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Cosmological consequences of the model of low-energy quantum gravity

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The model of low-energy quantum gravity by the author is based on the conjecture about an existence of the graviton background. An interaction of photons and moving bodies with this background leads to small additional effects having essential cosmological consequences. In the model, redshifts of remote objects and the dimming of supernovae 1a may be interpreted without any expansion of the Universe and without dark energy. Some of these consequences are discussed and confronted with supernovae 1a and long GRBs observations in this paper. It is shown that the two-parametric theoretical luminosity distance of the model fits observations with high confidence levels (100% for the SCP Union 2.1 and JLA compilations and 99.81% for long GRBs). These parameters are computable in the model.

I. INTRODUCTION

In contrast with classical electrodynamics in the XIX century or quantum electrodynamics in the XX century, at present we have a complete lack of experimental evidence to construct a theory of quantum gravity. From dimensional reasons only, if one assumes that the Newton constant is universal for any scales, the effects of quantum gravity are expected to be measurable over extremely small distances or very high energies. There are proposals how to detect some effects in a laboratory - for example, [1, 2], - or to observe a possible small violation of the Lorentz invariance for remote sources, but we have not any results in a frame of current paradigms which may pave us to the goal. Another constrain is, as I think, the common expectation that the future theory should be some symbiosis of the geometrical theory of general relativity and quantum mechanics. Geometry is useful for a description of the average motion of big bodies due to the universality of gravitation, but it is not the fact that quantum effects may be described geometrically. It is also necessary to keep in mind that the nature of gravity as well as the nature of quantum behavior of microparticles are unknown - we have remarkable descriptions in different languages but not understanding in both cases.

I describe here briefly some consequences of my approach to quantum gravity [3, 4], in which the phenomenon is a very-low-energy one and is caused by the background of super-strong interacting gravitons. The main quantum effect of this approach is the Newtonian attraction; its small effects enforce us to look at the known results of astrophysical observations from another point of view and give us the reasons to doubt in the validity of the current standard cosmological model.

II. THE MODEL OF LOW-ENERGY QUANTUM GRAVITY

The geometrical description of gravity in general relativity does not involve any mechanism of interaction. It is similar to the Newtonian model: we don't know how it works. In my model of low-energy quantum gravity [3, 4], gravity is considered as the screening effect. It is suggested that the background of super-strong interacting gravitons exists in the universe. Its temperature should be equal to the one of CMB. Screening this background creates for any pair of bodies both attraction and repulsion forces due to pressure of gravitons. For single gravitons, these forces are approximately balanced, but each of them is much bigger than a force of Newtonian attraction. If single gravitons are pairing, an attraction force due to pressure of such graviton pairs is twice exceeding a corresponding repulsion force if graviton pairs are destructed by collisions with a body. This peculiarity of the quantum mechanism of gravity leads to the difference of inertial and gravitational masses of a black hole. In such the model, the Newton constant is connected with the Hubble constant that gives a possibility to obtain a theoretical estimate of the last. We deal here with a flat non-expanding universe fulfilled with super-strong interacting gravitons; it changes the meaning of the Hubble constant which describes magnitudes of three small effects of quantum gravity but not any expansion or an age of the universe.

III. SMALL EFFECTS OF THE MODEL DUE TO ITS QUANTUM NATURE

There are two small effects for photons in the sea of super-strong interacting gravitons [3]: average energy losses of a photon due to forehead collisions with gravitons and an additional relaxation of a photonic flux due to non-forehead collisions of photons with gravitons. The first effect leads to the geometrical distance/redshift relation:

$$r(z) = ln(1+z) \cdot c/H_0, \tag{1}$$

where H_0 is the Hubble constant, c is the velocity of light. The both effects lead to the luminosity distance/redshift relation:

$$D_L(z) = c/H_0 \cdot \ln(1+z) \cdot (1+z)^{(1+b)/2} \equiv c/H_0 \cdot f_1(z), \tag{2}$$

where $f_1(z) \equiv \ln(1+z) \cdot (1+z)^{(1+b)/2}$; the "constant" b belongs to the range 0 - 2.137 [5] ($b = 2.137$ for very soft radiation, and $b \to 0$ for very hard one). For an arbitrary source spectrum, a value of the factor b should be still computed. It is clear that in a general case it should depend on a rest-frame spectrum and on a redshift. Because of this, the Hubble diagram should be a multivalued function of a redshift: for a given z, b may have different values for different kinds of sources. Further more, the Hubble diagram may depend on the used procedure of observations: different parts of rest-frame spectrum will be characterized with different values of the parameter b.

