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Nature does not play dice

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Abstract

Nature of the deBroglie-Bohm quantum potential is revealed. It is shown to be the energy of oscillating electromagnetic field coupled with moving charged particle. As an example, the zero - energy of harmonic oscillator is obtained from classical equations.

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"I think I can safely say that nobody understands quantum mechanics."
(Richard Feynman)

1 Introduction

Interpretation of quantum theory (both classical and relativistic) since its birth to the present day, for more than 100 years, is the subject of much debate. At first glance, the situation is reassuring, because now we have dozen of different formulations of quantum theory [1] and can apply these powerful techniques for computing.

However, there are a number of unresolved fundamental issues shows us something we do not fully understand the meaning of these formulations and quantum theory do not have her full interpretation.

It is well known that, among others, the Bohmian formulation, based on Louis de Broglie's pilot-wave theory, suggest particularly a great conceptual advantages in possible interpretation because it is causal and not local. Finally it leads to the quantum potential which is of great importance (see [2,3,4] and references therein).

The quantum potential plays a central role in the formalism of Bohm and is widely used in modern physics and chemistry. On the one hand the Bohmian formulation and quantum potential allow us deeply understand the basics of the theory.

On the other hand quantum potential has a great practical importance in different fields of knowledge (for example in the solid state physics, in theoretical chemistry etc.) because it is used to simulate different quantum effects without the involvement of the wave functions and without solving the Schrödinger equation. In this case, the Monte Carlo method is applied to the hydrodynamic calculations, which are sometimes the only opportunity to get the result, when the Shrödinger equation can not be solved exactly.

Recently the paper [5] was published, in which a new foundation has been proposed to unify the quantum theory and relativity. In this paper it was shown that quantization naturally appears as consequence of geometrical properties of our Universe and Planck's constant is just adiabatic invariant determined by the geometry of the Universe (by the Hubble constant and the cosmological constant). It was constructed self-consistent non-local quantum theory based on Einstein's generalized theory of gravity in the space of Riemann - Cartan. It should be stressed this theory does not required any initial assumptions, external to the theory. So we need not any axioms, wave functions or hidden variables.

Another paper in which a natural origin of quantum theory was suggested was published by Gonzales [6]. In this work the quantization of the action S was obtained as a consequence of thermodynamic theory (i.e. also without artificial axiomatic constructions).

These results suggest that the orthodox formulation of quantum theory based on the axiom of the wave function is not complete and should be reconsidered.

In present paper the quantum potential is shown to be formed by bounded electromagnetic field (virtual photon), which is a principle part and main participant for any bounded quantum system.

2 Enigma of quantum potential.

Usually quantum potential in the Bohmian formulation of quantum theory is defined this way (we consider here one - particle case because that for many particles is treated the same way). Shrödinger equation is

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = -\frac{\hbar^{\mathbf{2}}}{2m}\nabla^2\Psi(x,t) + U(x)\Psi(x,t) \tag{1}$$

Writing the wave function in form $\Psi(x,t) = R(x,t)\exp(iS(x,t)/\hbar)$ we immediately obtain two equations:

$$\frac{\partial S}{\partial t} = -H_{tot} = -\frac{(\nabla S)^2}{2m} - U(x) + \frac{\hbar^{\mathbf{2}}}{2m}\frac{\nabla^2 R}{R} \tag{2}$$

and

$$\frac{\partial R^2}{\partial t} + \nabla(R^2\frac{\nabla S}{m}) = 0 \tag{3}$$

The first one is a Hamilton - Jacobi equation written for a modified Hamiltonian:

$$H_{tot}(t,x,p,R) = \frac{(\nabla S)^2}{2m} + U(x) - Q(x,t) \tag{4}$$

where $Q(x,t) = \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}$ is so-called quantum potential, and second one is a continuity equation written for R^2. This modified Hamiltonian usually named as Ψ - dependent one, and additional term (the quantum potential) usually interpreted as "an internal energy associated with a certain region of phase space, absent in classical mechanics, but arising in quantum mechanics from the uncertainty principle" [2]. It is difficult to agree with this point of view for many reasons. First of all note that the discussed system, described by equation (1) is supposed to be isolated one, but it does not contains any variable electromagnetic field. Instead, it contains an external quantum potential.

It is clear, our system contain an oscillating electromagnetic field produced by electron, but we do not see it in equation (1). At the same time the Bohmian formulation has the following features:

- there is presence of hidden variables (it should be treated as a hint for presence of virtual electromagnetic field)
- it is causal (so, it should be a classical field theory)
- not local (presence of an electromagnetic field in theory)

This "lost" field was find in [5] and now we are able to reveal all hidden parameters of quantum theory and identify quantum potential with that non-local classical electromagnetic field mentioned above.

3 Nature does not have hidden variables.

Let us begin with non-relativistic case taking into account the fact the Bohmian formulation is non-relativistic by origin. Consider the hydrogen atom as an example and to simplicity sake we believe $m_e << m_p$ and can consider one-body equation (it is easy to show that introduction of second body will not change main properties of our result).

