# Modified Alcubierre Warp Drive II

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## Abstract:

A solution of general relativity is presented that describes an Alcubierre [1] propulsion system in which it is possible to travel at superluminal speed while reducing the components of the energy-impulse tensor (thus reducing energy density) by an arbitrary value. Here we investigate the negative energy in the Pfenning zone, and the quantum inequalities involved.

**1. Introduction:**

Alcubierre [1] in 1994 proposed a solution of the equations of general relativity which provides the only viable means to accelerate a spaceship up to superluminal velocities without using wormholes. A problem was soon identified: Pfenning [4] showed that the required energy is comparable to the total energy of the universe and that it is negative. Moreover he used quantum inequalities to show that this energy gets distributed at very short scale (about 100 times the Planck length) up to a multiplicative factor equal to the squared speed. In the previous part of this pubblication (part I), we presented a way to reduce the amount of energy involved and its spacial distribution within the warp bubble. In this second part we investigate the quantum inequalities.

Note: In the following we adopt the notation used by Landau and Lifshitz in the second volume ("The Classical Theory of Fields") of their well known Course of Theoretical Physics [15].

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### 1.1 Summary:

We start with the metric

$$ds^2=\left(1-v^2\frac{f(x,y,z-k(t))^2}{a(x,y,z-k(t))^2}\right)dt^2+2v\frac{f(x,y,z-k(t))}{a(x,y,z-k(t))}dt\,dz-dx^2-dy^2-dz^2 \qquad (1)$$

From the components of the Einstein tensor in contravariant form [11] for

- 1)-The Pfenning zone is the zone within the interval: $R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2}$ where $\Delta\ll 1$ $R$ is the radius of the Warp bubble and $\Delta$ is the wall thickness of the Warp bubble $R\gg\Delta$ .
- 2)- $r=(x^2+y^2+(z-k(t))^2)^{\frac{1}{2}}$ and $\frac{dk(t)}{dt}=v=const$
- 3)-In the Pfenning zone we let $a=a(r)=a(x,y,z-k(t))\gg 1$ (there is the source of esotic matter),and $D_i a\le a\,, D_{i\,,k}a\le a$ or $da(r)/dr\le a(r)$

can be reduced by an arbitrary value. $G^{tt}$ is [11]:

where $D_i=\frac{\partial}{\partial x^i}\,, x^i=x\,,y\,,z\,(i=1,2,3)$

$$G^{tt}=-\frac{1}{4}\frac{1}{a(x,y,z-k(t))^4}(v^2(f(x,y,z-k(t))^2\,\mathrm{D}_2(a)(x,y,z-k(t))^2+f(x,y,z-k(t))^2\,\mathrm{D}_1(a)(x,y,z-k(t))^2$$

$$+\mathrm{D}_1(f)(x,y,z-k(t))^2\,a(x,y,z-k(t))^2+\mathrm{D}_2(f)(x,y,z-k(t))^2\,a(x,y,z-k(t))^2$$

$$-2\,\mathrm{D}_1(f)(x,y,z-k(t))\,a(x,y,z-k(t))\,f(x,y,z-k(t))\,\mathrm{D}_1(a)(x,y,z-k(t))$$

$$-2\,\mathrm{D}_2(f)(x,y,z-k(t))\,a(x,y,z-k(t))\,f(x,y,z-k(t))\,\mathrm{D}_2(a)(x,y,z-k(t))))$$

Einstein Equations:

$$G^{ik}=\frac{8\pi G}{c^4}T^{ik} \quad [15] \quad T^{ik} \text{ (energy-impulse tensor)}$$

The functions $f(r)=f(x,y,z-k(t))$ and $a(r)=a(x,y,z-k(t))$ can assume the following values:

- 1)-inside the warp bubble $(0<r<R-\frac{\Delta}{2})$ $f(r)=1$ and $a(r)=1$
- 2)-outside the warp bubble $(r>R+\frac{\Delta}{2})$ $f(r)=0$ and $a(r)=1$
- 3)-in the Alcubierre warped region $(R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2})$ $0<f(r)<1$ , $f(r)$ is

$f(r)=-\frac{(r-R-\frac{\Delta}{2})}{\Delta}$ (Pfenning zone [4]) and $a=a(r)=a(x,y,z-k(t))\gg 1$

(possessing extremely large values) and $D_i a \le a, D_{i,k} a \le a$ or $da(r)/dr \le a(r)$

- 1)-Internal metric of the Warp bubble $(0<r<R-\frac{\Delta}{2})$ is:

$$ds^2=dt^2-(dz-vdt)^2-dx^2-dy^2 \tag{2}$$

moving with velocity v (multiple of the speed of light c) along the z-axis.

