On the Size of a Photon

Herbert Weidner [a]

Abstract: After discussing various examples, the diameter of a single photon is calculated by combining the formulas of quantum mechanics and wave theory. The experimentally known coherence length is the length of the photon.

History

Is light an electromagnetic wave or a stream of photons? About 350 years ago, Gassendi suspected a stream of corpuscles, Newton reaffirmed this idea. Although no one was able to provide experimental evidence, this assumption held for about 150 years. In 1800, the idea of light changed abruptly when Thomas Young showed first interference phenomena. Now light was a transverse wave whose electromagnetic character was discovered by Maxwell. This classic finding experienced a few years of success, because Max Planck recognized in 1900 that these waves could gain or lose energy only in finite amounts related to their frequency. Planck called these "lumps" of light energy "quanta". Within the framework of classical physics that was not only revolutionary, it was incomprehensible. Physics experienced a radical change in the way of thinking.

In 1905, Albert Einstein published an idea that came immediately with fierce resistance, but was nevertheless accepted 15 years later. He proposed that these light quanta had a "real" existence. Later they were called Photons. There are some experiments to measure the diameter of a photon by means of tiny holes in the metal layers[[1]].

Today, many physicists believe that photons actually exist, but they always emphasize that we must *not* imagine photons as small particles, such as spheres or bullets. Most properties are set by theory, but textbooks never mention the dimensions of photons and are very silent on this point. Nevertheless, some physicians claim *without experimental proof* that photons are point-like, without any relation to the wavelength. Do experimental facts support this view? Are there any observations which contradict this assertion?

Consider the famous double-slit experiment, carried out with *single* photons and not with an expanded beam of laser light. No broad bundle of phase-locked waves, several millimeters wide. How can a dot-shaped photon realize that there exists a second gap at a lateral distance of many wavelengths? Can the distance be any size, regardless of the wavelength and without spoiling the interference pattern? An answer to this question would allow a rough estimate of the size of a photon.

Interrelation of experiment and theory

Experiments alone provide unsystematic, incoherent garbage. A simple theory can establish relationships that were previously unknown. A good theory leads to at least one surprising and unexpected prediction that can be confirmed experimentally. The invention of theories without experimental basis and without testable predictions is worthless, a collection of unproven claims. Physics depends on observations, measurements and detection of patterns. Based on those results, theorists try to find useful formulas to calculate the results of future measurements. Every inexplicable and reproducible divergence may contribute to the improvement of the theory.

The requirement for manageable formulas requires to confine the theory to clearly defined parts of physics. Outside of their narrow range, they provide either no or wrong solutions. Here, we talk about electromagnetic waves in the region well below X-rays.

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a) v2, 24. September 2015, email: herbertweidner@gmx.de

On an atomic scale, Quantum mechanics (QM) allows to calculate the results of many experiments in advance with amazing accuracy. The results are correct, but often contradict common sense. Quantum Optics is a branch of physics describing the optical phenomena of light. Normally, the experiments are done near the wavelength of visible light, even though the total spectrum of electromagnetic radiation extends far beyond both sides. The limits of those experiments are determined by the size of atoms. On the one hand, the resonators and energy detectors used can not be smaller than several hundred atoms. Therefore, wavelengths shorter than 10 nm are seldom used. On the other hand, electronic rectifiers for frequencies above 10^{11} Hz still do not exist. At higher frequencies, neither amplitude nor phase of electromagnetic waves can be measured, only energies.

Below 10^{11} Hz there are a lot of experimental results, which can not be treated with QM. To construct a sensitive receiver for radar, radio astronomy or satellite communication, QM can not contribute a single solution, nothing. In high-frequency technology below 10^{11} Hz, everything is calculated with Maxwell's equations. This set of formulas provides accurate predictions in the field of electromagnetic waves. Not a single fact gets clearer if the word photon is used.

