Is it possible to make all the components of the energy-impulse tensor positive in the Alcubierre warp drive?

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Abstract:

A solution of general relativity is presented that describes an Alcubierre [1] propulsion system in which it is possible to travel at superluminal speed while reducing the components of the energy-impulse tensor, which turn out to be positive, by an arbitrary value.

1 Introduction:

Alcubierre [1] in 1994 proposed a solution of the equations of general relativity which provides the only viable means to accelerate a spaceship up to superluminal velocities without using wormholes. A problem was soon identified: Pfenning [4] showed that the required energy is comparable to the total energy of the universe and that it is negative. Moreover he used quantum inequalities to show that this energy gets distributed at very short scale (about 100 times the Planck length) up to a multiplicative factor equal to the squared speed.Questa pubblicazione affronta il tentativo di rendere positiva l'energia nella zona di Pfenning,pur mantenendone la riduzione arbitraria di questa energia nella cosidetta bolla warp.Later Hiscock [10] proved the existence of an event horizon for superluminal travels which would imply the presence of Hawking radiation responsible for the rapid destruction of the spaceship[7].

Note: In the following we adopt the notation used by Landau and Lifshitz in the second volume ("The Classical Theory of Fields") of their well known Course of Theoretical Physics [13].

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We start with the metric

$$ds^2=\left(1-v(t)^2\frac{k^2}{b(z-h(t))^2}\right)dt^2+2v(t)\frac{k}{b(z-h(t))}dz\,dt-b(z-h(t))dx^2-b(z-h(t))dy^2-b(z-h(t))dz^2 \qquad (1)$$

- 1)-The Pfenning zone is the zone within the interval: $R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2}$ where $\Delta \ll 1$ R is the radius of the Warp bubble and Δ is the wall thickness of the Warp bubble $R \gg \Delta$.
- 2)-where $r=((z-h(t))^2)^{\frac{1}{2}}$ and $\frac{dh(t)}{dt}=v(t)$
- 3)- In the Pfenning zone we let $b(z-h(t)) \gg 1$ (there is the source of esotic matter)

Einstein tensor in contravariant form in the Pfenning zone is:

where $D=\frac{d}{dz}$ and $D^{(2)}=\frac{d^2}{dz^2}$

$$G^{tt}=\frac{1}{(-b(z-h(t))^4+b(z-h(t))^2v(t)^2k^2-v(t)^2k^2)^2}\Bigg(2\,b(z-h(t))^3\,D^{(2)}(b)(z-h(t))\,v(t)^2k^2$$

$$-D(b)(z-h(t))^2\,b(z-h(t))^2\,v(t)^2k^2-2k^2v(t)^2\,b(z-h(t))\,D^{(2)}(b)(z-h(t))-2k^2\,D(b)(z-h(t))^2\,v(t)^2$$

$$+3\,D(b)(z-h(t))^2\left(\frac{d}{dt}h(t)\right)^2 b(z-h(t))^6-2\,b(z-h(t))^5\,D^{(2)}(b)(z-h(t))+D(b)(z-h(t))^2\,b(z-h(t))^4\Bigg)$$

Equation 1. The fifth term is dominant, $v=const$,(condition 1-a)

$$G^{tz} = -\frac{1}{(-b(z-h(t))^4 + b(z-h(t))^2 v(t)^2 k^2 - v(t)^2 k^2)^2\, b(z-h(t))} \Bigg(v(t)^3 k^3 \mathrm{D}(b)(z-h(t))^2$$

$$- 2 k^2 v(t)^2 b(z-h(t))^4 \mathrm{D}^{(2)}(b)(z-h(t)) \left(\frac{d}{dt} h(t)\right) + 2 k^2 v(t)^2 b(z-h(t))^2 \mathrm{D}^{(2)}(b)(z-h(t)) \left(\frac{d}{dt} h(t)\right)$$

$$+ 2 k^2 v(t)^2 b(z-h(t)) \mathrm{D}(b)(z-h(t))^2 \left(\frac{d}{dt} h(t)\right) - 3 k\, v(t) \mathrm{D}(b)(z-h(t))^2 \left(\frac{d}{dt} h(t)\right)^2 b(z-h(t))^4$$

$$+ \mathrm{D}(b)(z-h(t))^2 b(z-h(t))^2 v(t)\, k + 2 b(z-h(t))^6 \mathrm{D}^{(2)}(b)(z-h(t)) \left(\frac{d}{dt} h(t)\right)$$

$$- 2 b(z-h(t))^5 \mathrm{D}(b)(z-h(t))^2 \left(\frac{d}{dt} h(t)\right) \Bigg)$$

