A Preliminary Explanation for the Pentaquark P_c^+ found by LHCb

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Abstract

We propose that the two resonant states of the recently found pentaquark P_c^+ with masses of 4380 MeV and 4450 MeV are two states of the hadronic molecule $c\bar{c} \oplus uud$ with similar properties to those of the Karliner-Lipkin pentaquark. Applying the Morse molecular potential to the molecule some important numbers are obtained for its size.

I. INTRODUCTION

The idea of the pentaquark was firstly proposed by Strottman in 1979 [1]. In 2004 Karliner and Lipkin proposed a very important model for a pentaquark in the description of the Θ^+ [2]. They arrived at the conclusion that the bag model commonly used for hadrons may not be adequate for the pentaquark. In their model they propose that the pentaquark system is composed of two clusters, a diquark and a triquark, in a relative P-wave state, and the clusters can be separated by a distance larger than the range of the color-magnetic force which had been proposed by De Rujula, Georgi and Glashow [3].

The LHCb Collaboration has recently [4] announced the discovery of pentaquark-charmonium states which resulted from the Λ_b^0 exotic decay $\Lambda_b^0 \to P_c^+ K^-$, in which P_c^+ is a pentaquark with the quark content $u\bar{c}cud$. The two resonant states have masses of about 4380 MeV and 4450 MeV in opposite parities with the preferred values for $J = 3/2, 5/2$ and with the corresponding Γ's of $205 \pm 18 \pm 86$ MeV and $39 \pm 5 \pm 19$ MeV, which correspond to lifetimes $\tau_1 = (0.31 \pm 0.16) \times 10^{-22}$ s and $\tau_2 = (0.16 \pm 0.10) \times 10^{-21}$ s. All the data for the masses below were taken from the Particle Data Group [5].

II. A SIMPLE MODEL FOR THE LHCb P_c^+

We propose that the recently found LHCb P_c^+ is composed of two colorless clusters, a meson and a baryon. The quark content of the P_c^+ pentaquark, $u\bar{c}cud$ allows the two possibilities $c\bar{c} \oplus uud$ and $u\bar{c} \oplus cud$, but as the final decay is $J/\Psi p$ we should rule

out the second possibility. The symbol $\oplus$ is used to designate the binding between the two clusters. The two clusters should be weakly bound because hadrons are colorless, but as they are bound, they form a hadronic molecule.

The most famous hadronic molecule is the deuteron, also constituted of colorless particles. Because the two particles (nucleons) are colorless the binding is very weak, just about 2.2 MeV. This is a very important fact and tells us that in the case of the $c\bar{c} \oplus uud$ the binding energy should also be just a couple of MeVs. As the nuclear force is a residual effect of the more fundamental forces of the color field in the quark systems, the force responsible for the binding between each meson and the baryon in the molecule $c\bar{c} \oplus uud$ must also have the same nature. These residual forces are analogues of the London forces between neutral atoms, and between neutral molecules [6,7,8].

III. POSSIBILITIES FOR THE ANGULAR MOMENTA

As the final decay is $J/\Psi p$ we should consider the two possibilities for the molecule: $c\bar{c}(S=1) \oplus uud(\mathrm{S}=1/2)$ and $c\bar{c}(S=1) \oplus uud(\mathrm{S}=3/2)$. $c\bar{c}(S=1)$ is J/Psi and $uud(S=3/2)$ is the resonance N(1520) whose mass is actually about 1515 MeV. The sum of the masses 3096 MeV+1515 MeV = 4611 MeV is much higher than 4380 MeV and 4450 MeV. Therefore, we should consider only the first possibility. In order to reproduce the experimental values of $J=3/2,5/2$ we should have $L=1,2$ for the molecule. Taking into account the Karliner-Lipkin model [2] in which the clusters are in a relative P-wave state, we can propose that the P_c^+ system is in relative P-wave and D-wave states.

Let us now analyze the angular momenta of the molecule $c\bar{c}(S=1) \oplus uud(\mathrm{S}=1/2)$ and its components. Composing the two spins we obtain $S=1/2,3/2$. With $L=1$ and $S=1/2$ we have $J=1/2^-, 3/2^-$, and for $S=3/2$ we have $J=1/2^-, 3/2^-, 5/2^-$. With $L=2$ and $S=1/2$ we have $J=3/2^+, 5/2^+$, and for $S=3/2$ we have $J=1/2^+, 3/2^+, 5/2^+, 7/2^+$. We see that the most favored values for J are $1/2^-, 3/2^-, 3/2^+, 5/2^+$.

The sum of the masses of J/Ψ and the proton is 3096 MeV + 938,3 MeV = 4034.3 MeV. Let us call δE_1 and δE_2 the two differences in mass δE_1 = 4380 MeV – 4034.3 MeV = 345.7 MeV, and δE_2 = 4450 MeV – 4034.3 MeV = 415.7 MeV. As in the deuteron we expect the binding energy between the two colorless clusters of P_c^+ to be very small, of the order of a couple of MeVs. Thus, the binding energy is of the order of just 1% of δE_1 and δE_2, and therefore, these should come from rotational energy contributions.