Some very known experiments with light can only (or better) be explained by electromagnetic waves: In the field of light, no instrument can measure the amplitude (not intensity!) of electromagnetic waves. But without exception, *all results* in nonlinear optics are explained using the terms *amplitude* and *frequency.* Throughout the whole WP article Planck's constant is not even mentioned and the term "photon" is obviously superfluous in this context[b].

In 1956, the "Hanbury Brown and Twiss effect" was predicted and quite easily explained with the aid of Maxwell's equations and eventually discovered. From the perspective of QM, the HBT effect was doubted and treated with hostility, although it was experimentally demonstrated. The rules of QM prohibited explicitly that the HBT experiment could work. But QM was faulty and it took many years to expand the formula apparatus of QM so that finally the HBT effect could be explained[2]. The quantum mechanical explanation of the HBT effect is difficult to understand and considerably more complex than the explanation by means of electromagnetic waves. But anyhow, the QM is no longer left empty-handed. Probably future experimental facts will require further supplements of the QM.

The same applies to the double-slit experiment. It was first performed in 1801 and immediately explained convincingly by means of the wave theory. A QM explanation has been proposed only in 1995, 70 years after the construction of the QM. While reading, one thinks of Feynman resigned remark: “I couldn't reduce it to the freshman level. That means we really don't understand it."

The Limits of Theories

The four formulas of Maxwell's equations link only electrical measurement values and therefore can not provide statements about, for example, gravity or energy. They describe error free all phenomena of electromagnetic waves including generation (near field), propagation (far field) and detection by antennas.

Max Planck discovered that the energy of electromagnetic waves at very low energies may not be changed continuously, but only in steps. This enforced the development of quantum mechanics. QM focuses on energy levels, moments and probabilities, but contains no information on electric fields or gravity. (The “waves” of the Schrödinger equation have nothing to do with the electromagnetic waves. It is questionable whether they exist at all, or whether they are just a mathematical trick.)

Because the QM does not contain methods to describe electromagnetic waves, Einstein invented photons. Their description is very scarce: photons are electrically neutral particles, generated at *A* with the energy *hf* and absorbed with exactly the same energy in *B*. Point. (The redshift was not discovered in 1905). Not a word about how big they are and how they look. No description of route

b If light consists of electrically neutral photons, how can the concentration of a huge number of photons generate an electric field?

and speed between *A* and *B*. No distinction between near and far field. No instructions on how to deal with well known phenomena such as interference or refraction, because they were not required to explain the photoelectric effect. Einstein's speculation is a good approximation, if you transport visible light over very short distances inside a laboratory. But is it accurate enough at much lower frequencies and very large distances? Was that ever checked?

Because QM is limited to verifiable statements about birth and death of a photon, it provides no explanation for the SPDC, where occasionally a photon will be splitted halfway to the detector for a enigmatic an unknown reason. Not just anywhere, but only inside a carefully oriented BBO crystal. And this photon must be accompanied by many, many billions of other photons. Why does the conversion require a nonlinear BBO crystal and a very high number of photons? How are non-linearity and phase adjustment described in the language of QM? Actually, the splitting should also happen at much lower frequencies, because in the field of radio waves nonlinearities (called mixer) are very easy to implement and require much less power in the range of microwatts.

Probably the biggest shortcoming in the current rule set of QM is that it describes only what can be measured *after* the annihilation of a photon. In QM, “living” photons do not exist. QM resembles archeology: A collection of grave stone inscriptions hardly allows statements about the the living.

110 years after the invention of Photons, the question arises whether the reduction of electromagnetic waves on energy and angular momentum is not an oversimplification. For some tasks the few quantum numbers of photons may be enough, they are undoubtedly correct. But they may not be sufficient for many other tasks, because the QM does not have appropriate tools to perform calculations of speed, size and electromagnetic fields.

Two Worlds

When analog, finely stepped values of amplitude and phase can be measured or any kind of interference, use wave theory. The Maxwell equations are a set of partial differential equations, expecting readings without discontinuities. The wavelength should be much greater than the spacing of atoms. The wave theory provides detailed answers to questions like: How is the wave generated and propagates between transmitter and receiver and how is the path affected by obstacles. All properties of these transverse waves are well known and checked. But this formula set does not care about energy.

