# Remote sensing through stratified random media using pupil-plane interferometry

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## ABSTRACT

We investigate a novel method for the retrieval of an arbitrary amplitude-object which is illuminated from the far-field and sampled through a stratified random medium of unknown statistics. The setup includes two observation paths, a CCD-based imaging system and a multiaperture interferometer placed in a plane conjugate to the entrance pupil of the imaging system. The interferometric baselines are arranged in closed loops to make the closure phase insensitive to random refractive fluctuations. The method may be beneficial to applications such as surveillance, speckle interferometry and biomedical imaging.

**Keywords**: remote sensing, imaging through turbulence, pupil-plane interferometry, closure phase, digital filtering.

## 1. INTRODUCTION

A large variety of techniques related to the retrieval of embedded objects have been developed in recent years. They are applied in areas such as the detection of hidden targets using radar technology[20-25], imaging through atmospheric turbulence[7-10] and tissue imaging for biomedical research[26-29]. The overall performance of the retrieval process depends heavily on the ability to filter out both the random and the deterministic noise components present in the image (such as time dependent refractive fluctuations, residual scattering on inhomogeneities and absorption throughout the medium). In most cases, random screens[4] are introduced in computer simulations to model phase and amplitude distorsions. The statistics of the random noise is, however, seldom known in advance. This makes the object retrieval dependent on initial assumptions that may be partially invalid. We discuss in this work a dual detection protocol which does not require prior knowledge of noise statistics. The dual setup incorporates a CCD based imaging path and a multiaperture interferometric path, the latter using the phase-closure technique[7,9-11]to record the cross-spectral density of the radiated field. The retrieval strategy amounts to a four-step procedure, i.e: 1) locate a potential target, 2) measure the image spectrum from the CCD image, 3) evaluate the cross-spectral density using the interference fringe pattern 4) perform object matching.
The paper is organized as follows: we first model the stratified random medium as an equivalent phase filter and relate its transfer function to the unknown object and noise spectra. We then use the relationship between the cross-spectral density of the transmitted field at large distances from the object and the transfer function of the filter[3]. Using the phase-closure method, we establish a direct correlation between the fringe pattern and the object irradiance which is independent from the noise spectrum. Results are summarized in the last section.

## 2. THEORETICAL FRAMEWORK

With reference to fig.1, we consider a planar amplitude-object embedded inside a stratified medium with a randomly fluctuating refractive index distribution n(z,t) where "z" represents the stratification direction and "t" is the time. To simplify the following derivation, we assume that the average absorption properties of the stratified medium are stationary and homogeneous. Let the medium be illuminated from the far-field and viewed through the imaging path depicted in fig. 2.
The medium may then be simulated by an equivalent noisy spatial frequency filter[1,2], i.e.:

$$h(x,y)_m = h(x,y) + n(x,y)_m \tag{1}$$

where $h(x,y)$ is the undistorted point spread function , $n(x,y)_m$ is the additive noise created by random phase fluctuations and $h(x,y)_m$ the instantaneous point spread function. The CCD image is given by:

$$i(x,y) = h(x,y) * o(x,y) + n(x,y)_d \tag{2}$$

in which $o(x,y)$ represents the unknown object intensity function, $*$ is the convolution operator and $n(x,y)_d$ the generic detection noise introduced by optical aberrations, photon-noise and digitization. Taking the Fourier transform of (2) yields:

$$I(f_x , f_y) = [H(f_x , f_y).O(f_x , f_y) + N(f_x , f_y)_m.O(f_x , f_y)] + N(f_x , f_y)_d \tag{3}$$

where $f_x$ , $f_y$ are spatial frequency variables. In what follows we assume that the object is described as a secondary source of nearly monochromatic and unpolarized light that can be considered stationary at least in the wide sense[3,4]. The cross-spectral density function of the transmitted field in the half-space $z >> z_0 + h$ , where h represents the thickness of the stratified medium, is given by[3]:

$$w(r_1\mathbf{s}_1,r_2\mathbf{s}_2,\omega) = (2.\pi/k.r)^2.s_{1z}.s_{2z}.T^*(\mathbf{s}_{1p},\omega).T(\mathbf{s}_{2p},\omega).A(\mathbf{s}_{1p},\mathbf{s}_{2p},\omega) \tag{4}$$