Actually, the factor b describes an analog of the blurring effect of tired-light models. Due to the quantum nature of this effect in the model, non-forehead collisions of photons with gravitons should lead to relatively big average angles of deviations of photons of visible range:

$$\Delta\varphi \sim \frac{10^{-3}\ eV}{2.5\ eV} = 4 \cdot 10^{-4}\ rad,$$

where $10^{-3}\ eV$ and $2.5\ eV$ are average graviton and photon energies. By multiple collisions, deviated photons will not be recognized as emitted by a small-angle remote object. But images of high-z objects may be partly blurred due to a fraction of low-energy gravitons.

The third small effect of this model is the constant deceleration of massive bodies due to forehead collisions with gravitons. It is an analog of the redshift in this model. We get for the body acceleration w by a non-zero velocity v:

$$w = -ac^2(1 - v^2/c^2). \quad (3)$$

For small velocities we have for it: $w \simeq -H_0 c$. If the Hubble constant H_0 is equal to $2.14 \cdot 10^{-18} s^{-1}$ (it is the theoretical estimate of H_0 in this approach), a modulus of the acceleration will be equal to $|w| \simeq H_0 c = 6.419 \cdot 10^{-10}\ m/s^2$, that is of the same order of magnitude as a value of the observed additional acceleration $(8.74 \pm 1.33) \cdot 10^{-10} m/s^2$ for NASA probes Pioneer 10/11 [6].

IV. COSMOLOGICAL CONSEQUENCES OF THE MODEL

There are the two circumstances introduced in the model to rich the needed strength of gravitational attraction: 1) gravitons should be super-strong interacting, and 2) a part of gravitons should be paired and the pairs must be destructed by interaction with bodies. It leads to the very unexpected consequence: in the model, a black hole should have different gravitational and inertial masses, i. e. its possible existence contradicts to general relativity. Another unexpected feature of this approach is a necessity of "an atomic structure" of matter, because the considered mechanism doesn't work without it.

The property of asymptotic freedom of this model at very short distances leads to the important consequences, too. First, a black hole mass threshold should exist. A full mass of black hole should be restricted from the bottom with m_0; the rough estimate for it is: $m_0 \sim 10^7 M_\odot$. The range of transition to gravitational asymptotic freedom for a pair of protons is between $10^{-11} - 10^{-13}$ meter, and for a pair of electrons it is between $10^{-13} - 10^{-15}$ meter. This transition is non-universal; it means, second, that a geometrical description of gravity on this or smaller scales, for example on the Planck one, is not valid.

Any massive body moving relative to the graviton background should suffer in the model the constant deceleration of the order of $\sim H_0 c$, i. e. of the same order as an anomalous acceleration of the NASA's deep space probes (the Pioneer anomaly) [6]. Recently, it was shown by S. Turyshev et al [7], that the thermal origin of the Pioneer anomaly is very possible. From another side, the mass discrepancy in spiral galaxies appears at very low accelerations less than some a_0 and not much above a_0 [8], where the boundary acceleration a_0 has the same order. The need for dark matter in spiral galaxies appears at very low accelerations. A simple alternative to dark matter is MOND by

M. Milgrom [9], in which such the boundary acceleration is introduced by hand. The main feature of MOND is the strengthening of gravitational attraction in a case of low accelerations; I do not think that an exact form of this strengthening has been guessed in MOND. But MOND gives us a clear hint that general relativity may be not valid on galactic or bigger scales of distances, and its application in cosmology is in doubt. In my model, the universal deceleration of bodies should lead in any bound system to an additional acceleration of them relative to the system's center of inertia. Some additional strengthening of gravitation on a periphery of galaxies may be caused in the model by the destruction of graviton pairs flying through their central parts whereas pairs flying to the center are destructed in a less degree. The problem is open in this model.

The standard cosmological model is based on the assumption that redshifts of remote objects arise due to an expansion of the Universe. The model was re-builded a few times to save this base, the last innovation of it is an introduction of dark energy. Many people are searching for dark energy now or plan to do it, for example, with the help of big colliders. This basic cosmological assumption is considered by the community as a dogma, an invioalable sanctuary of present cosmology. For example, all observations of remote objects in the time domain are corrected for time dilation - but this effect is an attribute only of the standard model. In my model this assumption does not seem to be absolutely necessary. There exists a possibility in the model to interpret observations in another manner, without any expansion of the Universe.

A. The Hubble diagram of this model

In this model, the luminosity distance is given by $Eq.$ 2. The theoretical value of relaxation factor b for a soft radiation is $b = 2.137$. Let us consider the Hubble constant as a single free parameter to fit observations. The theoretical Hubble diagram of this model is compared with Supernovae 1a observational data by Riess et al. [10] (corrected for no time dilation as: $\mu(z) \rightarrow \mu(z) + 2.5 \cdot lg(1+z)$) in Fig. 1. As you can see, the theoretical diagram fits observations very well without any dark energy.

