

# The Division Of Zero By Zero

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## Abstract

Modern scientist come close to Einstein, the most prominent physicist of the twentieth century and may be of all time. Still, the question is justified, can there be ever another Einstein? Less well known, though of fundamental importance, are Einstein's contributions to the philosophy of science. Einstein's is well known for his conviction that scientist should trust simplicity. Einstein proclaimed that we can discover the true laws of nature or proof theorems by seeking those with the most simple mathematical formulation. Einstein's faith in the supreme power of mathematical simplicity was strong. Such an approach to scientific investigation is of strategic use, since by time the hypotheses from which scientists starts become ever more abstract and more remote from experience. Especially, under conditions where scientific investigations are moving steadily into domains ever further removed from direct contact with an experiment or observation, the starting point should be as simple as possible. This point of view, whose exact formulation while investigating the problem of the division of zero by zero may meet with great difficulties, may justify our trust that the problem of the division of zero by zero is solved.

## Keywords

Number theory; quantum theory; relativity theory; unified field theory; causality

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## 1. Introduction

Scientists and academic writers who attempt to discover the laws of nature often prefer or at least insist on the fact to avoid logical fallacies and rightly too. The knowledge of fallacies is needed to help us against the errors we might undertake and is thus far a criteria of good scientific reasoning. The present inquiry relies on the results in modern fallacy studies. In the following essay, a brief review of the core fallacies, especially as they appear frequently in introductory level textbooks, will not be given.

## 2. Definitions

### 2.1. Definitions.

#### Definition. Thought Experiments

Properly constructed (real or) thought experiments (as devices of scientific investigation) can be used for diverse reasons in a variety of areas. Thought experiments can help us to investigate some basic properties of nature even under conditions when it is too difficult or too expensive to run a real experiment. Furthermore, a thought experiment can provide some evidence against or in favour of a theory. However, a thought experiment is not a substitute for a real experiment.

#### Definition. Proof by contradiction (Reductio ad Absurdum)

The logical background of a proof by contradiction is Aristotle's law of non-contradiction. A rigorous proof by contradiction proof of a theorem follows the standard method of contradiction used in science and mathematics and should be convincing as much as possible. For the first, we assume that a claim / a theorem / a proposition / a statement et cetera which has to be proved, is true. One then proceeds to demonstrate that a conclusion drawn from such a claim / a theorem / a proposition / a statement et cetera leads to a contradiction. Hence, the supposed claim / theorem / proposition / statement et cetera is deemed to be false. Consequently, we are then led to conclude that it was wrong to assume the claim / the theorem / the proposition / the statement was true. Thus far, the claim / the theorem / the proposition / the statement is proved to be false. Reductio ad absurdum is a widely used technique to expose a fallacy. The logical form of reductio ad absurdum is the following:

**Assume P is true.**

From this assumption, deduce that **Q is true.**

Now **deduce** somehow that **Q is false.**

Thus, **P implies both Q and not Q** (a contradiction, which is necessarily false).

Therefore, **P itself** must be **false.**

#### Definition. Modus ponendo ponens

Modus ponendo ponens, a mechanisms for the construction of a deductive proof, is a valid rule of inference. Altogether modus ponendo ponens can be summarized as

P implies Q.

P is (asserted or proved to be) true

Therefore Q is true.

In mathematics and logic, modus ponendo ponens is a kind of a direct proof with the capacity to show the truth or falsehood of a given theorem/statement by a straightforward combination of established facts or axioms, or existing lemmas or other theorems and without making any further assumptions. In order to directly prove a conditional statement of the form '**If P, then Q**' it suffices to consider the situations in which the statement P is true.

#### Definition. Modus tollendo tollens

Modus tollens or modus tollendo tollens is a valid rule of inference. The modus tollens rule can be stated formally as:

If P, then Q.

If not Q, then not P.

Not Q.

Therefore, not P.

More complex rewritings involving modus tollendo tollens are often found in set theory.