Here k stands for the average vacuum wave number, $\mathbf{s} = (s_x,s_y,s_z)$ is the unit vector along a representative raypath, $\mathbf{s}_p = (s_z,s_y,0)$, $\mathbf{s}_1$ and $\mathbf{s}_2$ are direction vectors for two arbitrary field points, $\mathbf{r} = (x,y,z)$, r locates the midpoint :

$$r = |\,(\mathbf{r}_1 + \mathbf{r}_2)/2\,| \tag{5}$$

$\omega$ is the light average frequency, $T^*(\,)$ and $T(\,)$ are complex transmission functions through the medium and $A(\,)$ the angular correlation function of the field incident on the medium ($z = z_0$):

$$A(\mathbf{s}_{1p},\mathbf{s}_{2p},\omega) = \langle a^*(\mathbf{s}_{1p},\omega)\; a(\mathbf{s}_{2p},\omega)\rangle_\omega \tag{6}$$

in which $a^*(\,)$ and $a(\,)$ stand for the angular spectral amplitudes. The angular correlation function is directly related to the coherence properties of the object via:

$$A(\mathbf{s}_{1p},\mathbf{s}_{2p},\omega) = k^4.W^{(0)}(\mathbf{f}_1,\mathbf{f}_2,\omega) \tag{7}$$

where $W^{(0)}(\,)$ is the Fourier transform (FT) of the object cross-spectral density function at $z = 0$, i.e.:

$$W^{(0)}(\mathbf{f}_1,\mathbf{f}_2,\omega) = FT\,[W^{(0)}(\mathbf{r}_{10},\mathbf{r}_{20},\omega)] \tag{8}$$

In the above $\mathbf{r}_{10}$ and $\mathbf{r}_{20}$ locate two arbitrary object points at $z = 0$ and the spatial frequencies $\mathbf{f}_1$, $\mathbf{f}_2$ are given by:

$$\begin{aligned} \mathbf{f}_1 &= -k.\mathbf{s}_{1p} \\ \mathbf{f}_2 &= k.\mathbf{s}_{2p} \end{aligned} \tag{9}$$

To further simplify the derivation, we confine ourselves to the case of an incoherent and isotropically emitting object. It follows from (7) and (8) that , for a given $\omega$ and up to a multiplicative constant, the angular correlation function of such a source is identical with its Fourier spectrum, i.e.:

$$\begin{aligned} A^{incoh}(s_p) &= (const).O(f) \\ \mathbf{s}_{2p} &= -\mathbf{s}_{1p} \\ |\,\mathbf{f}_1\,| &= |\,\mathbf{f}_2\,| = f \end{aligned} \tag{10}$$

Replacing (10) in (4) we obtain:

$$w(r_1\mathbf{s}_1,r_2\mathbf{s}_2,\omega) = (const).s_{1z}.s_{2z}.T^*(\mathbf{s}_{1p},\omega).T(\mathbf{s}_{2p},\omega).O(f) \tag{11}$$

We now take the statistical average of (11) and note that, since the instantaneous transmission and the angular correlation functions are independent random variables, the average of the product equals the product of the averages. Hence we have:

$$< w(r_1 \mathbf{s}_1, r_2 \mathbf{s}_2, \omega) > = (\text{const}).s_{1z}.s_{2z}.< T^*(\mathbf{s}_{1p},\omega).T(\mathbf{s}_{2p},\omega) >.< O(f) > \tag{12}$$

According to the Wiener-Khinchin theorem, Fourier transforming the average spatial autocorrelation of the transmission function recovers the power spectral density of the phase filter[4,5]. It follows that, for a given $\omega$ , we obtain:

$$FT(< T^*(\mathbf{s}_{1p}).T(\mathbf{s}_{2p}) >) = FT(< \exp j.[\varphi(\mathbf{s}_{2p}) - \varphi(\mathbf{s}_{1p})] >) = (\text{const}).< | N(f)_m |^2 > \tag{13}$$

in which $\varphi$ represents the random phase shift introduced by the filter statistics. In (13) the noise spectrum is defined by:

$$N(f)_m = FT[n(\mathbf{s}_p)_m] = FT\{\exp[j.\varphi(\mathbf{s}_p)]\} \tag{14}$$