QM provides information about the *probability amplitudes* of position (no exact position!) and momentum and some other physical properties like the spin of a particle. But this theory does not tell us, how a photon looks like. There are experimental efforts to gain some insight. In contrast to the rectifiers of high frequency technology, the detection devices of Quantum Optics are nonlinear (square law detectors) and count individual photons, provided they exceed a certain energy threshold. This minimum energy is limited by the average thermal energy inside the detectors material. At room temperature, only photons with a maximum wavelength of about 10 µm may be measured, because - technically speaking - a photo-diode is a digital receiver with poor signal-to-noise ratio. QM makes no statement about the way of the photon between transmitter and receiver. When a photon is registered, the route can not be reconstructed. Some researchers conclude: That's impossible, because QM has no solution. But originally, QM had also no answer to the HBT effect.

Unfortunately, the two worlds do not overlap, which is why comparative measurements are impossible. Instead, there is a broad gap between these areas, the terahertz gap. Here, viable methods of measurement do not exist.

Estimating the Size of Photons

QM insists that Photons are uncharged elementary particles with zero rest mass. In a vacuum, they move at the speed of light. Size and shape could not be measured in the past 110 years and there are no substantiated estimates. In the following section, the term "electromagnetic wave" is avoided and replaced by "photon" (independent of frequency and wavelength).

Observation 1: A sieve is a simple tool to separate a mixture of different sized parts. A prerequisite is that the particles do not adhere to the sieve. Photons satisfy this requirement, because they are uncharged and can't be influenced by electrical currents or magnetic fields.

In almost every kitchen there is a microwave oven, containing two distinct photon sources: A weak lamp with low power and a magnetron transmitter with very high power. Both generate photons (NO electromagnetic waves!). The weak lamp generates much fewer photons per second than the magnetron because in the range of visible light each photon carries much more energy than one microwave photon.

The door of the cooking chamber usually has a window for easy viewing, with a layer of conductive mesh. Very few of the numerous microwave Photons can pass through the holes of the door, while the photons of visible light (with its much shorter wavelength) can. Why is there a difference when all the photons are point-shaped and only differ in the energy content? Whoever speaks of induced currents in the metal screen must also explain how *electrically neutral* photons can induce currents and may be influenced by them.

A simpler and more convincing explanation would be: The volume of photons increases with increasing wavelength and at a wavelength of 12 cm, the cross-sectional area of the photons is larger than the gaps of the metal sieve. Therefore, the “thick” photons can no longer slip through, only the much “thinner” photons of visual light.

Observation 2: Optical fibers permit data transmission over longer distances and at higher rates than wire cables. If light is stream of point-like photons, why should the required diameter of the core be larger than the wavelength? Why are the optimal diameter of core and cladding not calculated using the laws of quantum mechanics, but with the Maxwell equations? These differential equations do not meet the requirements of the QM. Does that mean that the fiber does not guide not photons but ordinary electromagnetic waves? QM does not even provide a reason for the total internal reflection.

Observation 3: A television transmitter (f = 0.6 GHz) generates a lot of photons, which are to be guided to the antenna by a long cylindrical waveguide. Why must the pipe diameter be larger than the minimum value 29.3 cm? If the diameter is too small, *all* the TV-photons are reflected to the transmitter, without exception. The tube does not contain any disturbing or absorbing elements or even mirrors. But not a single TV-photon arrives at the antenna 170 m above, although one can look through the voluminous tube. Where, why and how are the *uncharged* photons forced to turn back?

In none of the three examples, QM provides some sort of reason for the peculiar behavior of the photons. This theory has no methods to describe the path of a photon between source and destination. QM also has no way to describe size and shape of a photon. The reason for the construction of quantum mechanics was the handling of the quantized energy.

Calculating the Diameter of one Photon

In quantum optics, light may be considered not only as an electromagnetic wave but also as a "stream" of particles called photons which travel with c, the vacuum speed of light. A photon should not be considered to be a classical billiard ball, but as a quantum mechanical particle described by a wave function spread over a finite region. At this point it should be noted that there are two kinds of wave packets with very different time behavior.