### Definition. The fallacy of *circulus in demonstrando*

*Circulus in demonstrando* or circular reasoning is a logical fallacy. Circular reasoning is a type of reasoning in which the components are many times logically valid. The components of circular reasoning lead back and forth to each other, in a circle, each having only the other for support. Consequently, because the premises are true, the conclusion must be true. *Circulus in demonstrando* is a logical fallacy in which the proof begins with the conclusion.

#### Example I.

Our belief in the Bible is justified because the Bible it is the word of a God, which is existing.  
Our belief in a God, which is existing is justified because it is written in the Bible.

#### Example II.

Y is true because X is true.  
X is true because Y is true.

As a consequence the argument fails to persuade. Closely connected with the fallacy of circular reasoning (*circulus in probando*) is the fallacy of begging the question, a fallacy in which a prove is based on a premise which itself requires proof.

### Definition. The fallacy of *petitio principii* or ‘begging the question’

The fallacy of *petitio principii*, or ‘begging the question’ is a type of circular reasoning which is committed when a prove is based on a premise which itself requires proof.

### Definition. The time of a stationary observer $t_R$ and the time of a co-moving observer $t_O$

Time is dependent on the observer's reference frame. Especially, clocks moving at close to the speed of light  $c$  will slow down with respect to a stationary observer  $R$  (observer at rest). Thus far, let  $t_R$  denote the time as measured by a stationary observer, i. e. the relativistic time. Let  $t_O$  denote the time as measured by a moving observer. The relationship between the time  $t_O$  as measured by a clock moving at constant velocity  $v$  in relation to the time  $t_R$  as measured by a clock of a stationary observer is determined by Einstein's relativistic time dilation [1] as

$$t_O = t_R \times \sqrt{1 - \frac{v^2}{c^2}} \quad (1)$$

where  $t_O$  denotes time as measured by a moving observer,  $t_R$  denotes the time as measured by a stationary observer,  $v$  denotes the relative velocity and  $c$  denotes the speed of light in vacuum. Equally, it is

$$\frac{t_O}{t_R} = \sqrt{1 - \frac{v^2}{c^2}} \quad (2)$$

or

$$\frac{t_O}{c^2} \times \frac{c^2}{t_R} = \sqrt{1 - \frac{v^2}{c^2}} \quad (3)$$

#### Scholium.

Coordinate systems can be chosen freely, deepening upon circumstances. In many coordinate systems, an event can be specified by one time coordinate and three spatial coordinates. The time as specified by the time coordinate is denoted as coordinate time. Coordinate time is distinguished from proper time. The concept of proper time, introduced by Hermann Minkowski in 1908 and denoted as  $t_O$ , incorporates Einstein's time dilation effect. In principle, Einstein is defining time exclusively for every place where a watch measuring this time is located.

“... Definition ... der ... Zeit ... für den Ort, an welchem sich die Uhr ... befindet ...” [2]

In general, a watch is treated as being at rest relative to the place where the same watch is located.

“Es werde ferner mittels der **im ruhenden System** befindlichen **ruhenden** Uhren die Zeit  $t_{[Rt, author]}$  des ruhenden Systems ... bestimmt, ebenso werde die Zeit  $\tau_{[Ot, author]}$  des **bewegten Systems**, in welchen sich relativ zu letzterem **ruhende** Uhren befinden, bestimmt...” [3]

Only, the place where a watch at rest is located can move together with the watch itself. Therefore, due to Einstein, it is necessary to distinguish between clocks as such which are qualified to mark the time  $t_{Rt}$  when at rest relatively to the stationary system R, and the time  $t_{Ot}$  when at rest relatively to the moving system O.