The noise power spectrum $| N(f)_m |^2$ may be computed from (3) as:

$$| N(f)_m |^2 = N(f)_m.N^*(f)_m = [I(f).O^{-1}(f) - N_d(f).O^{-1}(f) - H(f)].[I^*(f).O^{*-1}(f) - N_d^*(f).O^{*-1}(f) - H^*(f)] \tag{15}$$

where:

$$O^*(f) = O(-f) \tag{16}$$

for real-valued object functions[6]. Using (12) we find:

$$< w(r_1 \mathbf{s}_1, r_2 \mathbf{s}_2) > = (\text{const}).s_{1z}.s_{2z}.IFT(< | N(f)_m |^2 >).< O(f) > \tag{17}$$

in which the average noise power spectrum is determined by (15) and IFT stands for the inverse Fourier transform. In the above, the average cross-spectral density of the outgoing field $< w(r_1 \mathbf{s}_1, r_2 \mathbf{s}_2, \omega) >$ is taken in the entrance pupil of the imaging path. Since the interferometric path records the cross-spectral density in the exit pupil, one needs to link the cross-spectral densities in the entrance and the exit pupils. The relationship between cross-spectral densities in the two planes may be expressed in symbolic form by the convolution[4]:

$$w(1,2) = w^0(1,2) * [\mathbf{K}(1,2).\mathbf{K}^*(1,2)] \tag{18}$$

where $w^0(1,2)$ represents the cross-spectral density in the exit pupil for the pair of field points (1, 2) and $\mathbf{K}(1,2)$ is the amplitude spread function of the imaging optics. Taking the average of (18) and subsituting in (17) gives:

$$< w^0(1,2) * [\mathbf{K}(1,2).\mathbf{K}^*(1,2)] > = (\text{const}).s_{1z}.s_{2z}.IFT(< | N(f)_m |^2 >).< O(f) > \tag{19}$$

The above relationship forms the basis for object retrieval. The process may be completed via one of the following routes:
1) solving (19) for the average object spectrum and computing its inverse transform:

$$< o(x,y) > = IFT(< O(f) >) \tag{20}$$

2) using (19) as a closure relationship for matching the average object spectrum.
Either one of these approaches requires measurement of the average cross-spectral density in the exit pupil $< w^0(1,2) >$. This is the topic of the next section.

## 3. THE PHASE CLOSURE METHOD

Pupil-plane interferometry is a high-resolution observation procedure for the restoration of images degraded by atmospheric turbulence[8,9-11]. To overcome randomly induced errors on the fringe pattern, the interferometric baselines are arranged in closed loops over the exit pupil plane. The closure phase is computed by adding the phases of detected fringes around these loops and it is shown to be free from additive distorsions produced by turbulence and input optics[9,10]. We consider below a three-aperture interferometer (fig. 3) with vector baselines $\mathbf{b}_{12}$ , $\mathbf{b}_{23}$ and $\mathbf{b}_{31}$. The fringe irradiance distribution near the optical axis of the interferometer has the following form[10]:

$$I(\mathbf{X}) = I_S(\mathbf{X}).[1+ | V_{ij} |.\cos(2.\pi.\mathbf{f}_{ij} \bullet \mathbf{X} + \phi_{ij})] \tag{21}$$

in which $\mathbf{X}$ is a spatial coordinate having a parallel orientation to the baseline $\mathbf{b}_{ij}$ , $I_S(\mathbf{X})$ is the sum of irradiances reaching the detector from the two apertures (i,j) (i,j = 1,2,3),$V_{ij}$ represents the complex fringe visibility:

$$V_{ij} =| V_{ij} |.\exp(j.\phi_{ij}) \tag{22}$$

$\phi_{ij}$ is the fringe phase and $\mathbf{f}_{ij}$ the vector spatial frequency defined by:

$$\mathbf{f}_{ij} = \mathbf{b}_{ij} /(\lambda.R^0) \tag{23}$$

where $R^0$ represents the distance between the image of the object produced by the input/CCD optics and the exit pupil plane and $\lambda$ is the average wavelength. For an isotropically emitting object it is reasonable to assume that each aperture is illuminated by equal irradiances. It can be shown that , in this case, the modulus of the complex fringe visibility is identical to the modulus of the normalized cross-spectral density[4], i.e.:

$$| V_{ij} | = | w^0(i,j) / [w^0(i,i).w^0(j,j)]^{1/2} | \tag{24}$$

The phase-closure technique estimates the overall fringe visibility:

$$V_{123} = V_{12}.V_{23}.V_{31} = | V_{12} |.| V_{23} |.| V_{31} |.\exp(j.\phi_{123}) \tag{25}$$

in which the closure phase is given by:

$$\phi_{123} = \phi_{12} + \phi_{23} + \phi_{31} + \phi_{12}^n + \phi_{23}^n + \phi_{31}^n \tag{26}$$

In (26) $\phi_{ij}^n$ represents the detection noise contribution (assumed to be known) and:

$$\begin{aligned} V_{ij} &= R_{ij} + j.I_{ij} \\ \phi_{ij} &= \mathrm{atan}(I_{ij} / R_{ij}) \end{aligned} \tag{27}$$

where $R_{ij}$ and $I_{ij}$ are the real and imaginary components of the detected fringe signal[10]. Three points need to be made regarding the use of (26) and (27):

a) phase unwrapping routines are required since $\phi_{ij}$ are known only modulo $2.\pi$.

b) the quality of the fringe signal may be first increased by smoothing with low-pass Gaussian filters[12] or by applying suitable routines pertaining to fringe-pattern analysis[13-19].

c) the rotationally symmetric object irradiance implies that:

$$| V_{ij} | = | V_c | = \text{constant} \tag{28}$$

Up to a multiplicative constant, the average cross-spectral density in the exit pupil may be derived from (28), (27), (25) and (24) as:

$$< w^0(1,2) > = < (\text{const}).\ (V_{123})^{1/3} > = < (\text{const}).\ | V_c |.\exp[j.(\phi_{123} /3)] > \tag{29}$$

## 4. SUMMARY OF STEPS

In what follows we summarize the sequence of steps involved in the object retrieval strategy:
1) locate a potential target.
2) compute the CCD image spectrum with:

$$I(f) = FT[i(x,y)] \tag{30}$$

3) compute the modulus of the complex fringe visibility $| V_c |$ from (27):

$$| V_c | = (R_{ij}^2 + I_{ij}^2)^{1/2} \tag{31}$$

4) compute the closure phase $\phi_{123}$ from (26) and (27).
5) compute the average cross-spectral density in the exit pupil $< w^0(1,2) >$ from (29).
6) compute the average cross-spectral density in the entrance pupil from (18).
7a) solve for the average object spectrum from (19) and (15).
7b) retrieve the average object irradiance with:

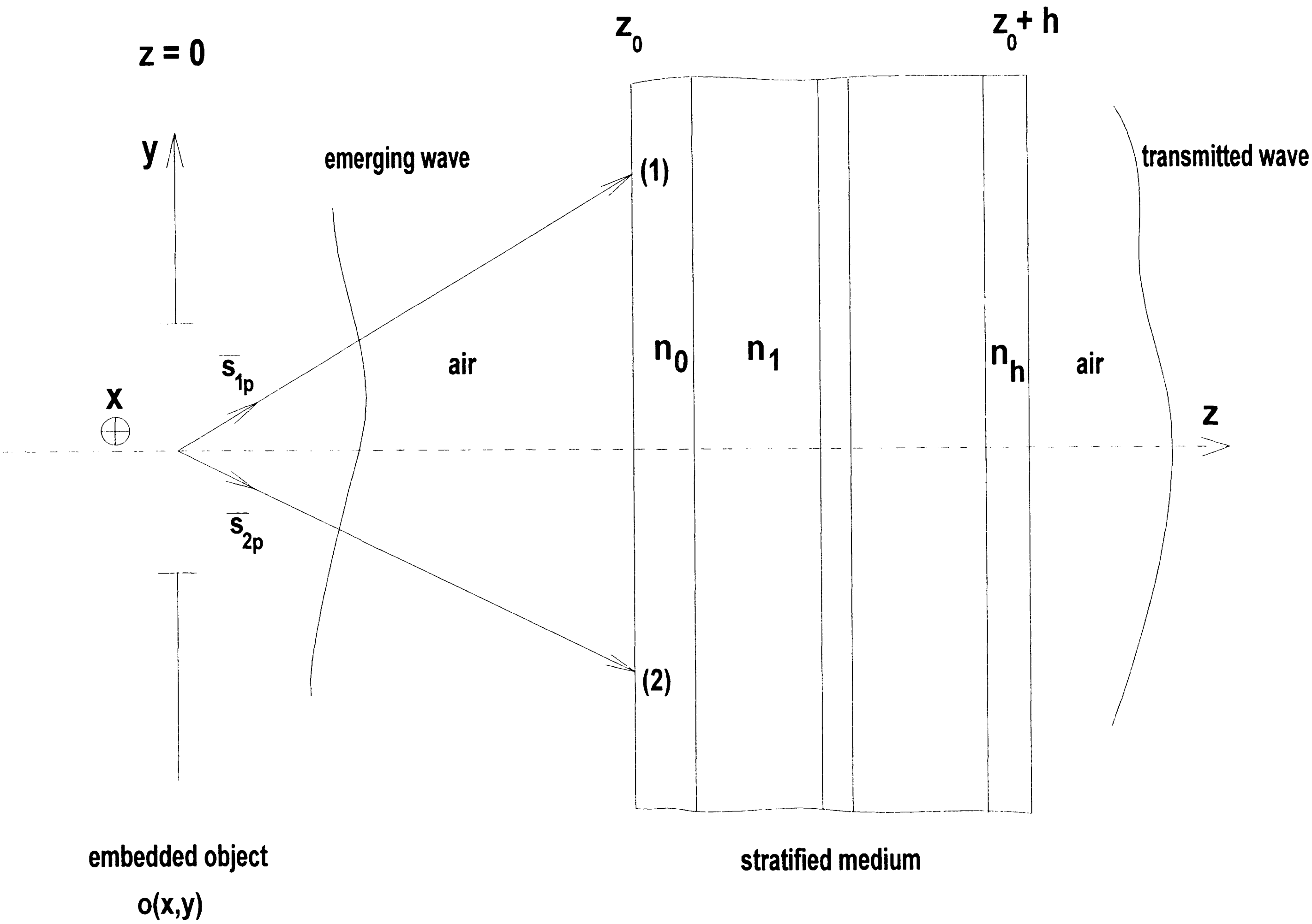
$$< o(x,y) > = IFT(< O(f) >) \tag{32}$$

or:
7c) match the average object spectrum based on (19) and (15).