To demonstrate how similar are predictions about distance moduli as a function of redshift of this model and of the concordance cosmology, the two theoretical Hubble diagrams are sown in Fig. 2: $\mu_0(z)$ of this model with $b = 1.137$ taking into account the effect of time dilation of the standard model (solid); and $\mu_c(z)$ for a flat Universe with the concordance cosmology by $\Omega_M = 0.27$ and $w = -1$ (dash). You can see a good accordance of this diagrams up to $z \approx 4$.

At present, two big compilations of SN 1a observations are available: the SCP Union 2.1 com-

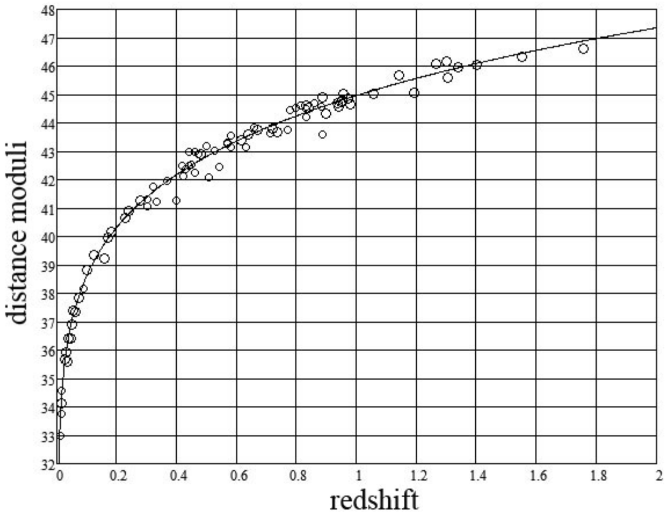

FIG. 1. The theoretical Hubble diagram $\mu_0(z)$ of this model (solid); Supernovae 1a observational data (circles, 82 points) are taken from Table 5 of [10] and corrected for no time dilation.

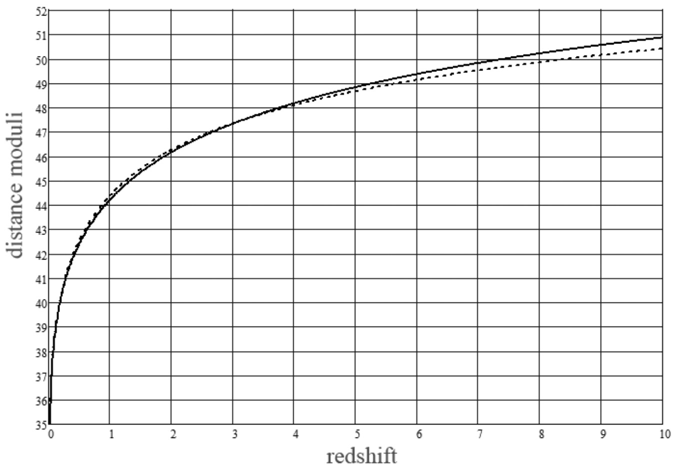

FIG. 2. The two theoretical Hubble diagrams: $\mu_0(z)$ of this model with $b = 1.137$ taking into account the effect of time dilation of the standard model (solid); $\mu_c(z)$ for a flat Universe with the concordance cosmology by $\Omega_M = 0.27$ and $w = -1$ (dash).

pilation (580 supernovae) [11] and the JLA compilation (740 supernovae) [12]. These compilations may be used to evaluate the Hubble constant in this approach. Using the definition of distance modulus: $\mu(z) = 5lgD_L(z)(Mpc) + 25$, we get from $Eq.$ 2 for the theoretical distance modulus $\mu_0(z)$: $\mu_0(z) = 5lgf_1(z) + k$, where the constant k is equal to:

$$k \equiv 5lg(c/H_0) + 25.$$

If the model fits observations, then we shall have for $k(z)$:

$$k(z) = \mu(z) - 5lg f_1(z), \quad (4)$$

where $\mu(z)$ is an observational value of distance modulus. The weighted average value of $k(z)$:

$$< k(z) >= \frac{\sum k(z_i)/\sigma_i^2}{\sum 1/\sigma_i^2}, \quad (5)$$

where σ_i^2 is a dispersion of $\mu(z_i)$, will be the best estimate of k. Here, σ_i^2 is defined as: $\sigma_i^2 = \sigma_{i\ stat}^2 + \sigma_{i\ sys}^2$. The average value of the Hubble constant may be found as:

