

# Fundamental Quantal Paradox And Its Resolution

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## Abstract

The postulate that coordinate and momentum representations are related to each other by the Fourier transform has been accepted from the beginning of quantum theory. As a consequence, coordinate wave functions of photons emitted by stars have cosmic sizes. This results in a paradox because predictions of the theory contradict observations. The reason of the paradox and its resolution are discussed.

PACS: 03.65.-w, 03.65.Sq, 03.65.Ta

Keywords: quantum theory, position operator, semiclassical approximation

## 1 Introduction

Although quantum theory exists for a rather long period of time, the problem of its interpretation is still widely debated. Let us first make a remark about the terminology of the theory. The term "wave function" has arisen at the beginning of quantum era in efforts to explain quantum behavior in terms of classical waves but now it is clear that no such explanation exists. The notion of wave is purely classical; it has a physical meaning only as a way of describing systems of many particles by their mean characteristics. The problem discussed in the present paper is a good illustration that the term "wave function" might be misleading since in quantum theory it defines not amplitudes of waves but only amplitudes of probabilities. So, although in our opinion the term "state vector" is more pertinent than "wave function" we use the latter in accordance with the usual terminology.

For example, the electron has an electric charge  $e$  which is indivisible. So for the electron the notion of the charge density is meaningless. Roughly speaking the electron is a point and its wave function (WF)  $\psi(\mathbf{r})$  describes only probabilities to find the electron at different points  $\mathbf{r}$ . If a decomposition  $\psi = \psi_1 + \psi_2$  is possible according to the superposition principle, this does not mean splitting the electron into two parts with the charges  $e_1$  and  $e_2$  such that  $e_1 + e_2 = e$ . Therefore classical description of elementary particles is not adequate. Those remarks will be important in Sec. 3.

Note also that standard quantum theory is based on classical mathematics involving infinitely small, infinitely large, continuity etc. The part of this mathematics which is used in quantum theory was developed mainly in the 19th century when people believed that every object can be divided into arbitrarily large numbers of arbitrarily small parts. However, in view of the existence of elementary particles, this is not the case. Hence a problem arises whether classical mathematics is pertinent on fundamental quantum level. In Refs. [1, 2] and references therein we argue that fundamental quantum theory should be based on finite mathematics. However, in the present paper we proceed from standard quantum theory.

One might say that the problem of coordinate description of elementary particles does not arise because *the results* of existing fundamental quantum theories describing interactions on quantum level (QED, electroweak theory and QCD) are formulated exclusively in terms of the S-matrix in momentum space without mentioning space-time at all.

For example, consider the problem of calculating the hydrogen energy levels. As follows from Feynman diagrams for the one-photon exchange, in the approximation  $(v/c)^2$  the electron in the hydrogen atom can be described in the potential formalism where the potential acts on the WF in momentum space. So for calculating energy levels one should solve the eigenvalue problem for the Hamiltonian with this potential. This is an integral equation which can be solved by different methods. One of them is to apply the Fourier transform and get standard Schrödinger or Dirac equation in coordinate representation. Hence the fact that the results for energy levels are in good agreement with experiment shows that QED defines the potential correctly and *standard coordinate Schrödinger and Dirac equations are only convenient mathematical ways of solving the eigenvalue problem*. For this problem the physical meaning of the position operator is not important. One can consider other transformations of the original integral equation and define other position operators.

However, since quantum theory is treated as a more general than classical one, in semiclassical approximation it should reproduce the motion of a particle along the classical trajectory defined by classical equations of motion. Hence the position operator is needed only in semiclassical approximation and it should be *defined* from additional considerations.

The postulate that the coordinate and momentum representations are related to each other by the Fourier transform has been accepted from the beginning of quantum theory from different considerations (see e.g. the discussion in Ref. [3]). A problem arises whether this postulate has been verified experimentally.

