A Preliminary Explanation for the Pentaquark P_c^+ found by LHCb

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Abstract

We propose that the two resonant states of the recently found pentaquark P_c^+ with masses of 4380 MeV and 4450 MeV are two states of the hadronic molecule $c\bar{c} \oplus uud$ with similar properties to those of the Karliner-Lipkin pentaquark. Applying the Morse molecular potential to the molecule its minimum size is estimated. If S states exist, the first two possible S states are suggested and their energies are estimated. It is shown that the coupling constant is close to that of charmonium, and this may mean Physics Beyond the Standard Model.

I. INTRODUCTION

The idea of the pentaquark was firstly proposed by Strottman in 1979 [1]. In 2004 Karliner and Lipkin proposed a very important model for a pentaquark in the description of the Θ^+ [2]. They arrived at the conclusion that the bag model commonly used for hadrons may not be adequate for the pentaquark. In their model they propose that the pentaquark system is composed of two clusters, a diquark and a triquark, in a relative P-wave state, and the clusters can be separated by a distance larger than the range of the color-magnetic force which had been proposed by De Rujula, Georgi and Glashow [3].

It is important to recall that many authors have considered the possibility of the binding between J/Ψ and nuclei. Among them we can cite [4,5,6,7,8,9,10,11]. They have raised some important observations concerning the interaction such as that meson exchange is suppressed by the OZI rule because J/Ψ and nuclei do not have quarks in common and also single gluon exchange is forbidden so that the interaction should happen due to multiple-gluon exchange.

The LHCb Collaboration has recently [12] announced the discovery of pentaquark-charmonium states which resulted from the Λ_b^0 exotic decay $\Lambda_b^0 \to P_c^+ K^-$, in which P_c^+ is a pentaquark with the quark content $u\bar{c}cud$. The two resonant states have masses of about 4380 MeV and 4450 MeV in opposite parities with the preferred values for $J = 3/2, 5/2$ and with the corresponding Γ's of $205 \pm 18 \pm 86$ MeV and

$39 \pm 5 \pm 19$ MeV. All the data for the masses below were taken from the Particle Data Group [13].

II. A SIMPLE MODEL FOR THE LHCb P_c^+

We propose that the recently found LHCb P_c^+ is composed of two colorless clusters, a meson and a baryon. The quark content of the P_c^+ pentaquark, $u\bar{c}ccud$ allows the two possibilities $c\bar{c} \oplus uud$ and $u\bar{c} \oplus cud$, but as the final decay is $J/\Psi p$ we should rule out the second possibility. The symbol $\oplus$ is used to designate the binding between the two clusters. According to QCD, we expect the two clusters to be weakly bound because hadrons are colorless.

The most famous hadronic molecule is the deuteron, also constituted of colorless particles. Because the two particles (nucleons) are colorless the binding is very weak, just about 2.2 MeV. But in the case of the $c\bar{c} \oplus uud$ we do not know what kind of binding energy we have. Between two nucleons the nuclear force is a residual effect of the more fundamental forces of the color field in the quark systems, but we still do not know the nature of the force responsible for the binding between J/Ψ and the proton in the molecule $c\bar{c} \oplus uud$. We can estimate below the minimum value for the binding energy.

III. POSSIBILITIES FOR THE ANGULAR MOMENTA

As the final decay is $J/\Psi p$ we should consider the two possibilities for the molecule: $c\bar{c}(S=1) \oplus uud(S=1/2)$ and $c\bar{c}(S=1) \oplus uud(S=3/2)$. $c\bar{c}(S=1)$ is J/Psi and $uud(S=3/2)$ is the resonance N(1520) whose mass is actually about 1515 MeV. The sum of the masses 3096 MeV+1515 MeV = 4611 MeV is much higher than 4380 MeV and 4450 MeV. Therefore, we should consider only the first possibility. In order to reproduce the experimental values of $J=3/2, 5/2$ we should have $L=1, 2$ for the molecule. Taking into account the Karliner-Lipkin model [2] in which the clusters are in a relative P-wave state, we can propose that the levels found by LHCb for the P_c^+ system are in relative P-wave and D-wave states. The possibility of S states is discussed below, in section V.