“Wir denken uns ferner eine der Uhren, welche **relativ zum ruhenden System ruhend** die Zeit  $t_{[Rt, author]}$ , **relativ zum bewegten System ruhend** die Zeit  $\tau_{[Ot, author]}$  anzugeben befähigt sind ...” [4]

In English:

<Further, we imagine one of the clocks which are qualified to mark **the time  $t_{[Rt, author]}$  when at rest relatively to the stationary system**, and **the time  $\tau_{[Ot, author]}$  when at rest relatively to the moving system** .... >

In other words, we have to take into account that both observers have at least one point in common, the stationary observer R and the moving observer O are at rest, but **at rest relative to what?** The stationary observer R is at rest relative to a stationary co-ordinate system R, the moving observer O is at rest relative to a moving co-ordinate system O. Both co-ordinate systems can but must not be at rest relative to each other. The time  $t_{Rt}$  of the stationary system R is determined by clocks which are at rest relatively to that stationary system R. Similarly, the time  $t_{Ot}$  of the moving system O is determined by clocks which are at rest relatively to that the moving system O. What is the time marked by the clock when viewed from the stationary system? What is the time marked by the clock when viewed from the moving system? In last consequence, due to Einstein's theory of special relativity, a moving clock ( $t_{Ot}$ ) will measure a smaller elapsed time between two events than that measured by a non-moving (inertial) clock ( $t_{Rt}$ ) between the same two events.

### Definition. The normalized relativistic time dilation

As defined above, due to Einstein's special relativity, it is

$$\frac{t_{Ot}}{t_{Rt}} = \sqrt{1 - \frac{v^2}{c^2}} \quad (4)$$

The normalized relativistic time dilation relation [5] follows as

$$\frac{t_{Ot}^2}{t_{Rt}^2} + \frac{v^2}{c^2} = 1 \quad (5)$$

## 2.2. Axioms

The following theory is based on the next axiom.

### Axiom I. (Lex identitatis)

$$+1 = +1 \quad (\text{Axiom I})$$

### 3. Results

Experimental mathematics is one of the many approaches to mathematics. As in experimental science, experimental mathematics can be used to investigate mathematical objects, to identify properties and patterns and to provide us with fundamental insights through the use of (properly constructed “thought”) experiments.

#### 3.1. Theorem. The normalization of the relationship between rest energy and relativistic energy

Let  ${}_R t$  denote the time as measured by a stationary observer and let  ${}_O t$  denote the time as measured by a moving observer. The relationship between the time as measured by a stationary observer and the time as measured by a moving observer can be normalized and generalized as

$$\frac{{}_R t - {}_O t}{{}_R t - {}_O t} = +1 \quad (6)$$

##### Direct proof.

Due to our Axiom I it is

$$+1 = +1 \quad (7)$$

Multiplying this equation with  ${}_R t$  the time as measured by a stationary observer, we obtain

$$1 \times {}_R t = {}_R t \times 1 \quad (8)$$

We subtract the rest energy  ${}_O t$  from the equation above. It is

$${}_R t - {}_O t = {}_R t - {}_O t \quad (9)$$

Now, we divide the term  $({}_R t - {}_O t)$  by the term  $({}_R t - {}_O t)$  itself and do obtain

$$\frac{{}_R t - {}_O t}{{}_R t - {}_O t} = +1 \quad (10)$$

##### Quod erat demonstrandum.

##### Scholium.

The prove above is not based on a premise or on an axiom which itself requires a proof since  $+1=+1$  is true. Consequently, the fallacy of *petitio principii* or ‘begging the question’ is not committed. The fallacy of *circulus in demonstrando* is not committed since the proof does not begin with the conclusion  $({}_R t - {}_O t)/({}_R t - {}_O t) = 1$ . The proof begins with the axiom that  $+1=+1$ , which is correct. Form this axiom the conclusion drawn is that  $({}_R t - {}_O t)/({}_R t - {}_O t) = 1$ . The above formula can be proofed by physical experiments. The above theorem justifies a transition from an axiom to an experiment and thus far to the testable consequences. Equally significant is the capacity of the above theoretical structure which leads to consequences that can be compared with experience.