In standard approaches to quantum theory the existence of space-time background is assumed from the beginning. Then the position operator for a particle in this background is the operator of multiplication by the particle radius-vector  $\mathbf{r}$ . As explained in textbooks (see e.g. Ref. [4]), the result  $-i\hbar\partial/\partial\mathbf{r}$  for the momentum operator can be justified from the requirement that quantum theory should correctly reproduce classical results in semiclassical approximation.

However, this requirement does not define the operator unambiguously. Indeed, if the operator  $B$  becomes zero in semiclassical limit then the operators  $A$  and  $A + B$  have the same semiclassical limit. In addition, as argued in Ref. [2], quantum theory should start not from classical space-time background but from a symmetry algebra. For example, on quantum level Poincare invariance means not that the space-time background is Minkowskian but that operators describing a system satisfy the commutation relations of the Poincare algebra. Then elementary particles are described by irreducible representations (IRs) of this algebra. Such IRs can be implemented in spaces of functions  $\chi(\mathbf{p})$  such that

$$\|\chi\|^2 = \int |\chi(\mathbf{p})|^2 d^3\mathbf{p} < \infty \quad (1)$$

where  $\|\dots\|$  is the norm and the momentum operator is the operator of multiplication by  $\mathbf{p}$ . At the same time, among the ten independent operators in such IRs there is no position operator.

One of the conditions characterizing semiclassical approximation is that the orbital angular momentum of the particle under consideration should be large. Then, as follows from explicit expressions for the representation operators (see e.g. Refs. [5, 3]), in this approximation spin effects can be neglected. A standard assumption is that the position operator in the representation (1) is  $i\hbar\partial/\partial\mathbf{p}$ . One of the arguments is that if the coordinate wave function contains  $\exp[iS(\mathbf{r}, t)/\hbar]$  where  $S(\mathbf{r}, t)$  is the classical action then the Schrödinger equation becomes the Hamilton-Jacobi one in the formal limit  $\hbar \rightarrow 0$ .

With this choice, an inevitable consequence of quantum theory is the effect of wave packet spreading (WPS) discussed in textbooks and many papers. At the beginning of quantum theory the WPS effect has been investigated by de Broglie, Darwin and Schrödinger. The fact that WPS is inevitable has been treated by several authors as unacceptable. For example, de Broglie has proposed to describe a free particle not by the Schrödinger equation but by a wavelet which satisfies a nonlinear equation and does not spread out (see e.g. Ref. [6]).

At the same time, it has not been explicitly shown that WPS is incompatible with experimental data. For example, as shown by Darwin [7], for macroscopic bodies this effect is negligible. Probably it is also believed that in experiments on the Earth with atoms and elementary particles spreading does not have enough time to manifest itself. Probably for those reasons the absolute majority of physicists do not treat WPS as a drawback of the theory.

However, a natural question arises what happens to photons which can travel from distant objects to Earth even for billions of years. To answer this question one needs to know the photon position operator. This problem has been discussed by many authors. Some of them state that the photon coordinate wave function does not exist. For example, the argument in Ref. [8] is that if the photon coordinate WF is defined in standard way then the photon electric and magnetic fields at any point

will depend not on the WF at this point but on integrals from the WF over a region with the dimension of the photon wave length.

This argument is problematic for several reasons. For example, as noted by Pauli (see p. 191 of Ref. [9]), the photon coordinates cannot have a physical meaning at distances less than the wave length. Since the wave length is defined as  $\lambda = 2\pi\hbar/p$  where  $p$  is the magnitude of momentum, then  $\lambda \rightarrow 0$  in the formal limit  $\hbar \rightarrow 0$ . Therefore on semiclassical level any WFs differing only by the behavior at distances less than the wave length are equivalent.

Another reason is that the notions of electric and magnetic fields of a single photon do not have a physical meaning. The description of a free particle is fully defined by its IR depending only on the mass and spin. It does not depend on interactions this particle participates in and, as noted above, in semiclassical approximation the particle spin is not important. Hence the description of an ultrarelativistic particle does not depend on the nature of the particle, i.e. on whether the particle is the photon, the proton, the electron etc.

