FUZZY NEUTROSOPHIC PRODUCT SPACE

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Abstract
In this paper we introduce the concept of fuzzy neutrosophic product spaces and investigate some of their properties.

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1 Introduction

The concept of neutrosophic set was introduced by Smarnandache [28, 29]. The traditional neutrosophic sets is characterized by the truth value, indeterminate value and false value. Neutrosophic set is a mathematically tool for handling problems involving imprecise, indeterminacy inconsistent data and inconsistent information which exits in belief system. The concept of neutrosophic set which overcomes the inherent difficulties that existed in fuzzy sets and intuitionistic fuzzy sets. Following this, the neutrosophic sets are explored to different heights in all fields of science and engineering.A.A.Salama [9] - [26]applied neutrosphic set in various prospects. Many researchers [3, 4, 5, 6, 7, 8, 30] applied the concept of fuzzy sets and intuitionistic fuzzy sets to topologies. In this paper we initiate the concept of fuzzy neutrosophic product and some of its properties are discussed.

2 Preliminary Notes

Definition 2.1. *[1] A Fuzzy Neutrosophic set A on the universe of discourse X is defined as* $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $x \in X$ *where* $T, I, F : X \longrightarrow [0, 1]$

and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.2. *[1] Let X be a non empty set, and* $A = \langle x, T_A(x), I_A(x), F_A(x) \rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x) \rangle$ *are fuzzy neutrosophic sets. Then A is a subset of B if* $\forall\ x \in X$

$T_A(x) \leq T_B(x), I_A(x) \leq I_B(x)), F_A(x) \geq F_B(x))$

Definition 2.3. *[1] Let X be a non empty set, and* $A = \langle x, T_A(x), I_A(x), F_A(x) \rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x) \rangle$ *are fuzzy neutrosophic sets. Then*

$A \cup B = \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x)) \rangle$

$A \cap B = \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x)) \rangle$

Definition 2.4. *[1] A Fuzzy neutrosophic set A over the non-empty set X is said to be empty fuzzy neutrosophic set if* $T_A(x) = 0$, $I_A(x) = 0$,$F_A(x) = 1$,$\forall\ x \in X$. *It is denoted by* 0_N.
A Fuzzy neutrosophic set A over the non-empty set X is said to be universe fuzzy neutrosophic set if $T_A(x) = 1$, $I_A(x) = 1$,$F_A(x) = 0$,$\forall\ x \in X$. *It is denoted by* 1_N.

Definition 2.5. *[1] The complement of Fuzzy neutrosophic set A denoted by* A^c *and is defined as*

$A^c(x) = \langle x, T_{A^c}(x) = F_A(x), I_{A^c}(x) = 1 - I_A(x), F_{A^c}(x) = T_A(x) \rangle$

Definition 2.6. *[2] Let X and Y be a non-empty sets and let f be a mapping from a set X to a set Y. Let* $A = \{\langle x, T_A(x), I_A(x), F_A(x) \rangle / x \in X\}$, $B = \{\langle y, T_B(y), I_B(y), F_B(y) \rangle / y \in Y\}$ *be fuzzy neutrosophic set in X and Y respectively,*

(a) *then the preimage of B under f denoted by* $f^{-1}(B)$ *is the fuzzy neutrosophic set in X defined by*
$f^{-1} = \{< x, f^{-1}(T_B)(x), f^{-1}(I_B)(x), f^{-1}(F_B)(x) > /x \in X\}$ *where* $f^{-1}(T_B)(x) = T_B(f(x))$, $f^{-1}(I_B)(x) = I_B(f(x))$ *and* $f^{-1}(F_B)(x) = F_B(f(x))$ *for all* $x \in X$.

(b) *the image of A under f, denoted by f(A) is the fuzzy neutrosophic set in Y defined by*
$f(A) = (f(T_A, f(I_A, f(F_A))$, *where for each* $y \in Y$.

$$f(T_A)(y) = \begin{cases} \bigvee_{x \in f^{-1}(y)} T_A(x) & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

$$f(I_A)(y) = \begin{cases} \bigvee\limits_{x \in f^{-1}(y)} I_A(x) & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

$$f(F_A)(y) = \begin{cases} \bigwedge\limits_{x \in f^{-1}(y)} F_A(x) & \text{if } f^{-1}(y) \neq \phi \\ 1 & \text{otherwise} \end{cases}$$

Proposition 2.7. *[2] Let A, $A_i(i \in I)$be fuzzy neutrosophic sets in X let B, $B_j(j \in J)$be fuzzy neutrosophic sets in Y and let $f : X \to Y$ a mapping. Then*

1. $A_1 \subset A_2$ *implies* $f(A_1) \subset f(A_2)$.
2. $B_1 \subset B_2$ *implies* $f^{-1}(B_1) \subset f^{-1}(B_2)$.
3. $A \subset f^{-1}(f(A))$. *If f is injective, then* $A = f^{-1}(f(A))$.
4. $f(f^{-1}(B)) \subset B$. *If f is surjective, then* $f(f^{-1}(B)) = B$.
5. $f^{-1}(\bigcup B_j) = \bigcup f^{-1}(B_j)$.
6. $f^{-1}(\bigcap B_j) = \bigcap f^{-1}(B_j)$.
7. $f(\bigcup A_i) = \bigcup f(A_i)$.
8. $f(\bigcap A_i) \subset \bigcap f(A_i)$.*If f is injective, then* $f(\bigcap A_i) = \bigcap f(A_i)$.
9. $f(1_N) = 1_N$, *if f is surjective and* $f(0_N) = 0_N$.
10. $f^{-1}(1_N) = 1_N$ *and* $f^{-1}(0_N) = 0_N$.
11. $[f(A)]^c \subset f(A^c)$ *if f is surjective.*
12. $f^{-1}(B^c) = [f^{-1}(B)]^c$.