$$< H_0 >= \frac{c \cdot 10^5}{10^{<k(z)>/5} \cdot Mpc}. \quad (6)$$

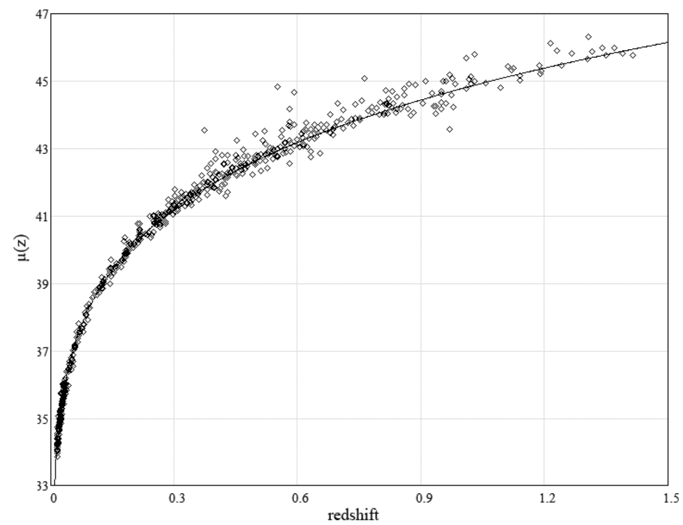

FIG. 3. The theoretical Hubble diagram $\mu_0(z)$ of this model (solid); Supernovae 1a observational data (580 points of the SCP Union 2.1 compilation) are taken from [11] and corrected for no time dilation.

For a standard deviation of the Hubble constant we have:

$$\sigma_0 = \frac{ln10 \cdot < H_0 >}{5} \cdot \sigma_k, \quad (7)$$

where σ_k^2 is a weighted dispersion of k, which is calculated with the same weights as $< k(z) >$.

The theoretical Hubble diagram $\mu_0(z)$ of this model with $< k(z) >$ which is calculated using the SCP Union 2.1 compilation [11] is shown in Fig. 3 together with observational points corrected for no time dilation. Values of $k(z)$ (580 points) and $< k(z) >$, $< k(z) > +\sigma_k$, $< k(z) > -\sigma_k$ (lines)

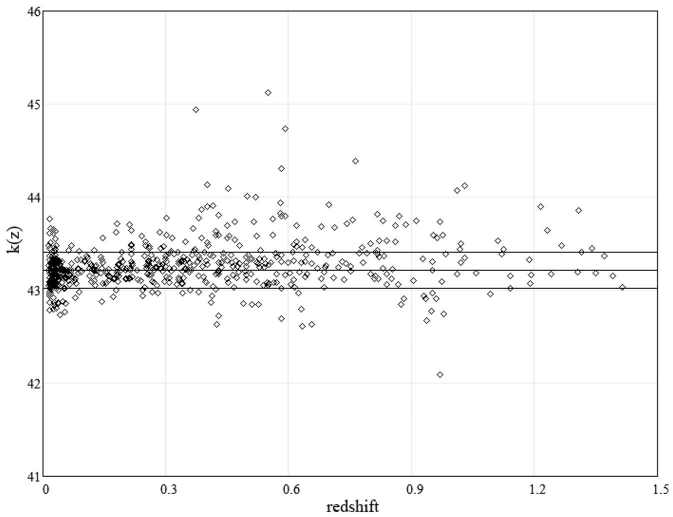

FIG. 4. Values of $k(z)$ (580 points) and $< k(z) >$, $< k(z) > +\sigma_k$, $< k(z) > -\sigma_k$ (lines) for the SCP Union 2.1 compilation.

are shown in Fig. 4. For this compilation we have: $< k > \pm\sigma_k = 43.216 \pm 0.194$. Calculating the χ^2 value as:

$$\chi^2 = \sum \frac{(k(z_i) - < H_0 >)^2}{\sigma_i^2}, \tag{8}$$

we get $\chi^2 = 239.635$. By 579 degrees of freedom of this data set, it means that the hypothesis that $k(z) = const$ cannot be rejected with 100% C.L. Using $Eqs.$ 6, 7, we get for the Hubble constant from the fitting:

$$< H_0 > \pm\sigma_0 = (2.211 \pm 0.198) \cdot 10^{-18}\ s^{-1} = (68.223 \pm 6.097) \frac{km}{s \cdot Mpc}.$$

The theoretical value of the Hubble constant in the model: $H_0 = 2.14 \cdot 10^{-18}\ s^{-1} = 66.875\ km \cdot s^{-1} \cdot Mpc^{-1}$ belongs to this range. The traditional dimension $km \cdot s^{-1} \cdot Mpc^{-1}$ is not connected here with any expansion.