Let us now analyze the angular momenta of the molecule $c\bar{c}(S=1) \oplus uud(S=1/2)$ and its components. Composing the two spins we obtain $S=1/2, 3/2$. With $L=1$ and $S=\frac{1}{2}$ we have $J=\frac{1}{2}^-, \frac{3}{2}^-$, and for $S=\frac{3}{2}$ we have $J=\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$. With $L=2$ and $S=\frac{1}{2}$ we have $J=\frac{3}{2}^+, \frac{5}{2}^+$, and for $S=\frac{3}{2}$ we have $J=\frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+$. We see that the most favored values for J are $\frac{1}{2}^-, \frac{3}{2}^-, \frac{3}{2}^+, \frac{5}{2}^+$.

IV. THE APPROXIMATE SIZE OF THE MOLECULE

We have no idea yet on the kind of effective potential that exists in the P_c^+ system. Assuming that there is a potential well we may model the system through a molecular potential and obtain some important numbers. It is important to notice that the components of the system are heavy and, thus, we can use the Schrödinger equation in the description of the system. Recently, de Souza [14,15] has used the Morse molecular potential in the description of charmonium and bottomonium states. The Morse molecular potential can be expressed as [16]

$$V(r) = D\left(e^{-2\alpha x} - 2e^{-\alpha x}\right) \tag{1}$$

where $-D$ is the minimum of the well, a is the distance where $V = -D$, and $x = (r-a)/a$.

For $|x| < 1$ this potential can be expanded around the minimum up to order 3 in x and the expression

$$V(x) = -D + \frac{1}{2}ka^2x^2 - \lambda ka^3x^3 \tag{2}$$

is obtained where $\lambda = \alpha/2a$.

For this potential the solution of the Schrödinger equation yields the expression [16,17]

$$E_{nl} = \hbar\omega\left(\upsilon + \frac{1}{2}\right) - A\left(\upsilon + \frac{1}{2}\right)^2 + B_L L(L+1)$$

$$-D_L L^2(L+1)^2 - C_{\upsilon L}\left(\upsilon + \frac{1}{2}\right)L(L+1) + ... \tag{3}$$

for the vibrational and rotational levels above the minimum of the potential, where the quantum numbers $\upsilon, L = 0, 1, 2, 3, ...$ In Eq. 3 the first term describes harmonic vibrations, the second term takes into account the anharmonicity of the potential, the third term describes rotations with constant moment of inertia, the fourth term represents the centrifugal distortion and the fifth term represents the coupling between vibration and rotation. The constant B_L is given by $B_L = \hbar^2/2\mu a^2$ where μ is the reduced mass of the system.

In the case of P_c^+ we only have two levels, one for L=1 and another for L=2 for $v = 0$ in Eq. 3 Therefore we can only determine two parameters of the above formula. The largest parameters in Eq. (3) are $\hbar\omega$ and B_L. In general A is not very small compared to $\hbar\omega$, but for $v = 0$ the 2nd term is $-A/4$ while the first one is $\hbar\omega/2$, and thus, the first term has a much larger contribution than the 2nd term. Moreover, the inclusion of A would not change the value of B_L in our fitting because the two levels 4380 MeV and 4450 MeV have $v = 0$. The inclusion of A would increase slightly the value of $\hbar\omega$. The 4th and 5th terms are much smaller than the 3rd term. So, due to lack of data we can only have

$$\hbar\omega\left(0+\frac{1}{2}\right) + B_L 1(1+1) = 4380 \pm 309$$

$$\hbar\omega\left(0+\frac{1}{2}\right) + B_L 2(2+1) = 4450 \pm 63$$

(4)

which yield $B_L = 17.5 \pm 61.5$ MeV and $\hbar\omega = 9110 \pm 864$ MeV. Using the relation $B_L = \hbar^2/2\mu a^2$ we find $a = 1.25$ fm but with a large uncertainty due to the large uncertainty of B_L.