3 Fuzzy Neutrosophic topological spaces and product spaces

Definition 3.1. *Let $p, q, r \in [0, 1]$ and $p + q + r \leq 3$. An fuzzy neutrosophic point $x_{(p,q,r)}$ of X is the fuzzy neutrosophic set in X defined by*

$$x_{(p,q,r)}(y) = \begin{cases} (p, q, r), & \text{if } x = y \\ (0, 0, 1), & \text{if } y \neq x \end{cases}, \text{ for each } y \in X.$$

A fuzzy neutrosophic point $x_{(p,q,r)}$ is said to belong to an fuzzy neutrosophic set $A =< T_A, I_A, F_A >$ in X denoted by $x_{(P,q,r)} \in A$ if $p \leq T_A(x)$, $q \leq I_A(x)$ and $r \geq F_A(x)$. We denote the set of all fuzzy neutrosophic points in X as FNP(X)

Theorem 3.2. *Let* $A =< T_A, I_A, F_A >$ *and* $B =< T_B, I_B, F_B >$ *be fuzzy neutrosophic sets in* X *, then* $A \subset B$ *if and only if for each* $x_{(p,q,r)} \in FNP(X)$, $x_{(p,q,r)} \in A$ *implies* $x_{(p,q,r)} \in B$.
Proof:

Let $A \subseteq B$ *and* $x_{(p,q,r)} \in A$, *Then* $p \leq T_A(x) \leq T_B(x)$, $q \leq I_A(x) \leq I_B(x)$ *and* $r \geq F_A(x) \geq F_B(x)$.*Thus* $x_{(p,q,r)} \in B$.
Conversely, Take and $x \in X$. *Let* $p = T_A(x)$, $q = I_A(x)$ *and* $r = F_A(x)$.*Then* $x_{(p,q,r)}$ *is a fuzzy neutrosophic point in* X *and* $x_{(p,q,r)} \in A$. *By the hypothesis,* $x_{(p,q,r)} \in B$. *Thus* $T_A(x) = p \leq T_B(x)$, $I_A(x) = q \leq I_B(x)$ *and* $F_A(x) = r \geq F_B(x)$. *Hence* $A \subseteq B$.

Theorem 3.3. *Let* $A =< T_A, I_A, F_A >$ *be a fuzzy neutrosophic set of X. Then* $A = \bigcup\{x_{(p,q,r)}\ x_{(p,q,r)} \in A\}$.

Definition 3.4. *Let* X *be a set and let* $p, q, r \in [0, 1]$ *with* $0 \leq p + q + r \leq 3$. *Then the fuzzy neutrosophic set* $C_{(p,q,r)} \in X$ *is defined by for each* $x \in x$,$C_{(p,q,r)}(x) = (p, q, r)$ *ie.,* $T_{C_{(p,q,r)}}(x) = p$, $I_{C_{(p,q,r)}}(x) = q$ *and* $F_{C_{(p,q,r)}}(x) = r$.

Definition 3.5. *Let* X *anon-empty set and let* $\mathscr{T} \subset FNS(X)$. *Then* $\mathscr{T}$ *is called a fuzzy neutrosophic topology (FNT) on X in the sense of Lowen, if it satisfies the following axioms:*

(i) *For each* $\alpha, \beta, \gamma \in [0, 1]$ *with* $\alpha + \beta + \gamma \leq 3$, $C_{(\alpha,\beta,\gamma)} \in \mathscr{T}$

(ii) *For any* $A_1, A_2 \in \mathscr{T}$, $A_1 \cap A_2 \in \mathscr{T}$

(iii) *For any* $\{A_k\}_{k \in K} \subset \mathscr{T}$, $\bigcup_{k \in K} \in \mathscr{T}$

Definition 3.6. *Let* A *be a fuzzy neutrosophic set in a fuzzy neutrosophic topological space* $(X, \mathscr{T})$ *then the induced fuzzy neutrosophic topology (IFNT in short) on A is the family of fuzzy neutrosophic sets in A which are the intersection with A of fuzzy neutrosophic open sets in X. The IFNT is denoted by* $\mathscr{T}_A$ *and the pair* $(A, \mathscr{T}_A)$ *is called a fuzzy neutrosophic subspace of* $(X, \mathscr{T})$.

Definition 3.7. *Let* $(X, \mathscr{T})$ *and* $(Y, \mathscr{U})$ *be two fuzzy neutrosophic topological spaces. A mapping* $f : (X, \mathscr{T}) \to (Y, \mathscr{U})$ *is said to be fuzzy neutrosophic continuous if the preimage of each fuzzy neutrosophic set in* $\mathscr{U}$ *is a fuzzy neutrosophic set in* $\mathscr{T}$, *and f is said to be fuzzy neutrosophic open if the image of each fuzzy neutrosophic set in* $\mathscr{T}$ *is a fuzzy neutrosophic set in* $\mathscr{U}$.

Definition 3.8. *Let* $(A, \mathscr{T}_A)$ *and* $(B, \mathscr{U}_B)$ *be fuzzy neutrosophic subspace of fuzzy neutrosophic topological spaces* $(X, \mathscr{T})$ *and* $(Y, \mathscr{U})$ *respectively and let* $f : (X, \mathscr{T}) \to (Y, \mathscr{U})$ *be a mapping. Then f is a mapping of* $(A, \mathscr{T}_A)$ *into* $(B, \mathscr{U}_B)$ *if f(A)* $\subset B$.
Furthermore f is said to be relatively fuzzy neutrosophic continuous if for each

fuzzy neutrosophic set V_B in $\mathscr{U}_B$, the intersection $f^{-1}(V_B) \cap A$ is a fuzzy neutrosophic set in $\mathscr{T}_A$ and f is said to be relatively fuzzy neutrosophic open if for each fuzzy neutrosophic set U_A in $\mathscr{T}_A$, the image $f(U_A)$ is the fuzzy neutrosophic set in $\mathscr{U}_B$

Proposition 3.9. *Let $(A, \mathscr{T}_A)$ and $(B, \mathscr{U}_B)$ be fuzzy neutrosophic subspace of fuzzy neutrosophic topological spaces $(X, \mathscr{T})$ and $(Y, \mathscr{U})$ respectively and let f be a fuzzy neutrosophic continuous mapping of $(X, \mathscr{T})$ into $(Y, \mathscr{U})$ such that $f(A) \subset B$ then f is relatively fuzzy neutrosophic continuous mapping of $(A, \mathscr{T}_A)$ into $(B, \mathscr{U}_B)$.*
Proof:

Let V_B be a fuzzy neutrosophic set in $\mathscr{U}_B$. Then there exist $V \in \mathscr{U}$ such that $V_B = V \cap B$. Since f is fuzzy neutrosophic continuous it follows that $f^{-1}(V)$ is a fuzzy neutrosophic set in $\mathscr{T}$. Hence $f^{-1}(V_B) \cap A = f^{-1}(V \cap B) \cap A = f^{-1}(V) \cap f^{-1}(B) \cap A = f^{-1}(V) \cap A$ is a fuzzy neutrosophic set in $\mathscr{T}_A$. Hence the proof.