- The wave packet of a *massive* particle (matter wave, proposed by Louis de Broglie) is dispersive. For non-relativistic velocities, the dispersion relation $\omega = h k^2/(4\pi m)$ applies. This non-linearity $\omega \propto k^2$ leads to wave-packet spreading.
- Since a photon has *no rest mass*, its wave packet is *non-dispersive*, the shape does *not* change. For plane waves in vacuum and no geometric constraint, the dispersion relation $\omega = c \cdot k$ is linear and therefore, the shape of the wave packet is preserved.

In physics, one photon of monochromatic light is described in two very different ways. From the perspective of quantum mechanics, a photon has the energy *hf*, the linear momentum *hf/c* and angular momentum (spin) *h/2π*.

From the perspective of the wave theory, the description is more complicated for two reasons.

- The Maxwell equations contain no equation for energy. This requires a detour via the energy density.
- The standard approach *A·sin(ωt)* for a wave is mathematically convenient, but it is a physically wrong starting point, because it corresponds to an *infinite* extended wave field with *infinite* energy. But a photon contains the smallest possible amount of energy and can't extend unlimited.

A wave packet with a limited volume solves both problems, but the shape is unknown. In the score of electromagnetic waves, the spin *h/2π* corresponds to a circularly polarized wave. The formulas for the moment and spin for this type of wave are known (A single photon can never be linearly polarized[3]). Since QM and wave theory shall describe the same object, the corresponding equations must be equated and yield a surprisingly simple result[4]. The radius of an elementary wave packet, called photon, is

$$R = \frac{3\sqrt{2}\,c}{4\pi f} = \frac{3\sqrt{2}\,\lambda}{4\pi} \approx 0.338 \cdot \lambda$$

As long as the wave packet is undisturbed, the angular momentum, the energy *hf* and the radius *R* are constant. This can be checked experimentally. In previous experiments[5] with microwaves, a value was measured, which is almost exactly twice. One possible cause for this deviation is the use of rectangular instead of circular metal apertures. In 1944, Bethe[6] proposed that for a plane wave, the transmission efficiency through a small hole should be proportional to $(R/\lambda)^4$. For the radius calculated above, a value below 1 % is expected. But the measured transmission coefficient of light through cylindrical holes in metal surfaces[7] is much higher than expected (~70 %). For structured holes of this size, the transmission is also a function of polarization[8]. From this we must conclude that the adoption of a planar light wave is obviously wrong.

The calculation of the radius is based on the assumption that the electric and magnetic field intensities are constant within the wave packet and vanish outside everywhere. This abrupt change is mathematically convenient, but probably not too realistic. Presumably, the field strength is at its maximum near the center of the wave packet maximum and gradually descends with increasing distance from the axis of symmetry. For this reason, the diameter of the wave packet can not be precisely defined, but the main fraction of the energy should be contained in a cylinder, whose diameter is smaller than a wavelength.

The very small diameter of *one* photon does not contradict the results of certain optical effects. The characteristic transverse dimensions of known diffraction phenomena as Arago spot, Fresnel zone plate and double-slit experiment are substantially larger than a wavelength. But the well known diffraction patterns are never observed using single photons. When illuminated with very many photons (coherent plane waves), additional properties of light emerge, which can not be observed with single photons.

Estimating the Length of one Photon

Unfortunately, the calculation above does not provide any information about the length of an electromagnetic wave packet. At high intensity, different wave packets overlap, whereby a determination of the coherence length is at least difficult (Exception is laser light with phase-locked wave packets of most photons). At low intensity, it is very unlikely that any wave packets overlap. Then, their length can be measured independently.

Relying on numerous experimental data, a simple relationship links the coherence length L of a wave packet named photon and the line width Δf of the spectral frequency: $L \cdot \Delta f = c$

Radio astronomers measure the Full width at half maximum of the well known HI line (λ = 21 cm), resulting in Δf = 20 kHz or less, corresponding to $L \approx$ 15000 m or more. It is puzzling how a lonely, tiny hydrogen atom in a hydrogen cloud of very low density can produce a coherent wave train, which is at least 71000 wavelengths long[9][10].