## 5. REFERENCES

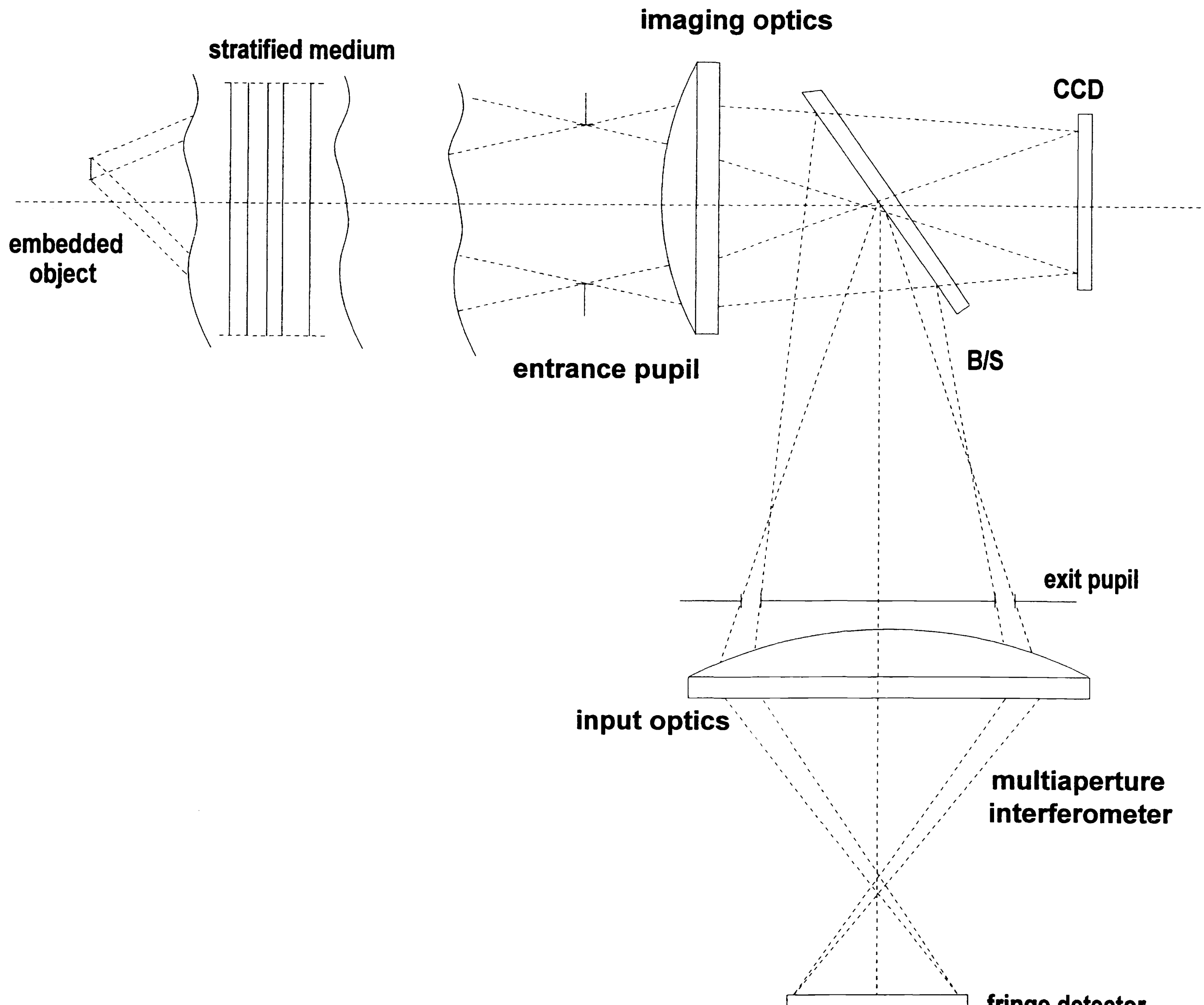

1. D. Sadot, S. R. Rotman, N. S. Kopeika, "Comparison between high-resolution restoration techniques of atmospherically distorted images", Opt. Eng. **34**, 144-153, 1995.
2. D. Sadot, A. Rosenfeld, G. Shuker, N. S. Kopeika, "High-resolution restoration of images distorted by the atmosphere, based on average atmospheric modulation transfer function", Opt. Eng. **34**, 1799-1807, 1995.
3. W. Wang, E. Wolf, "Effects of spatial coherence of incident field on reflection and transmission", Opt. Commun. **79**, 131-138, 1990.
4. J. W. Goodman, *Statistical Optics*, pp. 74-75, John Wiley & Sons, New York, 1985.
5. W. Lauterborn, T. Kurz, M. Wiesenfeldt, *Coherent Optics*, pp. 251-252, Springer-Verlag, Berlin, 1995.
6. B. Porat, *A Course in Digital Signal Processing*, pp. 11-12, John Wiley & Sons, New York, 1997.
7. E. Tolkova, "High-order correlation technique for imaging through turbulence", Opt. Eng. **36**, 2400-2408, 1997.
8. A. Chelli, J. M. Mariotti, "Visibility and phase analysis for image and pupil plane interferometry at optical wavelengths", Astron. Astrophys. **157**, 372-382, 1986.
9. F. Roddier, C. Roddier, "Phase closure with rotational shear interferometers", Opt. Commun. **60**, 350-352, 1986.
10. M. C. Roggemann, "Photon-noise limits to the detection of the closure phase in optical interferometry", Appl. Opt. **35** , 1809-1814, 1996.
11. W. T. Rhodes, J. W. Goodman, "Interferometric technique for recording and restoring images degraded by unknown aberrations", JOSA **63**, 647-657, 1973.
12. M. Rivera, R. Rodriguez-Vera, J. L. Marroquin, "Robust procedure for fringe analysis", Appl. Opt. **36**, 8391-8396, 1997.
13. K. Creath, "Phase-shifting speckle interferometry", Appl. Opt. **24**, 3053-3058, 1985.
14. M. Servin, J. L. Marroquin, D. Malacara, F. J. Cuevas, " Phase unwrapping with a regularized phase-tracking system", Appl. Opt. **37**, 1917-1923.
15. R. Windecker, H. J. Tiziani, " Semispatial, robust , and accurate phase evaluation algorithm", Appl. Opt. **34**, 7321-7326, 1995.
16. D. R. Schmitt, R. W. Hunt, "Optimization of fringe pattern calculation with direct correlations in speckle interferometry", Appl. Opt. **36**, 8848-8857, 1997.
17. J. B. Liu, P. D. Rooney, "Modified Fourier transform method for interferogram fringe pattern analysis", Appl. Opt. **36**, 6231-6241, 1997.