Definition 3.10. *A bijective mapping f of a fuzzy neutrosophic topological space $(X, \mathscr{T})$ into a fuzzy neutrosophic topological space $(Y, \mathscr{U})$ is a fuzzy neutrosophic homomorphism iff it is fuzzy neutrosophic continuous and fuzzy neutrosophic open. A bijective mapping f of a fuzzy neutrosophic subspace $(A, \mathscr{T}_A)$ of $(X, \mathscr{T})$ into a fuzzy neutrosophic subspace $(B, \mathscr{U}_B)$ of $(Y, \mathscr{U})$ is relative fuzzy neutrosophic homomorphism iff $f[A] = B$ and f is relatively fuzzy neutrosophic continuous and relatively fuzzy neutrosophic open.*

Proposition 3.11. *Let f be a fuzzy neutrosophic continuous (resp. fuzzy neutrosophic open) mapping of a fuzzy neutrosophic topological space $(X, \mathscr{T})$ into a fuzzy neutrosophic topological space $(Y, \mathscr{U})$ and g a fuzzy neutrosophic continuous (resp. fuzzy neutrosophic open)mapping of $(Y, \mathscr{U})$ into a fuzzy neutrosophic topological space $(Z, \mathscr{W})$. Then the composition $g \circ f$ is a fuzzy neutrosophic continuous (resp. fuzzy neutrosophic open) mapping of $(X, \mathscr{T})$ into $(Z, \mathscr{W})$.*
Proof:

Consider a fuzzy neutrosophic set W in $\mathscr{W}$, then $g^{-1}(W)$ is fuzzy neutrosophic open in $\mathscr{U}$ (since g is fuzzy neutrosophic continuous). Let $g^{-1}(W)$ be a fuzzy neutrosophic open in $\mathscr{U}$, then $f^{-1}(g^{-1}(W)) = (g \circ f)^{-1}(W)$ is a fuzzy neutrosophic open in $\mathscr{T}$ (since f is fuzzy neutrosophic continuous). Hence $g \circ f$ is a fuzzy neutrosophic continuous mapping of $(X, \mathscr{T})$ into $(Z, \mathscr{W})$. Similarly we can prove for fuzzy neutrosophic open mapping.

Proposition 3.12. *Let $(A, \mathscr{T}_A)$, $(B, \mathscr{U}_B)$ and $(C, \mathscr{W}_C)$ be a fuzzy neutrosophic subspaces of fuzzy neutrosophic topologies $(X, \mathscr{T})$, $(Y, \mathscr{U})$ and $(Z, \mathscr{W})$ respectively. Let f be a relatively fuzzy neutrosophic continuous (resp. relatively fuzzy neutrosophic open) mapping of $(A, \mathscr{T}_A)$ into $(B, \mathscr{U}_B)$ and g a relatively*

fuzzy neutrosophic continuous (resp. relatively fuzzy neutrosophic open) mapping of $(B, \mathscr{U}_B)$ *into* $(C, \mathscr{W}_C)$*. Then the composition* $g \circ f$ *is relatively fuzzy neutrosophic continuous (resp. relatively fuzzy neutrosophic open) mapping of* $(A, \mathscr{T}_C)$ *into* $(C, \mathscr{W}_C)$*.*
Proof:

Let $W_C \in \mathscr{W}_C$*. Since* g *is relatively fuzzy neutrosophic continuous,* $g^{-1}(W_C) \cap B \in \mathscr{U}_B$*. Since* f *is relatively fuzzy neutrosophic continuous* $f^{-1}[g^{-1}(W_C) \cap B] \cap A \in \mathscr{T}_A$*. Now* $f^{-1}[g^{-1}(W_C) \cap B] \cap A = f^{-1}[g^{-1}(W_C)] \cap f^{-1}(B) \cap A$ $= (g \circ f)^{-1}(W_C) \cap f^{-1}(B) \cap A = (g \circ f)^{-1}(W_C) \cap A$*(Since* $f(A) \subset B$*). Thus* $(g \circ f)^{-1}(W_C) \cap A \in \mathscr{T}_A$*. Hence* $g \circ f$ *is relatively fuzzy neutrosophic continuous.*

Let $U_A \in \mathscr{T}_A$*.Since* f *is relatively fuzzy neutrosophic open,* $f(U_A) \in \mathscr{U}_B$*. Since* g *is relatively fuzzy neutrosophic open* $g(f(U_A)) \in \mathscr{W}_C$ *and* $g(f(U_A)) = (g \circ f)(U_A)$*.Thus* $(g \circ f)(U_A) \in \mathscr{W}_C$*. Hence* $g \circ f$ *is relatively fuzzy neutrosophic open.*

Definition 3.13. *Let* $\mathscr{T}$ *be a fuzzy neutrosophic topology on a set X. A subfamily* $\mathscr{B}$ *of* $\mathscr{T}$ *is a base for* $\mathscr{T}$ *iff each member of* $\mathscr{T}$ *can be expressed as the union of members of* $\mathscr{B}$*.*

Definition 3.14. *Let* $\mathscr{T}$ *be a fuzzy neutrosophic topology on X and* $\mathscr{T}_A$ *the induced fuzzy neutrosophic topology on a fuzzy neutrosophic subset of A of X. A subfamily* $\acute{\mathscr{B}}$ *of* $\mathscr{T}_A$ *is a base for* $\mathscr{T}_A$ *iff each member of* $\mathscr{T}_A$ *can be expressed as the union of members of* $\acute{\mathscr{B}}$*.*

If $\mathscr{B}$ *is a base for a fuzzy neutrosophic topology* $\mathscr{T}$ *on a set X, then* $\mathscr{B}_A = \{U \cap A : U \in \mathscr{T}\}$ *is a base for the induced fuzzy neutrosophic topology* $\mathscr{T}_A$ *on the fuzzy neutrosophic subset A.*

Proposition 3.15. *Let f be a mapping from a fuzzy neutrosophic topological space* $(X, \mathscr{T})$ *to a fuzzy neutrosophic topological space* $(Y, \mathscr{U})$*. Let* $\mathscr{B}$ *be a base for* $\mathscr{U}$*. Then f is fuzzy neutrosophic continuous iff for each* $B \in \mathscr{B}$ *the inverse image* $f^{-1}(B)$ *is in* $\mathscr{T}$*.*
Proof: The only if part is obvious. Suppose the given condition is satisfied. Let $V \in \mathscr{U}$ *, then there exist* $V_{i \in I} \in \mathscr{B}$ *such that* $V = \bigcup_{i \in I} V_i$ *and* $f^{-1}(V_i) \in \mathscr{T}, i \in I$*. Hence* $f^{-1}(V) = f^{-1}(\bigcap V_i) = \bigcap f^{-1}(V_i) \in$*. So f is fuzzy neutrosophic continuous.*

