Toward a proof of Legendre conjecture

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Legendre conjecture states that there is always at least a prime between two consecutive squares.

Despite it looks quite simple, and even evident, proof is not (yet?) available.

One can conjecture it could be possible, and even easier, to proof an even more restrictive conjecture, implying the Legendre's one.

Examples of such conjectures do exist in literature. As far as we know, however, the following simple conjecture is not (yet?) popular:

there exist at least 2 primes between two consecutive squares

Such a conjecture would obviously imply Legendre's one, looking almost as simple and as evident as Legendre's one: it is just more restrictive, thus not necessary but sufficient to prove Legendre's one.

Moreover, at least for very low naturals n, it looks like such a pair of primes, existing (besides possible other primes) between n^2 and (n+1)^2, are symmetrical with respect to the product n(n+1).

This is even more restrictive (and of course not necessary for the proof of Legendre's conjecture), but it may be of help in the search of the proof.