Equation 2. The seventh therm is dominant $v = const$,(condition 1-a)

$$G^{xx} = \frac{1}{b(z-h(t))^4 (-b(z-h(t))^4 + b(z-h(t))^2 v(t)^2 k^2 - v(t)^2 k^2)^2} \Bigg(2 b(z-h(t))^9 \mathrm{D}(b)(z-h(t)) \left(\frac{d^2}{dt^2} h(t)\right)$$

$$- 2 \mathrm{D}^{(2)}(b)(z-h(t)) \left(\frac{d}{dt} h(t)\right)^2 b(z-h(t))^9 - v(t)^4 k^4 \mathrm{D}(b)(z-h(t))^2$$

$$- \mathrm{D}(b)(z-h(t))^2 \left(\frac{d}{dt} h(t)\right)^2 b(z-h(t))^8 + \mathrm{D}(b)(z-h(t))^2 \left(\frac{d}{dt} h(t)\right) b(z-h(t))\, v(t)^3 k^3$$

$$- \mathrm{D}(b)(z-h(t))^2 b(z-h(t))^6 - 2 \mathrm{D}(b)(z-h(t)) \left(\frac{d^2}{dt^2} h(t)\right) b(z-h(t))^7 v(t)^2 k^2$$

$$+ 2 b(z-h(t))^5 \mathrm{D}(b)(z-h(t)) \left(\frac{d^2}{dt^2} h(t)\right) v(t)^2 k^2 - k^3 b(z-h(t))^4 v(t)^3 \mathrm{D}^{(2)}(b)(z-h(t)) \left(\frac{d}{dt} h(t)\right)$$

$$+ b(z - h(t))^6\, D(b)(z - h(t)) \left(\frac{d}{dt} v(t) \right) k - D(b)(z - h(t))^2 \left(\frac{d}{dt} h(t) \right) b(z - h(t))^5\, v(t)\, k$$

$$+ b(z - h(t))^7\, D^{(2)}(b)(z - h(t)) + k^4\, D(b)(z - h(t))^2\, v(t)^4\, b(z - h(t))^2 - b(z - h(t))^5\, D^{(2)}(b)(z - h(t))\, v(t)^2\, k^2$$

$$+ 2\, D^{(2)}(b)(z - h(t)) \left(\frac{d}{dt} h(t) \right)^2 b(z - h(t))^7\, v(t)^2\, k^2$$

$$+ 2\, v(t)\, k^2\, D(b)(z - h(t)) \left(\frac{d}{dt} h(t) \right) b(z - h(t))^7 \left(\frac{d}{dt} v(t) \right)$$

$$+ 3\, v(t)^2\, k^2\, D(b)(z - h(t))^2 \left(\frac{d}{dt} h(t) \right)^2 b(z - h(t))^6 + b(z - h(t))^3\, D^{(2)}(b)(z - h(t))\, v(t)^2\, k^2$$

$$+ D(b)(z - h(t))^2\, b(z - h(t))^2\, v(t)^2\, k^2 + D^{(2)}(b)(z - h(t)) \left(\frac{d}{dt} h(t) \right) b(z - h(t))^2\, v(t)^3\, k^3$$

$$- 2\, D^{(2)}(b)(z - h(t)) \left(\frac{d}{dt} h(t) \right)^2 b(z - h(t))^5\, v(t)^2\, k^2 - 5\, D(b)(z - h(t))^2 \left(\frac{d}{dt} h(t) \right)^2 b(z - h(t))^4\, v(t)^2\, k^2$$

$$- 2\, b(z - h(t))^5\, D(b)(z - h(t)) \left(\frac{d}{dt} h(t) \right) v(t)\, k^2 \left(\frac{d}{dt} v(t) \right)$$

$$+ D^{(2)}(b)(z - h(t)) \left(\frac{d}{dt} h(t) \right) b(z - h(t))^6\, v(t)\, k - D(b)(z - h(t))^2\, v(t)^2\, k^2\, b(z - h(t))^4 \Bigg)$$

Equation 3. The second term is dominant $v = const$,(condition 1-a)

$$G^{yy} = \frac{1}{b(z - h(t))^4\, (-b(z - h(t))^4 + b(z - h(t))^2\, v(t)^2\, k^2 - v(t)^2\, k^2)^2} \Bigg(2\, b(z - h(t))^9\, D(b)(z - h(t)) \left(\frac{d^2}{dt^2} h(t) \right)$$

$$- 2\, D^{(2)}(b)(z - h(t)) \left(\frac{d}{dt} h(t) \right)^2 b(z - h(t))^9 - v(t)^4\, k^4\, D(b)(z - h(t))^2$$

$$- D(b)(z - h(t))^2 \left(\frac{d}{dt} h(t) \right)^2 b(z - h(t))^8 + D(b)(z - h(t))^2 \left(\frac{d}{dt} h(t) \right) b(z - h(t))\, v(t)^3\, k^3$$