To repeat the above calculations for the JLA compilation, I have used 31 binned points from Table F.1 of [12]. Given equal weights for points, we have for this compilation: $< k > \pm\sigma_k = 43.219 \pm 0.104$ with $\chi^2 = 0.337$. By 30 degrees of freedom of this data set, it means that the hypothesis that $k(z) = const$ cannot be rejected with 100% C.L., too. For the Hubble constant we have in this case:

$$< H_0 > \pm\sigma_0 = (2.207 \pm 0.106) \cdot 10^{-18}\ s^{-1} = (68.140 \pm 3.269) \frac{km}{s \cdot Mpc}.$$

Results of the fitting are shown in Figs. 5,6.

We have proved the important *mathematical* fact: SNe 1a observational data may be fitted with the two-parametric law of *Eq.* 2; it is impossible to decrease the number of parameters or to increase the confidence level of the fitting. It is strange that big teams of professionals in the field did not recognize this fact in the last 17 years.

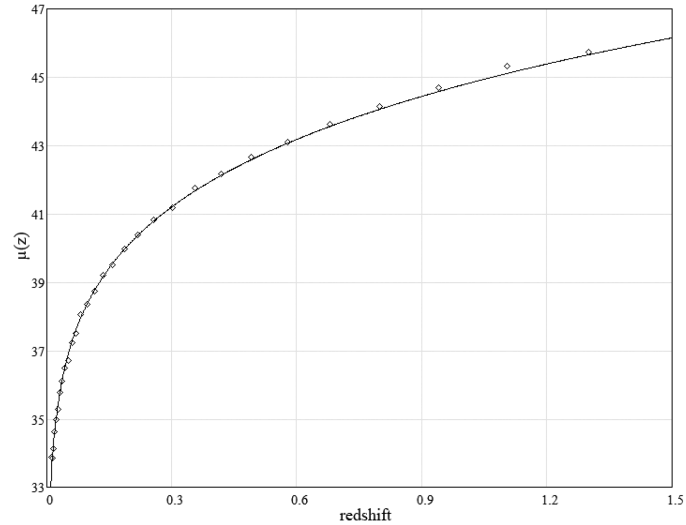

FIG. 5. The theoretical Hubble diagram $\mu_0(z)$ of this model (solid); Supernovae 1a observational data (31 binned points of the JLA compilation) are taken from Table F.1 of [12] and corrected for no time dilation.

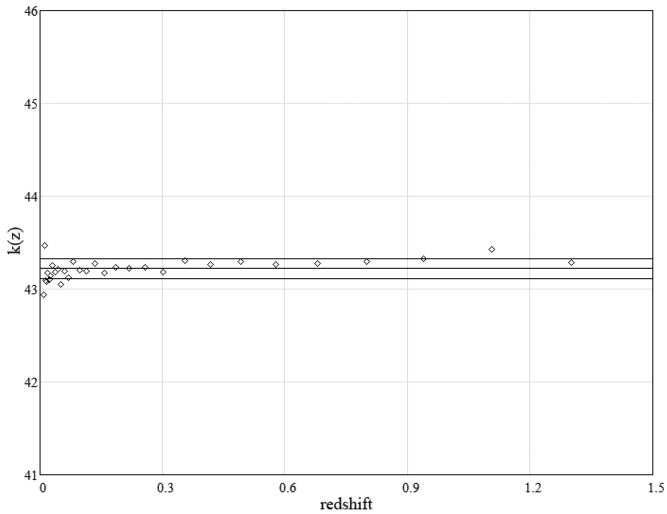

FIG. 6. Values of $k(z)$ (31 binned points) and $< k(z) >$, $< k(z) > +\sigma_k$, $< k(z) > -\sigma_k$ (lines) for the JLA compilation.

If observations of long Gamma-Ray Bursts (GRBs) for small z are calibrated using SNe 1a, observational points are fitted with this theoretical Hubble diagram, too [4]. But for hard radiation of GRBs, the factor b may be smaller, and the real diagram for them may differ from the one for SNe 1a. With this limitation, the long GRBs observational data (109 points) are taken from Tables 1,2 of [13] and fitted in the same manner with $b = 2.137$. In this case we have: $< k > \pm\sigma_k = 43.262\pm8.447$ with $\chi^2 = 70.39$. By 108 degrees of freedom of this data set, it means that the hypothesis that $k(z) = const$ cannot be rejected with 99.81% C.L. Results of the fitting are shown in Figs. 7,8.