Proposition 3.16. *Let* $(A, \mathscr{T}_A), (B, \mathscr{U}_B)$*, be fuzzy neutrosophic subspaces of fuzzy neutrosophic topologies* $(X, \mathscr{T})$ *and* $(Y, \mathscr{U})$ *respectively. Let* $\acute{\mathscr{B}}$ *be a base for* $\mathscr{U}_B$*. Then a mapping f of* $(A, \mathscr{T}_A)$ *into* $(B, \mathscr{U}_B)$ *is relatively continuous iff for each* $\acute{B}$ *in* $\acute{\mathscr{B}}$ *the intersection* $f^{-1}[\acute{B}] \cap A$ *is in* $\mathscr{T}_A$*.*
Proof: Straightforward.

Definition 3.17. *Given two fuzzy neutrosophic topologies* $\mathscr{T}_1$*,* $\mathscr{T}_2$ *on the same set X, then* $\mathscr{T}_1$ *is said to be finer than* $\mathscr{T}_2$ *(or* $\mathscr{T}_2$ *is coarser than* $\mathscr{T}_1$*) if the* *identity mapping of* $(X, \mathscr{T}_1)$ *into* $(X, \mathscr{T}_2)$ *is fuzzy neutrosophic continuous , ie.,* $(X, \mathscr{T}_2) \subset (X, \mathscr{T}_1)$

Definition 3.18. *Let f be a mapping of a set X into a set Y, and let* $\mathscr{U}$ *be a fuzzy neutrosophic topology on Y. Then the family* $\mathscr{T}_{f^{-1}} = \{f^{-1}(U) \in FNS(X); U \in \mathscr{U}\ \}$ *is called the inverse image of* $\mathscr{U}$ *under f.*$\mathscr{T}_{f^{-1}}$ *is the coarsest fuzzy neutrosophic topology on X for which* $f : (X, \mathscr{T}_{f^{-1}}) \to (Y, \mathscr{U})$ *is fuzzy neutrosophic continuous.*

Definition 3.19. *Let f be a mapping of a set X into a set Y, and let* $\mathscr{T}$ *be a fuzzy neutrosophic topology on X. Then the family* $\mathscr{U}_f = \{U \in FNS(Y); f^{-1}(U) \in \mathscr{T}\ \}$ *is called the image of* $\mathscr{T}$ *under f.*$\mathscr{U}_f$ *is the finest fuzzy neutrosophic topology on Y for which* $f : (X, \mathscr{T}) \to (Y, \mathscr{U}_f)$ *is fuzzy neutrosophic continuous.*

Definition 3.20. *Given a family* $\{(X_\lambda, \mathscr{T}_\lambda)\}_{\lambda\in\Lambda}$ *of fuzzy neutrosophic topologies and let* $X = \prod_{\lambda\in\Lambda} X_\lambda$ *, let* $(X, \mathscr{T})$ *a fuzzy neutrosophic topological space and let* $\mathscr{T}$ *the coarsest fuzzy neutrosophic topology on X for which* $\wp_\lambda : (X, \mathscr{T}) \to (X_\lambda, \mathscr{T}_\lambda)$ *is fuzzy neutrosophic continuous for each* $\lambda \in \Lambda$*, where* $\wp_\lambda$ *is the usual projection. Then* $\mathscr{T}$ *is called the fuzzy neutrosophic product topology on X and denoted by* $\prod_{\lambda\in\Lambda} X_\lambda$ *and* $(X, \mathscr{T})$ *a fuzzy neutrosophic product space.*

From the definition 3.13 and definition 3.20 we have the following proposition.

Proposition 3.21. *Let* $\{(X_\lambda, \mathscr{T}_\lambda)\}_{\lambda\in\Lambda}$ *be a family of fuzzy neutrosophic topological spaces and* $(X, \mathscr{T})$ *the fuzzy neutrosophic product space. Then* $\mathscr{T}$ *has a base the set of finite intersections of fuzzy neutrosophic sets in X of the form* $\wp_\lambda^{-1}[U_\lambda]$ *where* $U_\lambda \in \mathscr{T}_\lambda$ *for each* $\lambda \in \Lambda$*.*

Definition 3.22. *Let* $\{X_i\}$ *, i = 1,2,...n be a finite family of sets and for each i=1,2,....n, let* A_i *be a fuzzy neutrosophic set in* X_i*. We define the product* $A = \prod_{i=1}^{n} A_i$ *of the family* A_i*, i = 1, 2, ...n, as the fuzzy neutrosophic set in* $X = \prod_{i=1}^{n} X_i$ *that has membership function, indeterministic function and non-membership function given by : for each* $(x_1, x_2, ...x_n) \in X$
$T_A(x_1, x_2, ...x_n) = T_{A_1}(x_1) \wedge T_{A_2}(x_2)\wedge, ... \wedge T_{A_n}(x_n),$
$I_A(x_1, x_2, ...x_n) = I_{A_1}(x_1) \wedge I_{A_2}(x_2)\wedge, ... \wedge I_{A_n}(x_n), and$
$F_A(x_1, x_2, ...x_n) = F_{A_1}(x_1) \vee F_{A_2}(x_2)\vee, ... \vee F_{A_n}(x_n).$

Remark 3.23. *From the definition 3.20 and proposition 3.21 that if* X_i *has fuzzy neutrosophic topology* $\mathscr{T}_i$*, i = 1, 2, ...n, then the fuzzy neutrosophic product topology* $\mathscr{T}$ *on X has a the set of fuzzy neutrosophic product spaces of the form* $\prod_{i=1}^{n} U_i$ *where* $U_i \in \mathscr{T}_i$ *for each i = 1,2,...n.*

Proposition 3.24. *Let $\{X_i\}$, i= 1,2,....n, be a finite family of sets and let $A = \prod_{i=1}^{n} A_i$ the fuzzy neutrosophic product space in $X = \prod_{i=1}^{n} X_i$ where $A_i \in FNS(X_i)$ for each i = 1,2,...n. Then $\wp_i(A) \subset A_i$ for each i = 1,2,...n.*
Proof:

Let $x_i \in X_i$. Then $T_{\wp_i(A)}(x_i) = \wp_i(T_A)(x_i)$
$= \bigvee_{z_1,z_2,...z_n \in \wp_i^{-1}(x_i)} T_A(z_1, z_2, ...z_n) = \bigvee_{z_1,z_2,...z_n \in \wp_i^{-1}(x_i)} [T_{A_1}(z_1) \wedge T_{A_2}(z_2) \wedge, ... \wedge T_{A_1}(z_n)]$
$= \bigwedge\{ \bigvee_{z_1 \in X_1} (T_{A_1}(z_1), ... \bigvee_{z_n \in X_n} (T_{A_n}(z_n)\} \leq T_{A_i}(x_i)$. *Similarly we can prove that* $I_{\wp_i(A)}(x_i) \leq I_{A_i}(x_i)$ *and* $F_{\wp_i(A)}(x_i) \geq F_{A_i}(x_i)$. *Hence $\wp_i(A) \subset A_i$ for each i = 1,2,...n.*

Proposition 3.25. *Let $\{(X_i, \mathscr{T}_i)\}$, i = 1,2,...n be a finite family of fuzzy neutrosophic topological spaces, let $(X, \mathscr{T})$ the fuzzy neutrosophic product space and let $A = \prod_{i=1}^{n} A_i$ where A_i a fuzzy neutrosophic set in X_i for each i=1,2,...n. Then the induced fuzzy neutrosophic topology $\mathscr{T}_A$ on A has a base the set of fuzzy neutrosophic product spaces of the form $\prod_{i=1}^{n} \acute{U}_i$ where $\acute{U}_i \in (\mathscr{T}_i)_{A_i}$, i = 1,2, ...n.*
Proof:

By the above remark 3.23, $\mathscr{T}$ has a base $\mathscr{B} = \{\prod_{i=1}^{n} U_i : U_i \in \mathscr{T}_i, i = 1, 2, ...n\}$*.A base for $\mathscr{T}_A$ is therefore given by* $\mathscr{B}_A = \left\{ \left(\prod_{i=1}^{n} U_i \right) \cap A : U_i \in \mathscr{T}_i, i = 1, 2, ...n \right\}$.
But $\left(\prod_{i=1}^{n} U_i \right) \cap A = \left(\prod_{i=1}^{n} U_i \cap A_i \right)$ *and $U_i \cap A_i \in (\mathscr{T}_i)_{Ai}$ for i = 1,2,...n. Let $\acute{U}_i = U_i \cap A_i$ for each i = 1, 2, ...n.*
Then $\mathscr{B}_A = \left\{ \prod_{i=1}^{n} \acute{U}_i : \acute{U}_i \in (\mathscr{T}_i)_{A_i}, i = 1, 2, ...n \right\}$ *and we denote the fuzzy neutrosophic subspace $(A, \mathscr{T}_A)$ by* $\prod_{i=1}^{n} (A_i, (\mathscr{T}_i)_{A_i})$.

Proposition 3.26. *Let $\{(X_\lambda, \mathscr{T}_\lambda)\}_{\lambda \in \Lambda}$ be a family of fuzzy neutrosophic topological spaces, let $(X, \mathscr{T})$ the fuzzy neutrosophic product space, let $(Y, \mathscr{U})$ an fuzzy neutrosophic topological space and let $f : (Y, \mathscr{U}) \to (X, \mathscr{T})$. Then f is fuzzy neutrosophic continuous iff $\wp_\lambda \circ f : (Y, \mathscr{U}) \to (X_\lambda, \mathscr{T}_\lambda)$ is fuzzy neutrosophic continuous for each $\lambda \in \Lambda$.*
Proof:

Suppose $f : (Y, \mathscr{U}) \to (X, \mathscr{T})$ is fuzzy neutrosophic continuous. For each $\lambda \in \Lambda$., let $U_\lambda \in \mathscr{T}_\lambda$. By the definition of $\mathscr{T}$, $\wp_\lambda^{-1}(U_\lambda) \in \mathscr{T}$. By the hypothesis, $f^{-1}(\wp_\lambda^{-1}(U_\lambda)) \in \mathscr{U}$. But $f^{-1}(\wp_\lambda^{-1}(U_\lambda)) = (\wp_\lambda \circ f)^{-1}(U_\lambda)$. Thus $(\wp_\lambda \circ f)^{-1}(U_\lambda) \in \mathscr{U}$. Hence $\wp_\lambda \circ f : (Y, \mathscr{U}) \to (X_\lambda, \mathscr{T}_\lambda)$ is fuzzy neutrosophic

continuous.
Conversely, let the necessary condition holds and let $U \in \mathscr{T}$. By Proposition 3.21, there exist a finite subset $\acute{\Lambda}$ of Λ such that $U = \bigcap_{\lambda \in \acute{\Lambda}} \wp_\lambda^{-1}(U_\lambda)$ and $U_\lambda \in \mathscr{T}_\lambda$ for each $\lambda \in \acute{\Lambda}$. Since $\wp_\lambda \circ f : (Y, \mathscr{U}) \to (X_\lambda, \mathscr{T}_\lambda)$ is fuzzy neutrosophic continuous for each $\lambda \in \acute{\Lambda}$, $(\wp_\lambda \circ f)^{-1}(U_\lambda) \in \mathscr{U}$ for each $\lambda \in \acute{\Lambda}$. Thus $f^{-1}(\wp_\lambda^{-1}(U_\lambda))\mathscr{U}$ for each $\lambda \in \acute{\Lambda}$. So $\bigcap_{\lambda \in \acute{\Lambda}} f^{-1}(\wp_\lambda^{-1}(U_\lambda)) \in \mathscr{U}$. But $f^{-1}(\bigcap_{\lambda \in \acute{\Lambda}} \wp_\lambda^{-1}(U_\lambda)) = f^{-1}(U)$. So $f^{-1}(U) \in \mathscr{U}$. Hence f is fuzzy neutrosophic continuous.