In consequence with [5] we can write the energy equation $H = E$ for our reduced classical system (fixed proton and electron). In this equation there are no harmonic electromagnetic field due to electron oscillations. However the oscillating electron produce harmonic electromagnetic field $\varphi(k,x)$ which can be used to write opposite Fourier - transformation of the energy equation on coordinate x :

$$\int \hat{H}\varphi d^4x = i\hbar \int \frac{\partial}{\partial t}\varphi d^4x \tag{5}$$

Here integration is carried out over 4-volume , and $\hat{H}$ is operator of Liouville written for "reduced" non-relativistic system $\hat{H} = -\hbar^2\nabla^2/2m + U(x)$, which correspond to the problem of Sturm - Liouville with eigenfunctions $\Psi_n(x,t) = \exp(iS_n/\hbar)$. These functions, in turn, form complete basis and we can expand our virtual photon, and this way include it into consideration:

$$\varphi(p_\alpha, x^\alpha) = \sum R_n(p_\alpha)\Psi_n(x^\alpha) \tag{6}$$

Here we sum only over n , and index α just to mention the fact we are working with 4-vectors p_α and x^α in the Minkowsky space. By substitute this in (5) we have

$$\int \hat{H} \sum R_n(p_\alpha)\Psi_n(x^\alpha)d^4x = i\hbar \int \frac{\partial}{\partial t} \sum R_n(p_\alpha)\Psi_n(x^\alpha)d^4x \tag{7}$$

This actually are complete "quantum" non-local equations described our system in Minkowsky space, with clearly written non-local "hidden variables" of the virtual photon (coefficients $R_n(p_\alpha)$). In general case we should integrate this over 4-volume. However if we are interested in non-relativistic Shrödinger equation, we can evaluate these integrals by taking into account relation $v_e << c$, so, the main part of each integral is contributed by small region in vicinity of the electron and integration can be carried out easy. If we are interested in the case when our system stay in a defined state n, we can write:

$$\left[-\frac{\hbar^2}{2m}\nabla^2 R - \frac{i\hbar}{m}\nabla R \nabla S + \frac{R(\nabla S)^2}{2m} - \frac{i\hbar}{2m}R\nabla^2 S + RU(x)\right] = i\hbar\frac{\partial R}{\partial t} - R\frac{\partial S}{\partial t} \tag{8}$$

From this relation we immediately obtain the Hamilton - Jacobi equation (2) written for a complete Hamiltonian of our system (4), where "quantum potential" has clear sense and must be attributed to the virtual photon (electromagnetic "pilot-wave"). As to the continuity equation (3) written for R^2, it must be interpreted as continuity equation for energy of the virtual photon moving with velocity of the electron $\vec{v}_e$. It is actually an analog for well known Poynting's theorem. It should be stressed here in this continuity equation do not appears the Planck constant because it is classical equation for classical electromagnetic field coupled with electron:

$$\frac{\partial R^2}{\partial t} + \nabla(J) = 0 \tag{9}$$

where

$$R^2 = \frac{1}{8\pi}(E^2 + H^2) \tag{10}$$

$$\vec{J} = R^2\frac{\nabla S}{m} = R^2\vec{v}_e \tag{11}$$

To conclude this part it should be useful to make some comments on the Hamilton - Jacobi equation (2)

$$\frac{\partial S}{\partial t} = -H_{tot} = -\frac{(\nabla S)^2}{2m} - U(x) + \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} \tag{12}$$

Now, when the physical sense of "quantum potential" (as classical potential of coupled with electron oscillating electomagnetic field with energy $\hbar\omega$) became clear, we may definitely interpret limit $\hbar\omega \to 0$ as an hipotethical situation with absence of virtual photon (the energy of the oscillating electromagnetic field is zero). In this case we obtain a classical system with classical Hamilton function $H = \frac{(\nabla S)^2}{2m} + U(x)$ for our reduced, or incomplete system.

4 Discussion

Very fundamental and at the same time useful example suggests harmonic oscillator. We have discussed it before in [5], but it would be interesting to consider it briefly in respect to the quantum potential of this system. The Hamilton function for harmonic oscillator has a form:

$$H = -\frac{\hbar^{\mathbf{2}}}{2m}\nabla^2 + \frac{1}{2}m\omega^2 r^2 \tag{13}$$

Substituting wave function $\Psi = \exp(-m\omega r^2/2\hbar)$ into Shrödinger equation immediately give us quantum potential for harmonic oscillator:

$$Q = \hbar\omega/2 \tag{14}$$

which should be recognized as energy of virtual photon in zero-state of harmonic oscillator (remember here the frequency of electron oscillations is the same that has the virtual photon). So total Hamilton function for "quantum" (in reality classical) harmonic oscillator in ground state is

$$H_{tot}(t, x, p, R) = \frac{(\nabla S)^2}{2m} + U(x) - \hbar\omega/2 \tag{15}$$

with the oscillation frequency ω. One can see again - the total Hamilton function corresponds to the complete mechanical system (classical by nature), without any hidden variables, and so called "quantum potential" is merely virtual photon with frequency ω, which appears in classical Hamilton function like a "quantum" zero-state energy $\hbar\omega/2$. So, now we can definitely identify "quantum potential" and "hidden variables" with oscillating electromagnetic field (virtual photon) coupled with the moving electron.

In light of these results it becomes immediately obvious meaning of Bell's theorem, as a classical statement about the impossibility of motions with a speed faster than light in the framework of the relativistic theory.

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