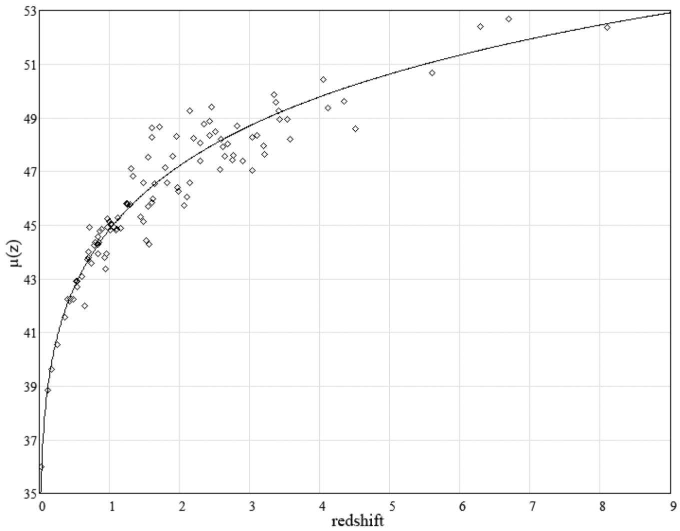

FIG. 7. The theoretical Hubble diagram $\mu_0(z)$ of this model (solid); long GRBs observational data (109 points) are taken from Tables 1,2 of [13] and corrected for no time dilation.

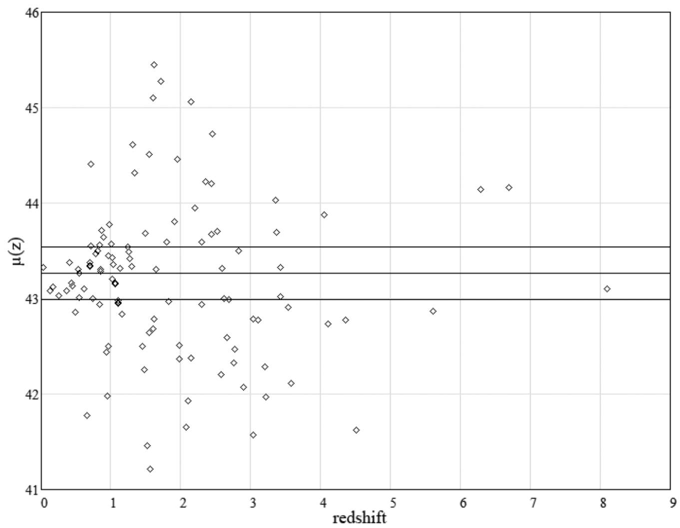

FIG. 8. Values of $k(z)$ (109 points) and $< k(z) >$, $< k(z) > +\sigma_k$, $< k(z) > -\sigma_k$ (lines) for long GRBs.

We may try to fit all three data sets using the theoretical values of b and H_0 - without free parameters. It is easy to do replacing $< k(z) >$ with $k = 43.259$ which corresponds to the theoretical value of H_0. For the first two data sets we get: $\chi^2 = 251.585$ and $\chi^2 = 0.386$ respectively, with 100% C.L. in both cases. For long GRBs we get: $\chi^2 = 70.416$ and 99.81% C.L.

B. The Hubble parameter $H(z)$ of this model

If the geometrical distance is described by $Eq.$ 1, for a remote region of the universe we may introduce the Hubble parameter $H(z)$ in the following manner:

$$dz = H(z) \cdot \frac{dr}{c}, \tag{9}$$

to imitate the local Hubble law. Taking a derivative $\frac{dr}{dz}$, we get in this model for $H(z)$:

$$H(z) = H_0 \cdot (1 + z). \tag{10}$$

It means that in the model:

$$\frac{H(z)}{(1+z)} = H_0. \tag{11}$$

The last formula gives us a possibility to evaluate the Hubble constant using observed values of the Hubble parameter $H(z)$. To do it, I use here 28 points of $H(z)$ from [14] and one point for $z < 0.1$ from [15]. The last point is the result of HST measurement of the Hubble constant obtained from observations of 256 low-z supernovae 1a. Here I refer this point to the average redshift $z = 0.05$. Observed values of the ratio $H(z)/(1 + z)$ with $\pm\sigma$ error bars are shown in Fig. 9 (points). The weighted average value of the Hubble constant is calculated by the formula:

$$< H_0 >= \frac{\sum \frac{H(z_i)}{1+z_i}/\sigma_i^2}{\sum 1/\sigma_i^2}. \tag{12}$$

The weighted dispersion of the Hubble constant is found with the same weights:

$$\sigma_0^2 = \frac{\sum (\frac{H(z_i)}{1+z_i} - < H_0 >)^2/\sigma_i^2}{\sum 1/\sigma_i^2}. \tag{13}$$

Calculations give for these quantities:

$$< H_0 > \pm\sigma_0 = (64.40 \pm 5.95)\ km\ s^{-1}\ Mpc^{-1}. \tag{14}$$

The weighted average value of the Hubble constant with $\pm\sigma_0$ error bars are shown in Fig. 9 as horizontal lines.