Precision measurements of the sodium D line[11] yield a FWHM of about 61.4 MHz or $L \approx$ 4.8 m, corresponding to $8.3 \cdot 10^6$ wavelengths.

In both cases a *single*, undisturbed atom generates a *single* photon. The Heisenberg uncertainty principle $\Delta E \cdot \Delta t \geqslant h$ says that an accurate energy information is necessarily linked with a large amount of time and vice versa. This principle interconnects the frequency tolerance $\Delta E = h \cdot \Delta f$ of a photon with the minimum length $L = n \lambda$ of the electromagnetic wave, leading to the result $n \geqslant \frac{f}{\Delta f}$, also known as Küpfmüller's uncertainty principle. The wave packet of a photon with a defined wavelength can not be shorter.

Lasers can create ultrashort pulses of light. The time duration of those electromagnetic pulses is of the order of a picosecond (10^{-12} second) or less, leading to a broadband optical spectrum. A laser pulse is not monochromatic, has no "color" and corresponds to white light without definable wavelength. The claim[5] that a photon with a defined wavelength is a soliton is physically untenable and contradicts all experimental observations.

Summary

A photon is born as an electromagnetic wave packet with the energy hf and the spin $\hbar$. During its flight, a photon is an approximately cylindrical, circularly polarized wave packet with the diameter of roughly 0.67 λ and a minimum length of about 100,000 λ. This elongated, very thin wave packet retains its shape until it is completely absorbed, leaving the energy hf. Even if the wave packet is modified while it is flying with the speed of light, it has all properties of an electromagnetic wave whose possible modifications (including nonlinear effects) are fully described with the aid of Maxwell's equations.

[1] T. W. Ebbesen, H. J. Lezec, H. F. Ghaemi, T. Thio, and P. A. Wolff, “Extraordinary optical transmission through sub-wavelength hole arrays,”Nature 391, 667–669 (1998).
[2] Silva, The Controversy on Photons and the Hanbury Brown & Twiss Experiment, 2010, http://quantum-history.mpiwg-berlin.mpg.de/news/workshops/hq3/hq3_talks/23_silva.pdf
[3] Alan M. Kadin, Steven B. Kaplan, Is a Single Photon Always Circularly Polarized?, IEEE J. Sel. Topics, http://arxiv.org/abs/1407.2605
[4] H. Weidner, The Size and Energy Loss of a Wave Packet, 2014 https://www.researchgate.net/publication/265013965_The_Size_and_Energy_Loss_of_a_Wave_Packet
[5] G. Hunter, M. Kowalski, C. Alexandrescu, Einstein’s Photon Concept Quantified by the Bohr Model of the Photon, http://arxiv.org/pdf/quant-ph/0506231.pdf
[6] H. Bethe, "Theory of diffraction by small holes," Physical Review, vol. 66, pp. 163-182, 1944
[7] F. Miyamaru, M. W. Takeda, T. Suzuki, C. Otani, Highly sensitive surface plasmon terahertz imaging with planar plasmonic crystals, OSA 2007, http://www.riken.jp/ExtremePhotonics/TH_imaging/s_no10.pdf
[8] A. Krasavin, A. S. Schwanecke, N. I. Zheludev, Polarization conversion and ”focusing” of light propagating through a small chiral hole in a metallic screen, http://arxiv.org/pdf/cond-mat/0501734.pdf
[9] P. Kalberla, The HI Distribution of the Milky Way, 2009 http://www.ifa.hawaii.edu/users/jpw/ism/reading/kalberla_mw_HI.pdf
[10] The HI 21 cm Line, http://www.cv.nrao.edu/course/astr534/HILine.html
[11] D. Steck, Sodium D Line Data, LANL 2003, http://steck.us/alkalidata/sodiumnumbers.1.6.pdf