IV. THE APPROXIMATE SIZES OF THE MOLECULE

We have no idea yet on the kind of effective potential that exists in the P_c^+ system. Assuming that there is a shallow potential well we may model the system through a molecular potential and obtain some important numbers. It is important to note that this system are heavy and, thus, we can use the Schrödinger equation for them. Recently, de Souza [9,10] has used the Morse molecular potential in the description of charmonium and bottomonium states. Let us try to model the molecule P_c^+ using the Morse molecular potential which can be expressed as [11]

$$V(r) = D\left(e^{-2\alpha x} - 2e^{-\alpha x}\right) \tag{1}$$

where $-D$ is the minimum of the well, a is the distance where $V = -D$, and $x = (r-a)/a$.

For $|x| < 1$ this potential can be expanded around the minimum up to order 3 in x and the expression

$$V(x) = -D + \frac{1}{2}ka^2x^2 - \lambda ka^3x^3 \tag{2}$$

is obtained where $\lambda = \alpha / 2a$.

For this potential the solution of the Schrödinger equation yields the expression [11,12]

$$\begin{aligned} E_{nl} &= \hbar\omega\left(\upsilon + \frac{1}{2}\right) - A\left(\upsilon + \frac{1}{2}\right)^2 + B_L L(L+1) \\ &- D_L L^2(L+1)^2 - C_{\upsilon L}\left(\upsilon + \frac{1}{2}\right)L(L+1) + \ldots \end{aligned} \tag{3}$$

for the vibrational and rotational levels above the minimum of the potential, where the quantum numbers $\upsilon, L = 0,1,2,3,\ldots$ In Eq. 3 the first term describes harmonic vibrations, the second term takes into account the anharmonicity of the potential, the third term describes rotations with constant moment of inertia, the fourth term represents the centrifugal distortion and the fifth term represents the coupling between vibration and rotation. The constant B_L is given by $B_L = \hbar^2 / 2\mu a^2$ where μ is the reduced mass of the system.

As the binding is very weak the first term in Eq. 3 for $\nu = 0$ is approximately equal to the 4032.3 MeV and the second term is small. Also the 4th and 5th terms are smaller than the third term. Thus, due to a lack of more information we can make the rough estimate

$$\delta E \approx \frac{\hbar^2}{2\mu a^2} L(L+1) \tag{4}$$

and estimate the sizes of the two molecules by means of a with μ equal to 720.1 MeV which is the reduced mass of the molecule of the $c\bar{c} \oplus uud$. Let us call a_1 the value of a for $L = 1$ and a_2 for $L = 2$.

From Eq. 5 we obtain the values below shown in Table 1 for *a*.

δE (MeV)	a (fm)
345.7	2.49
415.7	3.93

Table 1. Calculation of the possible values of *a* according to Eq. 4 for $L=1$ and $L=2$.

As a_2 is very large, the D-wave should be very unstable and thus the largest contribution should come from the P-wave. In the P-wave state the molecule is larger than the deuteron which has a charge radius of about 2.13 fm [13] and a matter radius of about 1.975 fm [14,15].

There are some available data for the radii of the proton and charmonium that allows us to have more information on the molecule $c\bar{c} \oplus uud$. The size of $c\bar{c}$ (J/Ψ) is about 0.35 ± 0.06 fm [10] for the first S state and the proton radius (charge radius) has been reported as being 0.8879 fm [16], 0.8775 fm [17], and 0.84087 fm [18]. Summing the radii of $c\bar{c}$ and the proton we obtain about 1.2 fm which means that the meson $c\bar{c}$ and the proton are more than a fermion apart. This means that the P_c^+ system is a very large system such as the deuteron. Let us recall that the deuteron is mostly in an S state. If it were in a P state it would be much larger and could have a size comparable with the values from Table 1. Between two nucleons the nuclear force becomes negligible at distances beyond 2.5 fm [19] but we do not know its range when it acts between a baryon and a meson. The pentaquark system is giving us this opportunity.

Let us analyze how good is our approximation for a_1 and a_2. As the binding is very weak the second term in Eq. 3 is very small. The 4th term, which is the centrifugal distortion, is in general much smaller than the 3rd term. In charmonium the 4th term is only about 10% of the 3rd term [10], and in bottomonium it is only 8% of the 3rd term [9]. It is reasonable to assume the same kind of trend for the molecule $c\bar{c} \oplus uud$. It is also important to notice that the 4th term is negative, and thus it lowers the values of a_1 and a_2, and thus the calculated values in Table 1 are upper limits, but should be close to the actual values.

V. CONCLUSION

It is proposed that the recently found pentaquark P_c^+ is a system composed of the molecule $c\bar{c} \oplus uud$ in the different relative angular momentum $L=1$ and $L=2$, and is similar to the pentaquark system proposed by Karliner and Lipkin [2]. It is shown how the observed angular momenta are generated, and approximate values for the sizes of the molecule are estimated.

Although still preliminary, the work is relevant and shows a consistent possibility for this recently discovered system of 5 quarks. More data is needed to improve the model and obtain more precise values for a_1 and a_2. It is important to find out, for example, if the system has or not excited states. This paper is a starting point for understanding this remarkable system.

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