Relativistic position operator has been first discussed by Newton and Wigner [10]. Their result is that if spin effects are neglected then the form of the position operator in the representation (1) is the same as in nonrelativistic case, i.e.  $i\hbar\partial/\partial\mathbf{p}$ . Position operators discussed by other authors differ from the Newton-Wigner one only by spin terms and by the behavior at distances less than the wave length (see Ref. [3] for a more detailed discussion). Therefore in semiclassical approximation all those operators are equivalent.

For this reason in Sec. 2 we calculate the coordinate WF of an ultrarelativistic particle proceeding from the usual assumption that the coordinate and momentum representations are related to each other by the Fourier transform. As argued in Sec. 3, the results contradict the usual experience on how we observe stars. The reason of the paradox and its resolution are discussed in Sec. 4.

## 2 Coordinate wave function of ultrarelativistic particle

Consider a model where the momentum WF is

$$\chi(\mathbf{p}) = f(\mathbf{p}/p)F(p)/p \quad (2)$$

Let  $f(\mathbf{p}/p) = \sum_{l\mu} c_{l\mu} Y_{l\mu}(\mathbf{p}/p)$  be the decomposition of the function  $f$  over spherical functions. Since the energy of the ultrarelativistic particle is  $E = pc$ , the dependence of this function on  $t$  is given by

$$\chi(\mathbf{p}, t) = \exp(-\frac{i}{\hbar}pct)\chi(\mathbf{p}) \quad (3)$$

Since we assume that the momentum and coordinate WFs are related to each other by the Fourier transform, the coordinate WF is

$$\psi(\mathbf{r}, t) = \int \exp\left(\frac{i}{\hbar}\mathbf{p}\mathbf{r}\right)\chi(\mathbf{p}, t)\frac{d^3\mathbf{p}}{(2\pi\hbar)^{3/2}} \quad (4)$$

For calculating this function we use the known decomposition:

$$\exp\left(\frac{i}{\hbar}\mathbf{p}\mathbf{r}\right) = 4\pi \sum_{l\mu} i^l j_l(pr/\hbar) Y_{l\mu}^*(\mathbf{p}/p) Y_{l\mu}(\mathbf{r}/r) \quad (5)$$

where  $r = |\mathbf{r}|$  and  $j_l$  is the spherical Bessel function. Its asymptotic expression when the argument is large is  $j_l(x) \approx \sin(x - \pi l/2)/x$ . Then, as follows from the orthogonality condition for spherical functions and Eqs. (2-5), at large distances

$$\psi(\mathbf{r}, t) = \frac{-i}{(2\pi\hbar)^{1/2}r} \sum_{l\mu} c_{l\mu} Y_{l\mu}(\mathbf{r}/r) [G(ct - r) - (-1)^l G(ct + r)] \quad (6)$$

where

$$G(\xi) = \int_0^\infty F(p) \exp\left(-\frac{i}{\hbar}\xi p\right) dp \quad (7)$$

For reasonable choices of  $F(p)$  we will have that at large distances and times  $G(ct - r) \gg G(ct + r)$ . Indeed if, for example, the quantities  $p_0$  and  $a$  are such that  $p_0 > 0$ ,  $a > 0$  and  $p_0 a \gg \hbar$  then possible ( $F, G$ ) choices are:

$$\begin{aligned} F(p) &= \exp\left(-\frac{|p - p_0|a}{\hbar}\right), & G(\xi) &= \frac{\exp(-ip_0\xi/\hbar)}{a^2 + \xi^2}; \\ F(p) &= \exp\left(-\frac{(|p - p_0|a)^2}{2\hbar^2}\right), & G(\xi) &= (2\pi)^{1/2} \frac{\hbar}{a} \exp\left(-\frac{ip_0\xi}{\hbar} - \frac{\xi^2}{2a^2}\right) \end{aligned} \quad (8)$$

