Closed and Symmetric Solution for Planetary Motion Equation of General Relativity

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Abstract: Although many scholars consider that original non-closed and non-symmetric solution for planetary motion equation of general relativity can be used to solve the problem of advance of planetary perihelion, it has some shortcomings really. This paper discusses the closed and symmetric solution for planetary motion equation of general relativity, and that means that general relativity cannot be used to solve the problem of advance of planetary perihelion. Also, if the solution for planetary motion equation of general relativity is unique, then the non-closed and non-symmetric solution including infinite terms should tend to the closed and symmetric solution. According to "partial and temporary unified theory of natural science so far" including all the equations so far related to natural science and its variational principle, the optimization method can be used to reach the optimal approximate closed and symmetric solution for problem of advance of planetary perihelion. On the basis of the closed and symmetric solution, the planetary orbit should be a rotating ellipse or a rotating approximate ellipse; namely the advance of planetary perihelion is the combined result of two motions: the first motion (the elliptical motion decided by law of gravity or the closed and symmetric approximate elliptical motion decided by planetary motion equation of general relativity) creates the perihelion, and the second motion (solar system's vortex motion) creates the advance of perihelion; while the parameters of advance will be decided by general relativity or accurate astronomical observation.

Key words: General relativity, planetary motion equation, closed and symmetric solution, partial and temporary unified theory of natural science so far, variational principle, advance of planetary perihelion, rotating ellipse

1 Introduction

Although general relativity can be used to solve the problem of advance of planetary perihelion, it contains some subjective factors; one of the most critical question is that for planetary motion equation of general relativity, the non-closed and non-symmetric solution is presented deliberately. Similarly, the closed and symmetric solution can also be presented deliberately. In this way, general relativity will be not available to deal with the problem of advance of planetary perihelion, and the new approach should be found.

2 Shortcomings of Solution for Planetary Motion Equation of General Relativity

According to general relativity, the planetary motion equation reads

$$u''+u=\frac{1}{p}+Ku^2 \quad (1)$$

where: $u=\frac{1}{r}$ **;** $K=\frac{3GM}{c^2}$ **; G – gravitational constant; M – mass of sun; c – speed of light; p - half normal focal chord, for ellipse: p = a(1-e^2).**

The non-closed and non-symmetric solution for Eq.(1) is as follows

$$u\approx\frac{GM}{a(1-e^2)c^2}[1+e\cos(1-\frac{3GM}{a(1-e^2)c^2})\varphi] \qquad (2)$$

It gives the value of advance of planetary perihelion for one circuit as follows

$$\varepsilon=\frac{24\pi^3a^2}{T^2c^2(1-e^2)} \qquad (3)$$

where: c is the speed of light; T, a, and e are orbital period, semi-major axis and eccentricity respectively.

From Eq.(2) we can see that, if considering the approximate solution including two terms only, general relativity can reach the better results as solving the problem of advance of planetary perihelion. But considering the approximate solution including more terms, whether or not general relativity can be used to deal with the problem of advance of planetary perihelion? This is a further topic needed to be studied carefully.

In contrast, according to Eq.(1), whether or not the closed and symmetric approximate solution can be reached? Now we will discuss this problem.

2 Closed and Symmetric Solution for Planetary Motion Equation of General Relativity

Referring to the planetary elliptical motion equation given by law of gravity, the closed and symmetric approximate solution for Eq.(1) can be taken as follows

$$u=\frac{1+e\cos\varphi}{p}+b_0+b_1\cos\varphi+b_2\cos2\varphi+.. \qquad (4)$$

Obviously, it satisfies the following symmetric condition

$$f(\varphi)=f(-\varphi) \qquad (5)$$

Firstly, we discuss the following closed and symmetric approximate solution including one undetermined constant only

$$u=\frac{1+e\cos\varphi}{p}+b_0 \qquad (6)$$

Substituting Eq.(6) into Eq.(1), omitting the higher order terms and keeping the linear terms only; then comparing the related coefficients, and it gives

$$b_0=\frac{K(1+e^2/2)}{p^2(1-2K/p)} \qquad (7)$$

Secondly, we discuss the following closed and symmetric approximate solution including two undetermined constants