Corollary 3.27. *Let $\{(X_\lambda, \mathscr{T}_\lambda)\}_{\lambda \in \Lambda}$, $\{(Y_\lambda, \mathscr{U}_\lambda)\}_{\lambda \in \Lambda}$ be two families of fuzzy neutrosophic topological spaces and let $(X, \mathscr{T})$, $(Y, \mathscr{U})$ the respectively fuzzy neutrosophic product spaces, where $X = \prod_{\lambda \in \Lambda} X_\lambda$ and $Y = \prod_{\lambda \in \Lambda} Y_\lambda$. For each $\lambda \in \Lambda$, let f_λ be a mapping of $(X_\lambda, \mathscr{T}_\lambda)$ into $(Y_\lambda, \mathscr{U}_\lambda)$. Then the product mapping $f = \prod_{\lambda \in \Lambda} f_\lambda : (X, \mathscr{T}) \to (Y, \mathscr{U})$ is fuzzy neutrosophic continuous iff f_λ is fuzzy neutrosophic continuous for each $\lambda \in \Lambda$, where $f(x) = (f_\lambda(\wp_\lambda(x))$ for each $x \in \prod_{\lambda \in \Lambda} X_\lambda$. Proof: The proof is obvious from the above proposition.*

Proposition 3.28. *Let $(X_i, \mathscr{T}_i)_i$, i=1,2...n be a finite family of fuzzy neutrosophic topological spaces and $(X, \mathscr{T})$ the fuzzy neutrosophic product space. For each i = 1,2,...n, let A_i be a fuzzy neutrosophic set in X_i and let $A = \prod_{i=1}^{n} A_i$ a fuzzy neutrosophic set in X. Let $(Y, \mathscr{U})$ be a fuzzy neutrosophic topological space and let B a fuzzy neutrosophic set in Y , and $f : (B, \mathscr{U}_B) \to (A, \mathscr{T}_A)$ is relatively fuzzy neutrosophic continuous iff $\wp_i \circ f : (B, \mathscr{U}_B) \to (A_i, (\mathscr{T}_i)_{A_i})$ is relatively fuzzy neutrosophic continuous for each i=1,2,..n.*
Proof:

Suppose $f : (B, \mathscr{U}_B) \to (A, \mathscr{T}_A)$ is relatively fuzzy neutrosophic continuous.$\wp :$ $(X, \mathscr{T}) \to (X_i, \mathscr{T}_i)$ is fuzzy neutrosophic continuous for each i=1,2,..n and by By Proposition 3.24 $\wp(A) \subset A_i$ for each i = 1, 2,....n. Then by By Proposition 3.9 $\wp_\lambda : (A, \mathscr{T}_A) \to (A_i, (\mathscr{T}_i)_{A_i})$ is relatively fuzzy neutrosophic continuous for each i= 1,2,....n. Hence $\wp_i \circ f : (B, \mathscr{U}_B) \to (A_i, (\mathscr{T}_i)_{A_i})$ is relatively fuzzy neutrosophic continuous for each i=1,2,..n.
Conversely, the necessary condition holds. Let $\acute{U} = \acute{U}_1 \times .... \times \acute{U}_n$ where $\acute{U}_i \in (\mathscr{T}_i)_{A_i})$, i = 1, 2...n. By Proposition the 3.25 set of $\acute{U}$ forms a base for $\mathscr{T}_A$ and $f^{-1}(\acute{U}) \cap B = f^{-1}[\wp_1^{-1}(\acute{U}_1) \cap ..... \cap \wp_n^{-1}(\acute{U}_n)] \cap B = \bigcap_{i=1}^{n}((\wp \circ f)^{-1}[\acute{U}_i] \cap B)$. Since $\wp_i \circ f : (B, \mathscr{U}_B) \to (A_i, (\mathscr{T}_i)_{A_i})$ is relatively fuzzy neutrosophic continuous for each i=1,2,..n, $f^{-1}(\acute{U}) \cap B \in \mathscr{U}_B$. Hence by Proposition 3.16 $f : (B, \mathscr{U}_B) \to (A, \mathscr{T}_A)$ is relatively fuzzy neutrosophic continuous.

Corollary 3.29. *Let $\{(X_i, \mathscr{T}_i)\}$, $\{(Y_i, \mathscr{U}_i)\}$, i=1,2,..n be two finite families of fuzzy neutrosophic topological spaces and $(X, \mathscr{T})$ and $(Y, \mathscr{U})$ the respective* *fuzzy neutrosophic product spaces. For each i = 1,2,...n let* A_i *be a fuzzy neutrosophic set in* X_i, B_i *a fuzzy neutrosophic set in* Y_i *and* $f_i : (A_i, (\mathscr{T}_i)_{A_i}) \to (B_i, (\mathscr{U}_i)_{B_i})$. *Let* $A = \prod_{i=1}^{n} A_i$, $B = \prod_{i=1}^{n} B_i$ *be the fuzzy neutrosophic product spaces in X, Y respectively. Then the product mapping* $f = \prod_{i=1}^{n} f_i : (A, \mathscr{T}_A) \to (B, \mathscr{U}_B)$ *is relatively fuzzy neutrosophic continuous if* f_i *is relatively fuzzy neutrosophic continuous for each i =1,2,....n.*

Proposition 3.30. *Let* $\{(X_i, \mathscr{T}_i)\}$, $\{(Y_i, \mathscr{U}_i)\}$, *i=1,2,...n be two finite families of fuzzy neutrosophic topological spaces and let* $(X, \mathscr{T})$, $(Y, \mathscr{U})$ *the respective fuzzy neutrosophic product spaces. For each i=1,2,...n , let* $f_i : (X_i, \mathscr{T}_i) \to (Y_i, \mathscr{U}_i)$.*Then the product mapping* $f = \prod_{i=1}^{n} f_i : (X, \mathscr{T}) \to (Y, \mathscr{T})$ *is fuzzy neutrosophic open if* f_i *is fuzzy neutrosophic open for each i =1, 2...n.*
Proof:

Let U be open in $\mathscr{T}$. *Let* $\mathscr{B}= \{\prod_{i=1}^{n} U_i$ *a fuzzy neutrosophic set in X:* $U_i \in \mathscr{T}_i$ *for each i=1,2,...n*$\}$. *Since* $\mathscr{B}$ *is a base for* $\mathscr{T}$, *there is a* $\acute{B} \subset \mathscr{B}$ *such that* $U = \bigcup \acute{B}$.*Since each member of* $\acute{B}$ *is of the form* $\prod_{i=1}^{n} U_i$, *we can consider* $\acute{B} = \{\prod_{i=1}^{n} U_{i\lambda}\}_{\lambda\in\Lambda}$. *Then* $U = \bigcup_{\lambda\in\Lambda} \prod_{i=1}^{n} U_{i\lambda}$. *Let* $y \in Y$ *such that* $f^{-1}(y) \neq \phi$.*Then* $T_{f(U)}(y) = f(T_U)(y) = \bigvee_{z\in f^{-1}(y)} T_U(z) = \bigvee_{z\in f^{-1}(y)} T_{\bigcup_{\lambda\in\Lambda} \prod_{i=1}^{n} U_{i\lambda}}(z)$

$$= \bigvee_{z\in f^{-1}(y)} \bigvee_{\lambda\in\Lambda} T_{\prod_{i=1}^{n} U_{i\lambda}}(z)$$

$$= \bigvee_{\lambda\in\Lambda} \bigvee_{z_1\in f_1^{-1}(y_1)} \ldots\ldots \bigvee_{z_n\in f_n^{-1}(y_n)} [T_{U_{1\lambda}}(z_1) \wedge \ldots\ldots \wedge T_{U_{n\lambda}}(z_n)]$$

$$= \bigvee_{\lambda\in\Lambda} \left[\bigvee_{z_1\in f_1^{-1}(y_1)} T_{U_{1\lambda}}(z_1) \wedge \ldots\ldots \wedge \bigvee_{z_n\in f_n^{-1}(y_n)} T_{U_{1\lambda}}(z_n) \right]$$

$$= \bigvee_{\lambda\in\Lambda} [T_{f_1(U_{1,\lambda})}(y_1) \wedge \ldots. \wedge T_{f_n(U_{n,\lambda})}(y_n)] = \bigvee_{\lambda\in\Lambda} T_{\prod_{i=1}^{n} f_i(U_{i\lambda})}(y) = T_{\bigcup_{\lambda\in\Lambda} \prod_{i=1}^{n} f_i(U_{i\lambda})}(y).$$

Similarly we can prove that $I_{f(U)}(y) = I_{\bigcup_{\lambda\in\Lambda} \prod_{i=1}^{n} f_i(U_{i\lambda})}(y)$

$$F_{f(U)}(y) = f(T_U)(y) = \bigwedge_{z\in f^{-1}(y)} F_U(z) = \bigwedge_{z\in f^{-1}(y)} F_{\bigcup_{\lambda\in\Lambda} \prod_{i=1}^{n} U_{i\lambda}}(z) = \bigwedge_{z\in f^{-1}(y)} \bigwedge_{\lambda\in\Lambda} F_{\prod_{i=1}^{n} U_{i\lambda}}(z)$$

$$= \bigwedge_{\lambda\in\Lambda} \bigwedge_{z_1\in f_1^{-1}(y_1)} \ldots\ldots \bigwedge_{z_n\in f_n^{-1}(y_n)} [F_{U_{1\lambda}}(z_1) \vee \ldots\ldots \vee F_{U_{n\lambda}}(z_n)]$$

$$= \bigwedge_{\lambda\in\Lambda} \left[\bigwedge_{z_1\in f_1^{-1}(y_1)} F_{U_{1\lambda}}(z_1) \vee \ldots\ldots \vee \bigwedge_{z_n\in f_n^{-1}(y_n)} F_{U_{1\lambda}}(z_n) \right]$$

$$= \bigwedge_{\lambda\in\Lambda} [F_{f_1(U_{1,\lambda})}(y_1) \vee \ldots. \vee F_{f_n(U_{n,\lambda})}(y_n)] = \bigwedge_{\lambda\in\Lambda} F_{\prod_{i=1}^{n} f_i(U_{i\lambda})}(y) = F_{\bigcup_{\lambda\in\Lambda} \prod_{i=1}^{n} f_i(U_{i\lambda})}(y).$$

Thus $f(U) = \bigcup_{\lambda\in\Lambda}\prod_{i=1}^{n} f_i(U_{i\lambda})$. *Since* f_i *is fuzzy neutrosophic open for each i=1,2...n,* $f_i(U_{i\lambda})$ *is fuzzy neutrosophic open is* X_i *for each i=1,2....n. Then* $\prod_{i=1}^{n} f_i(U_{i\lambda})$ *is fuzzy neutrosophic open in Y. So* $\bigcup_{\lambda\in\Lambda}\prod_{i=1}^{n} f_i(U_{i\lambda})$ *is a fuzzy neutrosophic open in Y. Hence f is fuzzy neutrosophic open.*

Proposition 3.31. *Let* $\{(X_i, \mathscr{T}_i)\}$, $\{(Y_i, \mathscr{U}_i)\}$, *i=1,2,...n be two finite families of fuzzy neutrosophic topological spaces and let* $(X, \mathscr{T})$, $(Y, \mathscr{U})$ *the respective fuzzy neutrosophic product spaces. For each i=1,2,...n , let* A_i *a fuzzy neutrosophic set in* X_i,B_i *a fuzzy neutrosophic set in* Y_i *and let* $A = \prod_{i=1}^{n} A_i$, $B = \prod_{i=1}^{n} B_i$ *be the fuzzy neutrosophic product spaces in X, Y respectively. If* $f_i : A_i \to B_i$ *is relatively fuzzy neutrosophic open for each i=1,2,...n, then the product mapping* $f = \prod_{i=1}^{n} f_i : (A, \mathscr{T}_A) \to (B, \mathscr{U}_B)$ *is relatively fuzzy neutrosophic open.*
Proof:

Let $\mathscr{B} = \{\prod_{i=1}^{n} U_i$ *a fuzzy neutrosophic set in A:* $U_i \in (\mathscr{T}_i)_{A_i}$ *for each i=1,2,...n*$\}$. *Then by Proposition 3.25 ,* $\mathscr{B}$ *is a base for* $\mathscr{T}_A$. *Let* $U \in \mathscr{T}_A$. *Then there is* $\acute{\mathscr{B}} \subset \mathscr{B}$ *such that* $\bigcup \acute{\mathscr{B}} = U$. *We can consider* $\acute{\mathscr{B}}$ *as* $\{\prod_{i=1}^{n} U_{i\lambda}\}_{\lambda\in\Lambda}$. *Then* $U = \bigcup_{\lambda\in\Lambda}\prod_{i=1}^{n} U_{i\lambda}$. *As in the above proposition 3.30 we get* $f(U) = \bigcup_{\lambda\in\Lambda}\prod_{i=1}^{n} f_i(U_{i\lambda})$. *Since* f_i *is relatively fuzzy neutrosophic open for each i=1,2...n,* $f(U) \in \mathscr{U}_B$. *Hence f is relatively fuzzy neutrosophic open.*