Then, as follows from Eq. (6), in those cases

$$\psi(\mathbf{r}, t) = \frac{-i}{(2\pi\hbar)^{1/2}r} f(\mathbf{r}/r) G(ct - r) \quad (9)$$

Therefore at large distances and times the coordinate WF has the following properties: a) at each moment of time  $t$  the radial part is not negligible only inside a thin sphere with the radius  $ct$  and the width of the order of  $a$ ; b) the angular part does not depend on time and is the same as the angular part of the momentum wave function. As shown in Ref. [3], similar conclusions can be obtained for Gaussian wave packets in the Fresnel approximation. Therefore those conclusions are rather general.

According to the present knowledge, photons emitted by stars belong to two categories. The first one consists of photons emitted from transitions between different atomic energy levels. For example, the most known case is the Lyman transition  $2P \rightarrow 1S$  in the hydrogen atom. For such photons the orbital angular momentum

is small, i.e. the sphere spreads almost uniformly in all directions. In this case the WF can be qualitatively described in the spherically symmetric approximation when  $f = \text{const}$ .

However, the main part of the radiation can be approximately described in the blackbody model such that typical photon energies are not close to energies of absorption lines for that matter. Then the photon WFs can be treated as wave packets (see Ref. [3] for a more detailed discussion). If the mean value of the momentum is directed along the positive direction of the  $z$ -axis then the function  $f$  is essentially different from zero only in the range where angles between momenta and the  $z$ -axis are small. If  $p_0$  is the magnitude of the mean momentum and  $p_\perp$  is the uncertainty of the transverse momentum such that  $p_\perp \ll p_0$  then  $\alpha = p_\perp/p_0$  is the angular uncertainty. Since the angular dependences of the coordinate and momentum WFs are the same, the transversal width of the packet spreads out with time as  $\alpha ct$ .

The estimation of typical values of  $\alpha$  is problematic because photons can be created in different processes. However, even if  $\alpha$  is small, after years or longer the quantity  $\alpha ct$  becomes much greater than sizes of stars and planets (see estimations in Ref. [3]). So, as a consequence of WPS, photons emitted by stars have WFs of cosmic sizes and the motion of such photons is not a motion along classical trajectories.

A problem arises whether this phenomenon is compatible with our experience in observations of stars. We will discuss this problem in Sec. 3. However, at least images of stars should not appear blurred. Indeed, suppose that we observe a star which has the radius  $R$  and is on the distance  $L$  from the Earth. Then the image of the star will not appear blurred if the angular resolution is or the order of  $R/L$  or better. However, for all stars the quantity  $R/L$  is typically much less than  $\alpha$ . Nevertheless, images of stars will not appear blurred for the following reasons. Suppose that a photon is created at the origin and the Earth is seen from the origin in the narrow angular range defined by the function  $\tilde{f}(\mathbf{r}/r)$ . Suppose that the support of  $\tilde{f}(\mathbf{r}/r)$  is within the range defined by  $f(\mathbf{r}/r)$ . Then the projection of the WF (9) onto Earth is given by the same expression where  $f(\mathbf{r}/r)$  is replaced by  $\tilde{f}(\mathbf{r}/r)$ . Since the angular WFs in coordinate and momentum representations are the same, the momenta measured on Earth will be in the angular range defined by the function  $\tilde{f}(\mathbf{p}/p)$ . This result is the generalization of the solution of the Mott-Heisenberg problem (see e.g. Ref. [3]). At the same time, as argued in the next section, the fact that photons emitted by stars have WFs with cosmic sizes does lead to a paradox in observations of stars.