$$u = \frac{1 + e\cos\varphi}{p} + b_0 + b_1 \cos\varphi \tag{8}$$

Substituting Eq.(8) into Eq.(1), omitting the higher order terms and keeping the linear terms only; then comparing the related coefficients, and it gives

$$b_0 = \frac{K(1 - e^2/2)}{p^2(1 - 2K/p + Ke^2/p)} \tag{9}$$

$$b_1 = -e/p - b_0 e \tag{10}$$

Similarly, the closed and symmetric approximate solution including more undetermined constants can be reached.

It should be noted that, the values of b_0 in Eq.(7) and Eq.(9) are different; therefore it can be inferred that, according to Eq.(1), if choosing the non-closed and non-symmetric approximate solution including more undetermined constants, the value of advance of planetary perihelion may be different from the one given by Eq.(3).

In addition, if the solution for planetary motion equation of general relativity (Eq.(1)) is unique, then the non-closed and non-symmetric solution including infinite terms should tend to the closed and symmetric solution Eq.(4).

3 Optimal Approximate Closed and Symmetric Solution for Planetary Motion Equation of General Relativity

According to "partial and temporary unified theory of natural science so far" including all the equations so far related to natural science and its variational principle, namely "partial and temporary unified variational principle of natural science so far", the optimization method can be used to reach the optimal approximate closed and symmetric solution for Eq.(1).

In order to deal with all natural science issues in a unified way, in reference [1], "partial and temporary unified theory of natural science so far" is presented with the least squares method, and it can be expressed by the following form of "partial and temporary unified variational principle of natural science so far".

$$\Pi_{NATURE} = \sum_1^n W_i \int_{\Omega_i} F_i^2 d\Omega_i + \sum_1^m W_j' S_j^2 = \min_0 \tag{11}$$

where: $\min_0$ was introduced in reference [2], indicating the minimum and its value should be equal to zero. W_i and W_j' are suitable positive weighted constants; for the simplest cases, all of these weighted constants can be taken as 1. If only a certain equation is considered, we can only make its corresponding weighted constant is equal to 1 and the other weighted constants are all equal to 0. The subscript NATURE denotes that the suitable scope is all of the problems of natural

science; all of the equations $F_i = 0$ denote so far discovered (derived) all of the equations related to natural science (they can be run the integral operation), and all of the equations $S_i = 0$ denote so far discovered (derived) all of the solitary equations related to natural science (they cannot be run the integral operation).

In this way, the theory of everything to express all of natural laws, described by Hawking that a single equation could be written on a T-shirt, is partially and temporarily realized in the form of "partial and temporary unified variational principle of natural science so far".

In variational principle Eq.(11), if we only consider the equation of planetary motion around the Sun according to the law of gravity, it gives the following variational principle

$$\Pi_1 = \int_0^{2\pi} (u''+u-\frac{1}{p})^2 \mathrm{d}\varphi = \min 0 \qquad (12)$$

Its accurate solution reads

$$u = \frac{1+e\cos\varphi}{p} = \frac{1+e\cos\varphi}{a(1-e^2)} \qquad (13)$$

In variational principle Eq.(11), if we consider the planetary motion equation of general relativity (Eq.(1)) and its closed and symmetric solution Eq.(4), it gives the following variational principle

$$\Pi_2 = \int_0^{2\pi} (u''+u-\frac{1}{p}-\frac{3GMu^2}{c^2})^2 = \min 0 \qquad (14)$$

According to this variational principle, with the help of optimization method, the optimal approximate closed and symmetric solution in the form of Eq.(4) can be reached.

For this method, it has been used in reference [2]. However, the optimal approximate non-closed and non-symmetric solution in the form of Eq.(2) was discussed in reference [2] (the integral interval is $[0, 2\pi+\varepsilon]$), while in this paper we discuss the optimal approximate closed and symmetric solution in the form of Eq.(4) (the integral interval is $[0, 2\pi]$).

4 Planetary Orbit Based on Closed and Symmetric Solution

In reference [3], we present that the advance of planetary perihelion is the combined result of two motions.