18. J. A. Quinoga, E. Bernabeu, "Phase-unwrapping algorithm for noisy phase-map processing", Appl. Opt. **33**, 6725-6731, 1994.
19. L. Guerriero, G. Nico, G. Pasquariello, S. Stramaglia, "New regularization scheme for phase unwrapping", Appl. Opt. **37**, 3053-3058, 1998.
20. R. M. Measures, *Laser Remote Sensing*, John Wiley & Sons, New York, 1984.
21. A. V. Jelalian, *Laser Radar Systems,* Artech House, Boston, London, 1992.
22. J. W. Goodman, "Comparative performance of optical-radar detection techniques", IEEE Trans. Aerosp. Electron. Syst., **AES-2**(5), 526-535, 1966.
23. M. I. Skolnik, *Introduction to Radar Systems*, 2nd ed., Mc.Graw-Hill, New York, 1980.
24. D. R. Wehner, *High-Resolution Radar*, 2nd ed., Artech House, 1994.
25. D. L. Mensa, *High-Resolution Radar Cross-Section Imaging*, Artech House, 1991.
26. B. Chance (editor), "Proceedings of Time-Resolved Spectroscopy and Imaging of Tissues", SPIE **1431**, 1991.
27. H. Podbielska, A. Katzir (editors), "Proceedings of Holography, Interferometry and Optical Pattern Recognition in Biomedicine", SPIE **1429** , 1991.
28. B. Chance, R. R. Alfano, A. Katzir (editors), "Proceedings of Optical Tomography, Photon Migration, and Spectroscopy of Tissues, and Model Media: Theory, Human Studies, and Instrumentation" SPIE **2389**, vol. 1 and vol. 2, 1995.
29. S. P. Morgan, M. G. Somekh, "Probabilistic method for phased array detection in scattering media", Opt. Eng. **37**, 1618-1626, 1998.



z = 0
$z_0$
$z_0$+ h
y
emerging wave
(1)
transmitted wave
$\bar{s}_{1p}$
air
$n_0$
$n_1$
$n_h$
air
x
z
$\bar{s}_{2p}$
(2)
embedded object
o(x,y)
stratified medium


FIG. 1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stratified medium
CCD
embedded
object
entrance pupil
B/S
exit pupil
input optics
multiaperture
interferometer
fringe detector