### 3 Paradox: standard choice of position operator contradicts observations of stars

The problem of explaining the redshift phenomenon has a long history. Different competing approaches can be divided into two big sets which we call Theory A and

Theory B. In Theory A the redshift has been originally explained as a manifestation of the Doppler effect but in recent years the cosmological and gravitational redshifts have been added to the consideration. In this theory the interaction of photons with the interstellar medium is treated as practically not important. On the contrary, in Theory B, which is often called the tired-light theory, the interaction of photons with the interstellar medium is treated as the main reason for the redshift. The majority of physicists believe that Theory A explains the astronomical data better than Theory B. Even some physicists working on Theory B acknowledged that any sort of scattering of light would predict more blurring than is seen (see e.g. the article "Tired Light" in Wikipedia).

As follows from these remarks, in Theory A it is assumed that with a good accuracy we can treat photons as propagating in empty space. In addition, simple arguments indicate (see e.g. Ref. [3]) that photons from distant stars practically do not interact with each other. Hence the effect of WPS can be considered for each photon independently.

As already noted, even if the function  $f(\alpha)$  describes a broad angular distribution, a star will be visible only in the angular range of the order of  $R/L$ . Hence one might think that the absence of classical trajectories does not contradict observations. We now consider this problem in greater details. For simplicity we first assume that the photon WF is spherically symmetric, i.e.  $f(\mathbf{r}/r) = \text{const}$ .

As already noted, the WF of the photon coming to Earth from a distant star is not negligible only within a thin sphere with the radius  $ct$  and the width of the order of  $a$ . On its way to Earth the sphere passes *all* stars, planets and other objects the distance from which to the star is less than  $L$  (in particular, even those objects which are from the star in directions opposite to the direction to Earth). A problem arises how to explain the fact that the photon was detected on Earth and escaped detection by those stars, planets etc.

One might think that the event when the photon was detected on Earth is purely probabilistic. The fact that the photon was not detected by the objects on its way to Earth can be explained such that since the photon WF has a huge size (of the order of light years or more) the probability of detection even by stars is extremely small and so it was only a favorable accident that the photon was detected on Earth.

However, if the photon passed stars, planets and other objects on its way to Earth then with approximately the same probability it can pass Earth and can be detected on the opposite side of the Earth. In that case we could see stars even through the Earth.

Moreover, consider the following experiment. Suppose that we first look at a star and then place a small screen between the eye and the star. Then the experiment shows that the star will not be visible. However, since the photon WF passed many big objects without interacting with them then with approximately the same probability it can pass the screen. In that case we could see the star through the screen.

Those phenomena are not unusual in view of our understanding of neutrino physics. It is known that neutrinos not only can pass the Earth practically without problems but even neutrinos created in the center of the Sun can easily reach the Earth. The major neutrino detectors are under the Earth surface and, for example, in the OPERA and ICARUS experiments neutrinos created at CERN reached Gran Sasso (Italy) after traveling 730km under the Earth surface. The explanation is that the probability of the neutrino interaction with particles comprising the Sun and the Earth is very small.

At low energies the electromagnetic interaction is much stronger than the weak one but, as noted in Sec. 2, the probability of interaction for photons having cosmic sizes contains the factor  $|f/f|^2$ . Therefore it is reasonable to expect that for such photons the probability of interaction with particles comprising an object is even much less than in Earth experiments with neutrinos.

In my discussions with physicists some of them proposed to avoid the above paradoxes by using an analogy with classical diffraction theory. Here it is assumed that in optical phenomena a wave falling on an object cannot penetrate inside the object. Then the wave far from the object does not change, right after the object the wave has a hole but when its length is much greater than the Rayleigh one the hole disappears and the wave is practically the same as without diffraction. Those results are natural from the point of view that classical waves consist of many almost pointlike particles.

Let us now consider an experiment where a photon encounters a classical object and the transversal width of the photon coordinate WF is much greater than the size of the object. By analogy with diffraction theory one might represent the photon WF as  $\psi = \psi' + \psi''$  where the support of  $\psi'$  is outside the object and the support of  $\psi''$  is inside the object. In contrast to diffraction theory, this decomposition is ambiguous because coordinates of the object have uncertainties. However, one might assume that the decomposition is valid with some accuracy. Then one might expect that after interaction with the object  $\psi'$  will not change and  $\psi''$  will be absorbed by the object. This statement can be formalized as follows.

