get with Einstein’s formula.

References

- ARP, H. (1998): *Seeing Red, Redshifts, Cosmology and Academic Science*. Montral: Aperia.
- AUGUSTI, A. T., AND A. RADOSZ (2006): “An Observation on the Congruence of the Escape Velocity in Classical Mechanics and General Relativity in a Schwarzschild metric,” *European Journal of Physics*, pp. 331–335.
- EDDINGTON, A. S. (1922): *The Mathematical Theory of Relativity*. Cambridge University Press.
- EINSTEIN, A. (1905): “Zur Elektrodynamik Bewegter Körper,” *Annalen der Physik*, (17).
- (1916): “Näherungsweise Integration der Feldgleichungen der Gravitation,” *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften Berlin*.
- HAUG, E. G. (2016a): “The Gravitational Constant and the Planck Units. A Deeper Understanding of the Quantum Realm,” *www.ViXra.org March 13 2016*.
- (2016b): “Planck Quantization of Newton and Einstein Gravitation,” *www.ViXra.org March 19 2016*.
- HAWKINS, M. R. S. (2010): “On Time Dilation in Quasar Light Curves,” *Mon. Not. R. Astron. Soc.*, 405, 1940–1946.
- NEWTON, I. (1686): *Philosophiae Naturalis Principia Mathematica*. London.
- PLANCK, M. (1901): “Ueber das Gesetz der Energieverteilung im Normalspectrum,” *Annalen der Physik*, 4.
- SCHWARZSCHILD, K. (1916a): “Über das Gravitationsfeld einer Kugel aus Inkompressibler Flüssigkeit nach der Einsteinschen Theorie,” *Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse für Mathematik, Physik, und Technik*, p. 424.
- (1916b): “Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie,” *Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse für Mathematik, Physik, und Technik*, p. 189.