FIG. 2

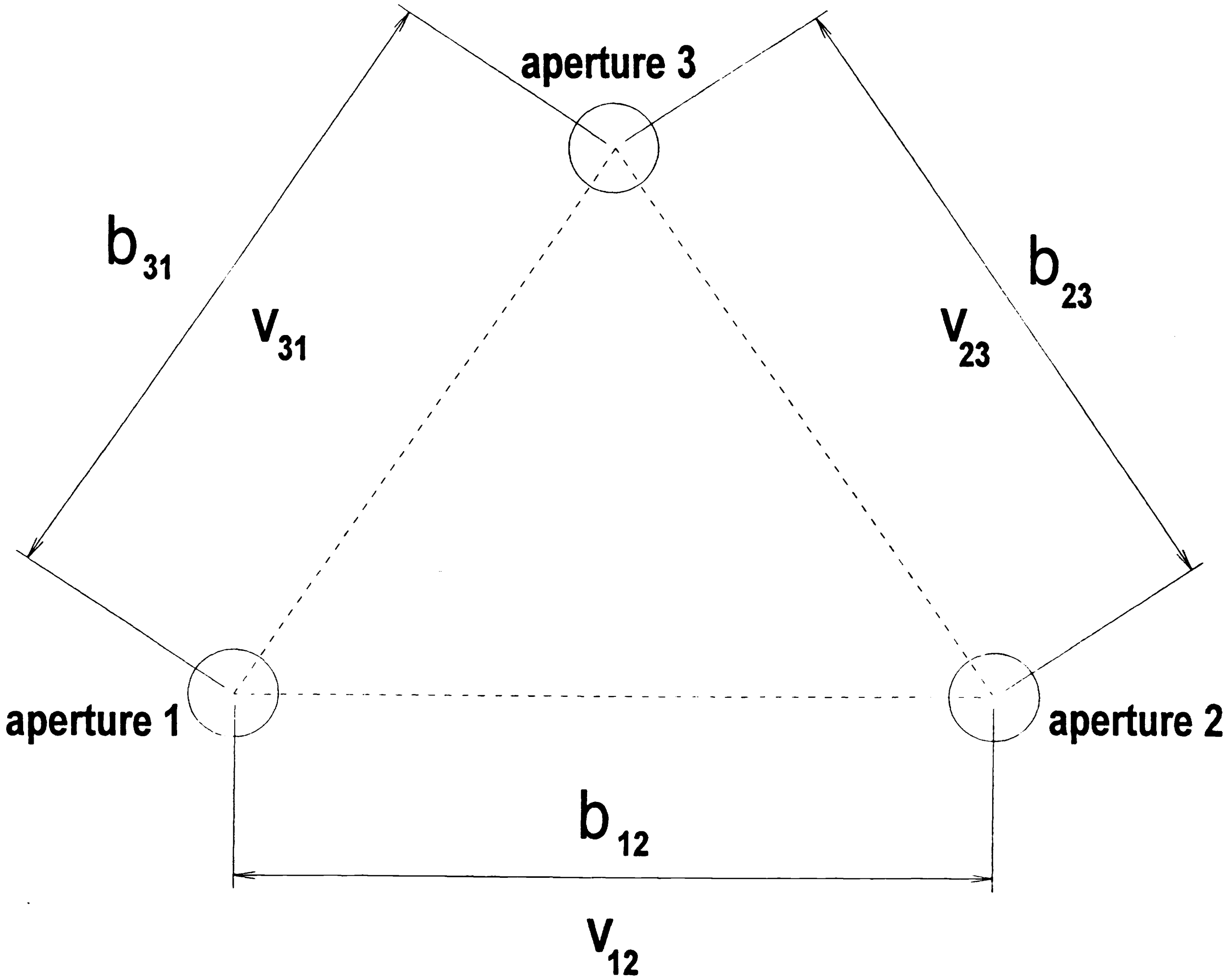
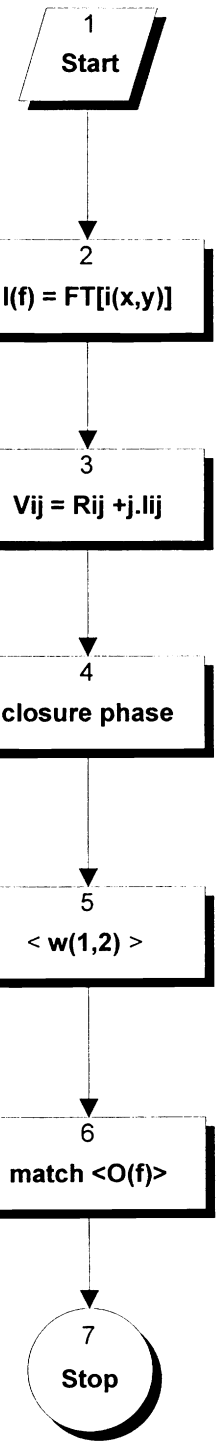



FIG. 3


1
Start
2
I(f) = FT[i(x,y)]
3
Vij = Rij +j.Iij
4
closure phase
5
< w(1,2) >
6
match <O(f)>
7
Stop