Let the object be initially in the ground state  $\Psi_g$ . Then the initial WF of the system photon+object is  $\psi\Psi_g$ . The S-matrix acts on this state as

$$S(\psi\Psi_g) = S(\psi'\Psi_g) + S(\psi''\Psi_g) = \psi'\Psi_g + (\dots) \quad (10)$$

where (...) consists of states emerging after interaction. This expression describes the situation when the photon always interacts with the object but only a small part  $\psi''$  of the initial WF interacts while the major part  $\psi'$  remains intact. As a result of interaction, the photon will be either absorbed by the object or will pass the object. In the latter case the photon WF  $\psi'$  will have a hole by analogy with the behavior of waves after diffraction. Therefore in any case the photon cannot be detected in the geometrical shadow of the object.

Understanding whether or not Eq. (10) is acceptable is crucial for drawing



a conclusion on the above paradoxes. This expression can be justified if evolution is described by a Hamiltonian where interaction of the photon with the object is local. In fundamental quantum theories elementary particles are described by states in the Fock space. Let  $a(\mathbf{p})$  and  $a(\mathbf{p})^*$  be the annihilation and creation operators for a photon with the momentum  $\mathbf{p}$ . They satisfy the commutation relations

$$[a(\mathbf{p}'), a(\mathbf{p}'')] = [a(\mathbf{p}')^*, a(\mathbf{p}'')^*] = 0, \quad [a(\mathbf{p}'), a(\mathbf{p}'')^*] = \delta^{(3)}(\mathbf{p}'' - \mathbf{p}') \quad (11)$$

and if  $\Phi_0$  is the vacuum state then  $a(\mathbf{p})\Phi_0 = 0$ . If  $\chi(\mathbf{p})$  is the momentum WF then the particle state in the Fock space is  $\Phi_1 = \int \chi(\mathbf{p})a(\mathbf{p})^*d^3\mathbf{p}\Phi_0$ .

If  $\psi$  is the coordinate WF defined by Eq. (4) then the decomposition  $\psi = \psi' + \psi''$  corresponds to the decomposition  $\chi = \chi' + \chi''$  where

$$\chi'(\mathbf{p}) = \int \exp(-\frac{i}{\hbar}\mathbf{p}\mathbf{r})\psi'(\mathbf{r})\frac{d^3\mathbf{r}}{(2\pi\hbar)^{3/2}}, \quad \chi''(\mathbf{p}) = \int \exp(-\frac{i}{\hbar}\mathbf{p}\mathbf{r})\psi''(\mathbf{r})\frac{d^3\mathbf{r}}{(2\pi\hbar)^{3/2}} \quad (12)$$

Therefore the photon state in the Fock space can be represented as

$$\Phi_1 = \int [\chi'(\mathbf{p}) + \chi''(\mathbf{p})]a(\mathbf{p})^*d^3\mathbf{p}\Phi_0 \quad (13)$$

In quantum mechanics particles exist during the whole time interval  $t \in (-\infty, \infty)$  and evolution is defined by the interaction operator acting on the particle WF. This operator is local in coordinate space if in momentum space it acts on the particle WF  $\chi$  as the convolution operator. However, if annihilation and creation of particles are possible then evolution is described by operators acting not on particle WFs but on the operators  $a(\mathbf{p})$  and  $a(\mathbf{p})^*$ . In approximations when annihilation and creation is not important, evolution can be reformulated in terms of  $\chi$  only. For example, as noted in Sec. 1, in the approximation  $(v/c)^2$  the electron in the hydrogen atom can be described by the Dirac or Schrödinger equation.