### 3.1. Theorem. The division of zero by zero.

Let  ${}_R t$  denote the time as measured by a stationary observer and let  ${}_O t$  denote the time as measured by a moving observer. If Einstein's special relativity is correct and valid, then under conditions where  ${}_R t = {}_O t$  it is

$$\frac{{}_R t - {}_O t}{{}_R t - {}_O t} = \frac{+0}{+0} = +1 \quad (11)$$

#### Proof by modus ponendo ponens.

Due to our Axiom I it is

$$+1 = +1 \quad (12)$$

Due to the theorem before this is equivalent to

$$\frac{{}_R t - {}_O t}{{}_R t - {}_O t} = +1 \quad (13)$$

Einstein's special **relativity covers** even **the case** if  ${}_R t = {}_O t$ . In this case, the relative velocity between the stationary observer and the moving observer is  $v = 0$ . Thus far, it is  ${}_R t - {}_O t = 0$ . We obtain

$$\frac{{}_R t - {}_O t}{{}_R t - {}_O t} = \frac{+0}{+0} = +1 \quad (14)$$

Consequently due to the requirements of a proof by modus ponendo ponens **if P then Q**, we obtain the following premise: **if** Einstein's special relativity is valid, **then**

$$\frac{+0}{+0} = +1 \quad (15)$$

Einstein's special theory of relativity predicts a lot of phenomena that seem weird. But special theory of relativity has passed a huge number of experimental tests. The experimental observations are still consistent with the predictions of special relativity. The results of all known experiments are that Einstein's special theory of relativity is correct and valid. Consequently, **P** (or Einstein's special relativity) **is** (asserted or proved to be) **true**. Therefore **Q** (i.e.  $0/0=1$ ) **is true** or

$$\frac{+0}{+0} = +1 \quad (16)$$

#### Quod erat demonstrandum.

##### Scholium.

A number of experiments can be performed to prove the above relationship. It is known that clocks on orbiting satellites move slower by a certain amount. Further, atomic clocks on planes move slower too compared to identical stationary clocks on earth. An atomic clock on the moon will run slower compared to an identical atomic clock on the earth.

## 4. Discussion

Opponents of this approach to the problem of the division of 0 by 0 may attempt to discredit this contribution in both personal and professional ways. A common technique used by opposing authors is to create the impression that a proof is based on a logical fallacy or the result of a proof is grounded on a logical fallacy. With this in mind, there is a lot that can be done to minimize the harmful effects of possible opponents.

The theorem 3.1 is not based on the division of zero by zero. In theorem 3.1 a term ( $\mathbf{r}t - \mathbf{o}t$ ) is divided by itself, i. e. by ( $\mathbf{r}t - \mathbf{o}t$ ). Due to the rules of mathematics and based on the achievements of special theory of relativity the result is

$$\frac{\mathbf{r}t - \mathbf{o}t}{\mathbf{r}t - \mathbf{o}t} = +1 \quad (17)$$

nothing more but nothing less too. In accordance with special theory of relativity, the term ( $\mathbf{r}t - \mathbf{o}t$ ) can take many different values. Since it is possible and allowed in real life that the time  $\mathbf{r}t$  as measured by a stationary observer is identical with the time  $\mathbf{o}t$  as measured by a co-moving observer, it is natural, possible and allowed that  $\mathbf{r}t = \mathbf{o}t$  and thus far that  $(\mathbf{r}t - \mathbf{o}t) = 0$ . This is a real-life situation. In this case and of course **under conditions where Einstein special theory of relativity is still valid**, we obtain  $(\mathbf{r}t - \mathbf{o}t) / (\mathbf{r}t - \mathbf{o}t) = 0/0 = 1$ . Real life experiments or practice as such and not a highly abstract theoretical framework difficult to understand and much more difficult to proof tell us exactly what happens, if we are faced with situations, where zero is divided by zero.

## 5. Conclusions

The general problem of the division of zero by zero is solved. In general, under conditions of special relativity, it is  $(0/0) = 1$ .

## Acknowledgements

None.

## Appendix

None.

## References

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