Calculating the χ^2 value as:

$$\chi^2 = \sum \frac{(\frac{H(z_i)}{1+z_i} - < H_0 >)^2}{\sigma_i^2}, \tag{15}$$

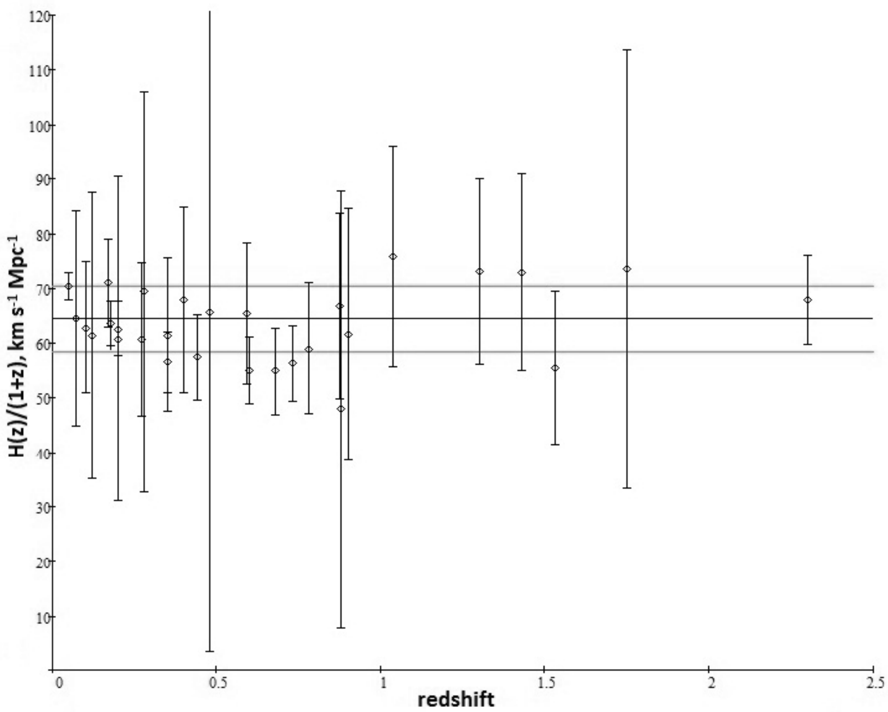

FIG. 9. The ratio $H(z)/(1+z) \pm \sigma$ and the weighted value of the Hubble constant $H_0 \pm \sigma_0$ (horizontal lines). Observed values of the Hubble parameter $H(z)$ are taken from Table 1 of [14] and one point for $z < 0.1$ is taken from [15].

we get $\chi^2 = 16.491$. By 28 degrees of freedom of our data set, it means that the hypothesis described by $Eq.$ 11 cannot be rejected with 95% C.L.

If we use another set of 21 cosmological model-independent measurements of H(z) based on the differential age method [16], we get (see Fig. 10):

$$< H_0 > \pm\sigma_0 = (63.37 \pm 4.56)\ km\ s^{-1}\ Mpc^{-1}. \tag{16}$$

The value of χ^2 in this case is smaller and equal to 3.948. By 21 degrees of freedom of this new data set, it means that the hypothesis described by $Eq.$ 11 cannot be rejected with 99.998% C.L.

Some authors try in a frame of models of expanding universe to find deceleration-acceleration transition redshifts using the same data set (for example, [14]). The above conclusion that the ratio $H(z)/(1+z)$ remains statistically constant in the available range of redshifts is model-independent. For the considered model, it is an additional fact against dark energy as an admissible alternative to the graviton background.