However, Feynman diagrams contain only vertices with one photon. Therefore in any interaction the photon is first absorbed as a whole, in the intermediate state there is no photon, and in the case when the photon is reemitted this is a new photon. So in the case of interactions the evolution of the photon cannot be described by an equation where the photon WF exists during the whole time interval  $t \in (-\infty, \infty)$ , and the action of the evolution operator on photon states can be defined only in terms of  $a(\mathbf{p})$  and  $a(\mathbf{p})^*$ . As follows from Eq. (11), those operators are not local in coordinate space. So it is not possible that they act only on  $\psi''$  and do not act on  $\psi'$ .

In general, if  $\Psi$  is the WF of a system, and  $\Psi = \Psi_1 + \Psi_2$  is a decomposition of this function then evolutions of  $\Psi_1$  and  $\Psi_2$  will be independent of each other if the states  $\Psi_1$  and  $\Psi_2$  have at least one different conserved quantum number (e.g. angular momentum). However, in the decomposition  $\psi\Psi_g = \psi'\Psi_g + \psi''\Psi_g$  the states are not characterized by a different conserved quantum number and therefore evolutions of the different parts of the decomposition will not be independent.

The crucial difference between diffraction theory and the given case follows. In diffraction theory it is always known where different parts of the wave are. However, the photon does not have parts (roughly speaking, it is a point) and its WF describes only probabilities to find the photon at different points. Hence the fact that  $\psi'' \neq 0$  does not mean that the part  $\psi''$  of the photon is inside the object but means only that the probability to find the photon inside the object is not zero because this probability equals  $||\psi''||^2$ . Since this quantity is very small then with the probability very close to unity the photon will not interact with the object.

This expectation is also in the spirit of QED. Since in any interaction the initial photon will be first absorbed as a whole and there will be no photon in the intermediate state, the sizes of reemitted photons (if they are created) will be defined by the absorber, and after any interaction the WF of the object will not be  $\Psi_g$ . So there is no part of the photon which does not participate in the interaction, and after any interaction WFs of final photons will not have large transverse sizes anymore.

This is an illustration of the WF collapse: if the photon WF has a large size before interaction then, as a result of the WF collapse, after any interaction the WF cannot have a large size. The WF collapse is a pure quantum phenomenon and there is no analog of WF collapse in diffraction theory. The problem of the WF collapse is widely discussed in the literature (see e.g. Ref. [11] and references therein).

A possible reason why Eq. (10) might seem to be acceptable is that the decomposition  $\psi = \psi' + \psi''$  is implicitly (and erroneously) understood as breaking the photon into two photons with the WFs  $\psi'$  and  $\psi''$ . However, as noted in Sec. 1, such a decomposition does not mean that a particle is broken into two parts. Mathematically this is clear from the fact that the two-photon state

$$\Phi_{12} = const \int \int \chi'(\mathbf{p}') \chi''(\mathbf{p}'') a(\mathbf{p}')^* a(\mathbf{p}'')^* d^3 \mathbf{p}' d^3 \mathbf{p}'' \Phi_0 \quad (14)$$

fully differs from the state (13).

We conclude that the photon WF after interaction cannot be  $\psi'$  and, instead of Eq. (10), the result is

$$S(\psi \Psi_g) = c \psi \Psi_g + (\dots) \quad (15)$$

where  $1 - |c|^2$  is the probability of interaction. Since the probability is small, the quantity  $c$  is very close to unity and the photon will probably pass the objects without any interaction. In rare cases when interaction happens, the WF of any final photon will not have a cosmic size anymore. Such a photon can reach Earth only if its momentum considerably differs from the original one but this contradicts Theory A. So the assumption that the above paradoxes can be explained by analogy with diffraction theory is not justified.

If  $f(\mathbf{r}/r) \neq const$  then, as follows from Eqs. (8) and (9), the radial part of the WF is the same as in the spherically symmetric case and, as follows from the above discussion, the coordinate WF of the initial photon still has a cosmic size.

Therefore on its way to Earth the photon WF will also encounter stars, planets and other objects (even if they are far from the line connecting the star and Earth) and the same inconsistencies arise.