Lemma 3.32. *Let* $(X_1, \mathscr{T}_1)$, $(X_2, \mathscr{T}_2)$ *be fuzzy neutrosophic topological spaces. Then the constant mapping* $f : (X_2, \mathscr{T}_2) \to (X_1, \mathscr{T}_1)$ *given by* $f(x_2) = x_0 \in X_1$ *for each* $x_2 \in X_2$, *is fuzzy neutrosophic continuous.*
Proof:

Let $U \in \mathscr{T}_1$ *and let* $x_2 \in X_2$. *Then* $T_{f^{-1}(U)}(x_2) = f^{-1}(T_U)(x_2) = T_U f(x_2) = T_U(x_0)$.*Similarly we have* $I_{f^{-1}(U)}(x_2) = I_U(x_0)$ *and* $F_{f^{-1}(U)}(x_2) = F_U(x_0)$. *Let* $T_U(x_0) = \alpha$, $I_U(x_0) = \beta$ *and* $F_U(x_0) = \gamma$. *Consider* $C_{\alpha,\beta,\gamma}$. *Since* $U \in FNS(X_1)$, $\alpha + \beta + \gamma \leq 3$. *Then* $C_{(\alpha,\beta,\gamma)}$ *is fuzzy neutrosophic open in* X_2. *Thus* $T_{f^{-1}(U)}(x_2) = \alpha = T_{C_{(\alpha,\beta,\gamma)}}(x_2)$, $I_{f^{-1}(U)}(x_2) = \beta = I_{C_{(\alpha,\beta,\gamma)}}(x_2)$*and* $F_{f^{-1}(U)}(x_2) = \gamma = F_{C_{(\alpha,\beta,\gamma)}}(x_2)$ *implies* $f^{-1}(U) = C_{(\alpha,\beta,\gamma)}$. *So* $f^{-1}(U)$ *is fuzzy neutrosophic open in* X_2. *Hence f is fuzzy neutrosophic continuous.*

Proposition 3.33. *Let* $(X_1, \mathscr{T}_1)$, $(X_2, \mathscr{T}_2)$ *be fuzzy neutrosophic topological spaces and let* $(X, \mathscr{T})$ *the fuzzy neutrosophic product space. Then for each* $x_1 \in X_1$*the mapping* $i : (X_2, \mathscr{T}_2) \to (X, \mathscr{T})$ *defined by* $i(x_2) = (x_1, x_2)$ *for each* $x_2 \in X_2$ *is fuzzy neutrosophic continuous.*
Proof:

By Lemma 3.32 the constant mapping $i_1 : (X_2, \mathscr{T}_2) \rightarrow (X_1, \mathscr{T})$given by $i(x_2) = x_1$ for each $x_2 \in X_2$ is fuzzy neutrosophic continuous. The identity mapping $i_2 : (X_2, \mathscr{T}_2) \rightarrow (X_2, \mathscr{T}_2)$ is fuzzy neutrosophic continuous. Hence by Proposition 3.26 i is fuzzy neutrosophic continuous.

Proposition 3.34. *Let $(X_1, \mathscr{T}_1)$, $(X_2, \mathscr{T}_2)$ be fuzzy neutrosophic topological spaces and let $(X, \mathscr{T})$ the fuzzy neutrosophic product space. Let A_1, A_2 be fuzzy neutrosophic sets in X_1, X_2 respectively and let A the fuzzy neutrosophic product space in X. Let $a_1 \in X_1$ such that $T_{A_1}(a_1) \geq T_{A_2}(x_2)$, $I_{A_1}(a_1) \geq I_{A_2}(x_2)$ and $F_{A_1}(a_1) \leq F_{A_2}(x_2)$ for each $x_2 \in X_2$. Then the mapping $i : (A_2, (\mathscr{T}_2)_{A_2}) \rightarrow (A_1, \mathscr{T}_A)$ given by $i(x_2) = (a_1, x_2)$ for each $x_2 \in X_2$ is relatively fuzzy neutrosophic continuous.*

Proof:Let $(x_1, x_2) \in X$. Then

$$T_{i(A_2)}(x_1, x_2) = \begin{cases} \bigvee_{x'_2 \in i^{-1}(x_1,x_2)} T_{A_2}(x'_2) & \text{if } i^{-1}(x_1, x_2) \neq \phi \\ 0 & \text{otherwise} \end{cases} = \begin{cases} T_{A_2}(x_2) & \text{if } x_1 = a_1 \\ 0 & \text{otherwise} \end{cases}$$

$$I_{i(A_2)}(x_1, x_2) = \begin{cases} \bigvee_{x'_2 \in i^{-1}(x_1,x_2)} I_{A_2}(x'_2) & \text{if } i^{-1}(x_1, x_2) \neq \phi \\ 0 & \text{otherwise} \end{cases} = \begin{cases} I_{A_2}(x_2) & \text{if } x_1 = a_1 \\ 0 & \text{otherwise} \end{cases}$$

$$F_{i(A_2)}(x_1, x_2) = \begin{cases} \bigwedge_{x'_2 \in i^{-1}(x_1,x_2)} F_{A_2}(x'_2) & \text{if } i^{-1}(x_1, x_2) \neq \phi \\ 1 & \text{otherwise} \end{cases} = \begin{cases} F_{A_2}(x_2) & \text{if } x_1 = a_1 \\ 1 & \text{otherwise} \end{cases}$$

and $T_A(x_1, x_2) = T_A(x_1) \wedge T_A(x_2)$, $I_A(x_1, x_2) = I_A(x_1) \wedge I_A(x_2)$ and $F_A(x_1, x_2) = F_A(x_1) \vee F_A(x_2)$. By the assumption, $T_A(x_1, x_2) \geq T_{A_2}(x_2)$, $I_A(x_1, x_2) \geq I_{A_2}(x_2)$ and $F_A(x_1, x_2) \leq F_{A_2}(x_2)$. Thus $T_A(x_1, x_2) \geq T_{i(A)}(x_1, x_2)$, $I_A(x_1, x_2) \geq I_{i(A)}(x_1, x_2)$ and $F_A(x_1, x_2) \leq F_{i(A)}(x_1, x_2)$. Hence $i(A) \subset A$. The proof of relative continuity of i is similar to the proof of fuzzy neutrosophic continuity of i in Proposition 3.33.

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