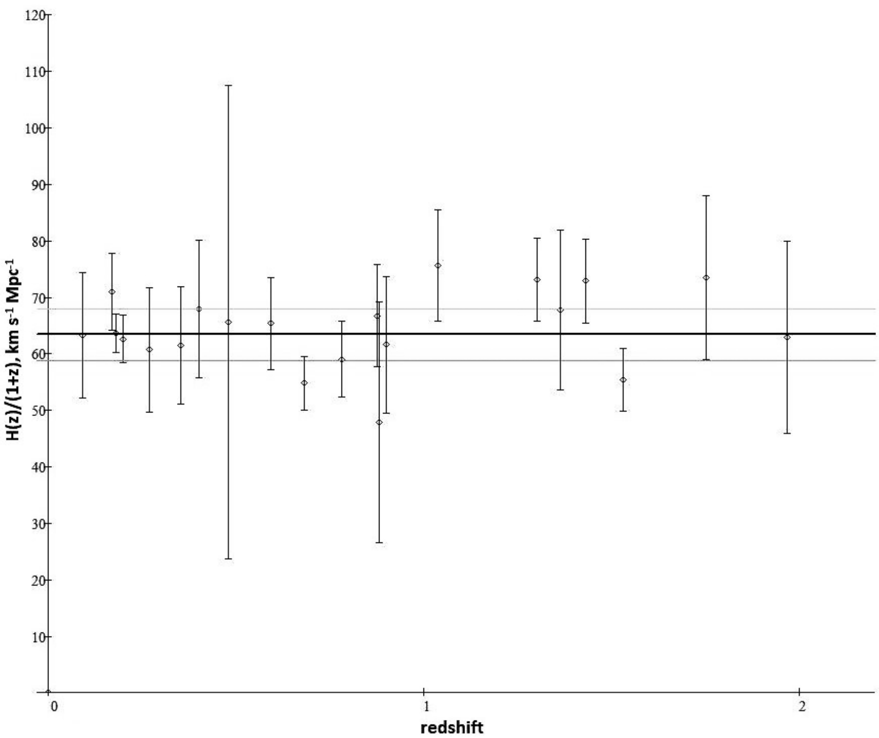

FIG. 10. The ratio $H(z)/(1+z) \pm \sigma$ and the weighted value of the Hubble constant $H_0 \pm \sigma_0$ (horizontal lines). Observed values of the Hubble parameter $H(z)$ are taken from [16].

C. The Alcock-Paczynski test of this model

The Alcock-Paczynski cosmological test consists in an evaluation of the ratio of observed angular size to radial/redshift size [17]. Recently, this test has been carried out for a few cosmological models by Fulvio Melia and Martin Lopez-Corredoira [18]. They used new model-independent data on BAO peak positions from [19] and [20]. For two mean values of z ($< z >= 0.57$ and $< z >= 2.34$), the measured angular-diameter distance $d_A(z)$ and Hubble parameter $H(z)$ give for the observed characteristic ratio $y_{obs}(z)$ of this test the values: $y_{obs}(0.57) = 1.264 \pm 0.056$ and $y_{obs}(2.34) = 1.706 \pm 0.076$. In this model we have: $d_{com}(z) = d_A(z) = r(z)$, where $d_{com}(z)$ is the cosmological comoving distance. Because the Universe is static here, the ratio $y(z)$ for this model is defined as:

$$y(z) = \frac{r(z)}{z \cdot \frac{d}{dz} r(z)} = \frac{r(z) \cdot H(z)}{cz} = (1 + \frac{1}{z}) \cdot \ln(1+z), \tag{17}$$

where $H(z)$ is defined by $Eq.$ 10. This function without free parameters characterizes any tired light model (model 6 in [18]). We have only two observational points to fit them with this function.

Calculating the χ^2 value as:

$$\chi^2 = \sum \frac{(y_{obs}(z_i) - y(z_i))^2}{\sigma_i^2}, \tag{18}$$

we get $\chi^2 = 0.189$, that corresponds to the confidence level of 91% for two degrees of freedom.

V. CONCLUSION

The key cosmological question is a mechanism of redshift: is it local or non-local one? If this mechanism is local as in the considered model, the quantum gravitational nature of redshifts may be verified in a ground-based laser experiment [21].

As it is shown above, the Hubble diagram of supernovae 1a corrected for no time dilation and GRBs (when their luminosity is calibrated with the help of SN 1a observations), the Hubble parameter $H(z)$ and the ratio of observed angular size to radial/redshift size are well fitted in this model. The Hubble diagram for GRBs may differ in the model from the diagram for SNe 1a, but one should calibrate the GRB luminosity independently of supernovae 1a to discover this difference. In the model, space-time is flat, and the geometrical distance as a function of the redshift coincides with the angular diameter distance. Given that a galaxy number density is constant in the no-evolution scenario, theoretical predictions for galaxy number counts in this model have been found using only the luminosity and geometrical distances defined by $Eqs.1,2$ [22]. The geometrical distance $r(z)$ of this model is very different from the one of the standard model; for example, GRB 090429B with $z = 9.4$ [23] took place 24.6 Gyr ago in a frame of this model; the age of the Universe of the standard model: ~ 13.5 Gyr corrseponds here to $z \simeq 2.6$.

At present this model is not a full cosmological one; it is necessary to develop many open problems to bring it closer to the pursuable completeness. But even now it has an interesting advantage: one can describe the observed Hubble diagram of supernovae 1a by $Eq.$ 2 with the computable parameter $b = 2.137$ without dark energy.

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