*In summary, since according to standard theory photons emitted by stars have coordinate wave functions with cosmic sizes, the above arguments indicate that the theory contradicts observational data.*

## 4 The reason of the paradox and its resolution

As noted in Sec. 1, *the results* of fundamental quantum theories are formulated only in momentum space without mentioning space-time at all. So for those theories the position operator is not needed. It is needed only in semiclassical approximation, and the choice  $i\hbar\partial/\partial\mathbf{p}$  is usually substantiated from the requirement of compatibility with this approximation. Let  $A$  be a physical quantity,  $\bar{A}$  be its mean value in some state and  $\Delta A$  be its uncertainty in this state. Then  $A$  is semiclassical if  $\Delta A \ll |\bar{A}|$ . In particular,  $A$  cannot be semiclassical if  $\bar{A} = 0$ .

Consider first the one-dimensional case. As argued in textbooks (see e.g. Ref. [4]), if the mean value of the  $x$  component of the momentum  $p_x$  is rather large, the choice  $i\hbar\partial/\partial p_x$  can be justified but it does not have a physical meaning in situations when  $p_x$  is small. This is clear even from the fact that if  $p_x$  is small then  $\exp(ip_x x/\hbar)$  is not a rapidly oscillating function of  $x$ .

Consider now the three-dimensional case. If all the components  $p_j$  ( $j = 1, 2, 3$ ) are rather large then all the operators  $i\hbar\partial/\partial p_j$  can have a physical meaning. A semiclassical wave function  $\chi(\mathbf{p})$  in momentum space describes a narrow distribution around the mean value  $\mathbf{p}_0$ . Suppose that coordinate axes are chosen such  $\mathbf{p}_0$  is directed along the  $z$  axis. Then the mean values of the  $x$  and  $y$  components of the momentum operator equal zero and the operators  $i\hbar\partial/\partial p_j$  cannot be physical for  $j = 1, 2$ , i.e. in directions perpendicular to the particle momentum. The situation when a definition of an operator is physical or not depending on the choice of coordinate axes is not acceptable. Hence standard definition of the position operator is not physical.

The effect of WPS in transverse directions discussed in Sec. 2 is a consequence of standard definition of the position operator in these directions. As shown in Sec. 3, this results in fundamental paradoxes in observations of stars. Therefore the inconsistent definition of the position operator in these directions is not a pure academic problem.

In Ref. [3] a new definition of the position operator has been proposed. In contrast to standard definition, the new operator does not depend on the choice of coordinate axes and is expected to be physical only if the *magnitude* of the momentum is rather large. As a consequence, WPS in directions perpendicular to the particle momentum is absent regardless of whether the particle is nonrelativistic or relativistic. Moreover, for an ultrarelativistic particle the effect of WPS is absent at all.

In this approach uncertainties of each component of the photon momentum and each component of the photon coordinate do not change with time. If those quantities can be treated as small then the photon can be (approximately) treated as a pointlike particle moving along classical trajectory. So uncertainties of photon coordinates never have a cosmic size and there can be no paradoxes discussed in Ref. [3] and Sec. 3.

As noted in Secs. 1 and 3, a very important point in understanding the paradoxes is that the wave function describes only amplitudes of probabilities. This is important in discussing other quantum phenomena. As an example, consider the double-slit experiment which is usually treated as a strong confirmation of standard quantum theory. A standard explanation is that parts of the wave function of an elementary particle projected on the slits pass the screen and interfere, and the remaining part is absorbed by the screen. However, in view of the above consideration, it is not consistent to treat the elementary particle by analogy with the classical wave different parts of which interact with the screen differently. The problem of understanding the experiment is very difficult because here the WF of the elementary particle does not have an anomalously large size and the particle strongly interacts with the screen.

### Acknowledgments

I am very grateful to Gregory Keaton for numerous important discussions and constructive criticism. The present work is essentially based on paper [3] and while working on that paper numerous discussions with Anatoly Kamchatnov and his criticism were very important. I am also grateful to Volodya Netchitailo for important remarks.

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