The first elliptical motion creates the perihelion, and the second vortex motion creates the advance of perihelion.

In the first motion of planet-sun system, under the action of law of gravity, the planetary orbit is a closed ellipse, and consistent with the law of conservation of

energy. Meanwhile, the planet also participates in the vortex motion of solar system taking the sun as center; the long-term trend of the vortex is the further topic, but in the short-term may be considered that due to the inertia the planetary perihelion will run circular motion in vortex and lead to the advance of perihelion, thus also without acting against the law of conservation of energy. Based on the result of general relativity, the approximate angular velocity of advance of perihelion is given.

In a word, the proposed new explanation of combined motion in reference [3] does not run counter to the law of conservation of energy from start to finish.

Now we discuss the angular velocity of advance of planetary perihelion according to the result of general relativity.

According to Eq.(3), taking the sun as center, the angular velocity of advance of planetary perihelion is as follows

$$\omega = \frac{\varepsilon}{T} = \frac{24\pi^3 a^2}{T^3 c^2 (1-e^2)} \qquad (15)$$

According to Kepler's third law, it gives

$$\frac{T^2}{a^3} = \frac{4\pi^2}{GM}$$

where: G is the gravitational constant, and M is the solar mass.

Then Eq. (15) can be rewritten as

$$\omega = \frac{3G^{3/2} M^{3/2}}{a^{5/2} c^2 (1-e^2)} \qquad (16)$$

According to this expression we can see that, the angular velocity of advance of planetary perihelion is inversely proportional to $a^{5/2}$, and the velocity of advance of planetary perihelion is inversely proportional to $a^{3/2}$.

For the results of Eq.(3), there are small differences compared with accurate astronomical observations, so we say that the result of Eq.(16) is the approximate angular velocities of advance of perihelion based on the related result of general relativity.

If based on accurate astronomical observation, we can reach the accurate angular velocity of advance of perihelion as follows.

$$\omega' = \frac{\varepsilon'}{T}$$

where: ε' is the accurate astronomical observation of advance of perihelion.

Now the rotate transformation in Cartesian coordinate system is applied to derive the planetary orbit equation including the advance of perihelion.

For the sake of convenience, we only discuss the case of elliptical orbit, but the method is also applicable to discuss the case of approximate elliptical orbit as shown by Eq.(4).

In the planet-sun system, taking the solar center as the origin of coordinate, the planetary elliptical orbit equation reads

$$\frac{(x-k)^2}{a^2}+\frac{y^2}{b^2}=1$$

where: k is the semi-focal length of ellipse.

According to the rotate transformation in Cartesian coordinate system, it gives

$$x=x'\cos\theta-y'\sin\theta$$

$$y=x'\sin\theta+y'\cos\theta$$

where: θ is the angle of rotation (namely the angle of advance), $\theta=\omega t$ or $\theta=\omega' t$.

Thus, after considering the vortex motion, the planetary rotation orbit equation is as follows

$$\frac{(x'\cos\theta-y'\sin\theta-k)^2}{a^2}+\frac{(x'\sin\theta+y'\cos\theta)^2}{b^2}=1$$

5 Conclusions

For the planetary motion equation of general relativity, the closed and symmetric solution is existed, and therefore general relativity cannot be used to deal with the problem of advance of planetary perihelion. But the result of the value of advance of planetary perihelion given by general relativity can be combined with the closed and symmetric solution derived by planetary motion equation of general relativity to reach valuable result. Namely, the advance of planetary perihelion is the combined result of two motions: the first motion (the elliptical motion decided by law of gravity or the approximate elliptical motion decided by the planetary motion equation of general relativity) creates the perihelion, and the second motion (solar system's vortex motion) creates the advance of perihelion; while the parameters of advance will be decided by general relativity or accurate astronomical observation.

The further topics are as follows: More detail discussion on the closed and symmetric solution for planetary motion equation of general relativity; Deriving the new planetary motion equation to replace the one given by general relativity; Finding the method to partially replace relativity (the preliminary results can be seen in reference [4]); and so on.

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