$$- D(b)(z - h(t))^2\, b(z - h(t))^6 - 2\, D(b)(z - h(t)) \left(\frac{d^2}{dt^2} h(t) \right) b(z - h(t))^7\, v(t)^2\, k^2$$

$$+ 2\, b(z - h(t))^5\, D(b)(z - h(t)) \left(\frac{d^2}{dt^2} h(t) \right) v(t)^2\, k^2 - k^3\, b(z - h(t))^4\, v(t)^3\, D^{(2)}(b)(z - h(t)) \left(\frac{d}{dt} h(t) \right)$$

$$+ b(z - h(t))^6\, D(b)(z - h(t)) \left(\frac{d}{dt} v(t) \right) k - D(b)(z - h(t))^2 \left(\frac{d}{dt} h(t) \right) b(z - h(t))^5\, v(t)\, k$$

$$+ b(z - h(t))^7\, D^{(2)}(b)(z - h(t)) + k^4\, D(b)(z - h(t))^2\, v(t)^4\, b(z - h(t))^2 - b(z - h(t))^5\, D^{(2)}(b)(z - h(t))\, v(t)^2\, k^2$$

$$+ 2\, D^{(2)}(b)(z - h(t)) \left(\frac{d}{dt} h(t) \right)^2 b(z - h(t))^7\, v(t)^2\, k^2$$

$$+ 2\, v(t)\, k^2\, D(b)(z - h(t)) \left(\frac{d}{dt} h(t) \right) b(z - h(t))^7 \left(\frac{d}{dt} v(t) \right)$$

$$+ 3\, v(t)^2\, k^2\, D(b)(z - h(t))^2 \left(\frac{d}{dt} h(t) \right)^2 b(z - h(t))^6 + b(z - h(t))^3\, D^{(2)}(b)(z - h(t))\, v(t)^2\, k^2$$

$$+ D(b)(z - h(t))^2\, b(z - h(t))^2\, v(t)^2\, k^2 + D^{(2)}(b)(z - h(t)) \left(\frac{d}{dt} h(t) \right) b(z - h(t))^2\, v(t)^3\, k^3$$

$$- 2\, D^{(2)}(b)(z - h(t)) \left(\frac{d}{dt} h(t) \right)^2 b(z - h(t))^5\, v(t)^2\, k^2 - 5\, D(b)(z - h(t))^2 \left(\frac{d}{dt} h(t) \right)^2 b(z - h(t))^4\, v(t)^2\, k^2$$

$$- 2\, b(z - h(t))^5\, D(b)(z - h(t)) \left(\frac{d}{dt} h(t) \right) v(t)\, k^2 \left(\frac{d}{dt} v(t) \right)$$

$$+ D^{(2)}(b)(z-h(t))\left(\frac{d}{dt}h(t)\right) b(z-h(t))^6\, v(t)\, k - D(b)(z-h(t))^2\, v(t)^2\, k^2\, b(z-h(t))^4 \Bigg)$$

Equation 4. The second term is dominant $v = const$,(condition1-a).

$$G^{zz} = -\frac{1}{b(z-h(t))^2\,(-b(z-h(t))^4 + b(z-h(t))^2\, v(t)^2\, k^2 - v(t)^2\, k^2)^2}\Bigg(v(t)^4\, k^4\, D(b)(z-h(t))^2$$

$$- 2\, D^{(2)}(b)(z-h(t))\left(\frac{d}{dt}h(t)\right)^2 b(z-h(t))^5\, v(t)^2\, k^2 + 2\, b(z-h(t))^5\, D(b)(z-h(t))\left(\frac{d^2}{dt^2}h(t)\right) v(t)^2\, k^2$$

$$- 3\, D(b)(z-h(t))^2\left(\frac{d}{dt}h(t)\right)^2 b(z-h(t))^4\, v(t)^2\, k^2 - 2\, k^2\, v(t)^2\, D(b)(z-h(t))\left(\frac{d^2}{dt^2}h(t)\right) b(z-h(t))^3$$

$$+ 2\, k^2\, v(t)^2\, D^{(2)}(b)(z-h(t))\left(\frac{d}{dt}h(t)\right)^2 b(z-h(t))^3 + 2\, k^2\, v(t)^2\, D(b)(z-h(t))^2\left(\frac{d}{dt}h(t)\right)^2 b(z-h(t))^2$$

$$- 2\, b(z-h(t))^5\, D(b)(z-h(t))\left(\frac{d}{dt}h(t)\right) v(t)\, k^2\left(\frac{d}{dt}v(t)\right)$$

$$+ 2\, k^2\, v(t)\, D(b)(z-h(t))\left(\frac{d}{dt}h(t)\right) b(z-h(t))^3\left(\frac{d}{dt}v(t)\right) + 2\, D(b)(z-h(t))\, b(z-h(t))^4\left(\frac{d}{dt}v(t)\right) k$$

$$- 2\, D(b)(z-h(t))\left(\frac{d^2}{dt^2}h(t)\right) b(z-h(t))^7 + 2\, D^{(2)}(b)(z-h(t))\left(\frac{d}{dt}h(t)\right)^2 b(z-h(t))^7$$

$$+ D(b)(z-h(t))^2\left(\frac{d}{dt}h(t)\right)^2 b(z-h(t))^6 - D(b)(z-h(t))^2\, b(z-h(t))^4 \Bigg)$$