There are some available data for the radii of the proton and charmonium that allows us to have more information on the molecule $c\bar{c} \oplus uud$. The size of $c\bar{c}$ (J/Ψ) is about 0.35 ± 0.06 fm [14] and the proton radius (charge radius) has been reported as being 0.8879 fm [18], 0.8775 fm [19], and 0.84087 fm [20,21]. Summing the radii of $c\bar{c}$ and the proton we obtain about 1.2 fm which is very close to the value of a. But P_c^+ in states with L=1 and L=2 is larger than 1.25 fm, of course, because the wave functions of L states contain the term r^L. It is only worth making the calculation when we obtain a much smaller value for the Γ of the 4380 MeV level, and thus, as a consequence, a more precise value for B_L.

V. THE FIRST LEVELS OF THE MOLECULE

In our model above ν corresponds to $n-1$ in QCD models, that is, n=$\nu+1$. Thus, the two states 4380 MeV and 4450 MeV are the states $1P$ and $1D$, respectively.

As we saw above we could not calculate the value of A whose inclusion would increase slightly the value of $\hbar\omega$. This means that $\hbar\omega$ is a little larger than 9110 MeV. As B_L is very small ($B_L = 17.5$ MeV) and is, indeed, much smaller than the Γ of the level 4380 MeV which is $205 \pm 18 \pm 86$ MeV, if the $1S$ level exists, it would have an energy around 4.3 GeV. This would explain why the level 4380 MeV is much broader than the 4450 MeV level, and would mean that the 4380 MeV level is, actually, degenerate levels $1S$ and $1P$. Of course, considering the spins of J/Ψ and the proton, the values of J for S states would be only $\frac{1}{2}^+$ and $\frac{3}{2}^+$.

By the same token, if the $2S$ state ($\nu = 1$) exists, it would have an energy around $\hbar\omega(1+1/2) \sim 9110 \times 3/2 \sim 14$ GeV. Thus we can propose that, if S states exist, the LHCb pentaquark would have the levels shown in Table 1.

VI. THE BINDING ENERGY

For a square well we have that in order to have a bound state (for an S wave) we should have a depth of

$$D > \frac{\pi^2 \hbar^2}{8\mu a^2} \qquad (5)$$

which for $a = 1.25$ fm and $\mu = 720.1$ MeV yields $D > 0.278$ Mev.

Level (nL)	Energy (MeV)
1S	~ 4300
1P	4380 ± 309
1D	4449.8 ± 4.2
2S	~ 14000

Table 1. Possible levels of the pentaquark P_c^+ if S states exist.

The coupling constant is not small at all. From the value of $\hbar\omega = 9110 \pm 864$ MeV we obtain $k = \mu\omega^2 / \hbar^2 = \mu c^2 (\hbar\omega)^2 / (\hbar c)^2 \approx (1.52 \pm 0.29) \times 10^3$ GeV/fm^2 which is close to that for charmonium, $k \approx 1.44 \times 10^3$ GeV/fm^2 [15]. Taking into account the OZI rule, we can say that such a strong coupling may be due to Physics Beyond the Standard Model.

VII. CONCLUSION

It is proposed that the observed levels of the recently found pentaquark P_c^+ are states of the molecule $c\bar{c} \oplus uud$ in the different relative angular momentum $L = 1$ and $L = 2$ states, and is similar to the pentaquark system proposed by Karliner and Lipkin [2]. It is shown how the observed angular momenta are generated, and an approximate minimum value for the size of the molecule is calculated. It is shown that the coupling constant is not small and has the same order of magnitude of that for charmonium. This may mean Physics Beyond the Standard Model. Possible S states are proposed for the system.

Although still preliminary, the work is relevant and shows a consistent possibility for this recently discovered system of 5 quarks. More data is needed to improve the model and calculate other parameters of Eq. 3. It is important to find out, for example, if the system has or not excited states. This paper should be taking as one of the starting points for understanding this remarkable system.

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