Equation 5. The twelfth term is dominant $v = const$,(condition1-a)

Being $b(z-h(t)) \gg 1$,the components of Einstein Tensor, and therefore of the components of the impulse-energy tensor, can be reduced by an arbitrary value.

` If:

` $D^{(2)} b(z-h(t))<0$, $|(D^{(2)} b(z-h(t)))\, b(z-h(t))| > (D\, b(z-h(t)))^2$ (1-a)

and $v = constant$, then:

$$G^{tt} > 0 \tag{2}$$

$$G^{xx} > 0 \tag{3}$$

$$G^{yy} > 0 \tag{4}$$

$$G^{zz} > 0 \tag{5}$$

- 1)-Internal metric of the Warp bubble $\left(0 < r < R - \frac{\Delta}{2}\right)$ is:

$$ds^2 = dt^2 - (dz - v dt)^2 - dx^2 - dy^2 \tag{6}$$

moving with velocity v (multiple of the speed of light c) along the z-axis.

- 2)-Metric outside of the bubble beyond the Pfenning zone $\left(r > R + \frac{\Delta}{2}\right)$ is:

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 \tag{7}$$

Einstein Equations:

$$G^{ik} = \frac{8 \pi G}{c^4} T^{ik} \qquad T^{ik} \text{ (inpulse-energy tensor)} \tag{8}$$

2 General considerations:

The general metric is:

$$ds^2 = (1 - v(t)^2 f(z-h(t))^2) dt^2 + 2 v(t) f(z-h(t)) dz\, dt - b(z-h(t)) dx^2 - b(z-h(t)) dy^2 - b(z-h(t)) dz^2 \quad (9)$$

The functions $f(r) = f(z-h(t))$ and $b(r) = b(z-h(t))$ assume the following values:

- 1)-inside the warp bubble $(0 < r < R - \frac{\Delta}{2})$, $b(r) = k$ and $f(r) = 1$
- 2)-outside the warp bubble $(r > R + \frac{\Delta}{2})$, $b(r) = 1$ and $f(r) = 0$
- 3)-in the Alcubierre warped region $(R - \frac{\Delta}{2} < r < R + \frac{\Delta}{2})$,is $f(r) = \frac{k}{b(r)}$ and

  $b(r) \gg 1$ possessing extremely large values.

From metric (9) we find again all results of (1), (6) and (7).

3 Consclusions: The calculations presented in this paper suggest that the Alcubierre propulsion system and its modification can be built even at superluminal speeds without giving rise to problems in the energy density, with positive components of the energy-impulse tensor. There is a problem: the light gets strongly slowed down while moving from the outside into the Pfenning zone, which contains exotic matter, and into the spaceship. This paper presents a possible, very simplified, modification of the Alcubierre metric based on a single spacial coordinate that addressed this problem.

4 Appendix:

The components of the Einstein tensor proportional to the energy-impulse tensor have been calculated with reference to an observer whose gravitational field is very weak and whose speeds are far lower than the speed of light, observing the spaceship and the warp bubble moving at speed v, i.e., an inertial reference frame in which the spaceship is moving at speed v If we want to calculate in the Eulerian reference frame, that is moving with the spaceship, we get for each component of the energy-impulse tensor in implicit form the following:

$$(energy\ density)=const\,G^{tt} \quad (9)$$

$$(impulse\,density\ z)=const\left[G^{tt}\,v\,f-G^{tz}\right] \quad (10)$$

$$(stress\ xx)=const\,G^{xx} \quad (11)$$

$$(stress\ yy)=const\ G^{yy} \quad (12)$$

$$(stress\,zz)=const\left[G^{tt}\,v^2\,f^2-2\mathrm{v}\,f\,G^{tz}+G^{zz}\right] \quad (14)$$

The various stress xx, stress yy, stress zz are the components of the stress tensor [12], and $const=\frac{c^4}{8\pi G}$; c is the speed of light and G is Newton's gravitational constant; where f in Pfenning zone is

$$\frac{k}{b(z-h(t))} \quad (19)$$

where the energy-impulse tensor is not zero.

As can be seen, the components can be reduced arbirarily by choosing $b(z-h(t))\gg 1$. The function $b(z-h(t))$ is not discussed in this paper, as its purpose is only to illustrate a quite simplified and theoretica possibility which is only valid in an extreme solution.

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