- 2)-Metric outside of the bubble beyond the Pfenning zone $(r>R+\frac{\Delta}{2})$ is:

$$ds^2=dt^2-dx^2-dy^2-dz^2 \tag{3}$$

**2 Computation of the negative energy in the Pfenning zone, its comparison with the Casimir effect (plane-parallel condenser) and quantum inequalities.**

The energy is:

$$E=\int\int\int(-g)^{1/2}T^{tt}\,dx^{3} \tag{4}$$

where the triple integral extends over all the volume

and the energy density is

$$T^{tt}=k\,G^{tt} \tag{5}$$

$(g\ \textit{determinant of the spacial metric})$ and $k=c^{4}/8\pi G$ .

In our case we get, in xyz-coordinates:

$$G^{tt}=-\frac{1}{4}\frac{1}{a(x,y,z-k(t))^{4}}\left(v^{2}\left(f(x,y,z-k(t))^{2}\,D_2(a)(x,y,z-k(t))^{2}+f(x,y,z-k(t))^{2}\,D_1(a)(x,y,z-k(t))^{2}\right.\right.$$

$$+D_1(f)(x,y,z-k(t))^{2}\,a(x,y,z-k(t))^{2}+D_2(f)(x,y,z-k(t))^{2}\,a(x,y,z-k(t))^{2}$$

$$-2\,D_1(f)(x,y,z-k(t))\,a(x,y,z-k(t))\,f(x,y,z-k(t))\,D_1(a)(x,y,z-k(t))$$

$$\left.\left.-2\,D_2(f)(x,y,z-k(t))\,a(x,y,z-k(t))\,f(x,y,z-k(t))\,D_2(a)(x,y,z-k(t))\right)\right)$$

which, written in Alcubierre form becomes:

$$G^{tt}=-\frac{1}{4}v^{2}\frac{x^{2}+y^{2}}{r^{2}}g(r) \tag{6}$$

where $g(r)$ is given by:

$$g(r)=\left[\frac{1}{a(r)^{2}}\left(\frac{df(r)}{dr}\right)^{2}+\left(\frac{f(r)^{2}}{a(r)^{4}}\right)\left(\frac{da(r)}{dr}\right)^{2}-2\frac{df(r)}{dr}\frac{f(r)}{a(r)^{3}}\frac{da(r)}{dr}\right] \tag{7}$$

In the simplified case:

- 1)-inside the warp bubble $(0<r<R-\frac{\Delta}{2})$ $f(r)=1$ and $a(r)=1$
- 2)-outside the warp bubble $(r>R+\frac{\Delta}{2})$ $f(r)=0$ and $a(r)=1$
- 3)-in the Alcubierre warped region $(R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2})$ $0<f(r)<1$ , $f(r)$ is

  $f(r)=-\frac{(r-R-\frac{\Delta}{2})}{\Delta}$ and $a(r)=a(x,y,z-k(t))=A=constant\gg 1$ possessing extremely large values

**2.1 The energy E in the Pfenning zone in our case (see [4] for an example):**

The energy given by (4), taking into account the conditions set there in, becomes:

$$E=-\frac{8\pi k v^2}{32\pi}\int\frac{x^2+y^2}{r^2}g(r)dx^3 \tag{8}$$

$$g(r)\approx\left(\frac{-1}{a(r)\Delta}\right)^2 \tag{9}$$

((9) is dominant term of (7) for $\Delta\ll 1$ , $a=a(r)\gg 1$ and $D_i a\leq a, D_{i,k} a\leq a$ or $da(r)/dr\leq a(r)$ , $a(r)>1/\Delta$

$$E=-\frac{8\pi k v^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} r^2 g(r)dr=-\frac{8\pi k v^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} r^2\left(\frac{-1}{a(r)\Delta}\right)^2 dr \tag{10}$$

$$E=-\frac{8\pi k v^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} r^2\left(\frac{-1}{A\Delta}\right)^2 dr \tag{11}$$

$$E=-\frac{8\pi k}{12}v^2\frac{\left(\frac{R^2}{\Delta}+\frac{\Delta}{12}\right)}{A^2} \tag{12}$$

where $R \gg \Delta$ (very likely), R being the radius of the warp bubble. If $\Delta$ is about

$1.6 \bullet 10^{-35} m$ (Planck lenght) as an absurd, setting $A = 10^{50}$ and R = 100 m, the energy E gets

reduced significantly. Using the chosen data the energy is $E = -3 \bullet 10^{-17} v^2 \, joule$ quite smaller

than the U(d) due the the Casimir effect.

**2.2 The energy U(d) between the two plates in a plane-parallel condenser in empty space, due to the Casimir effect, is:**

$$U(d) = -\pi^2 \left[ \left(\frac{h}{2\pi}\right) \frac{c}{720 \mathrm{d}^3} \right] L^2 \tag{13}$$

where d is the distance between the plates and L is the side of the square conducting plate. As can

be seen, the energy is negative and this implies that the force (equal to the opposite of the derivative

with respect to d) is attractive, as has been experimentally found ("Lamoreaux" [12]). In the case

$d = 1 \mu m$ and L = 1 m (L is chosen to be quite large, but not as large as the Pfenning zone) it is

found that $U(d) = -4\, 10^{-10} \, joule$ .

**2.3 Quantum inequalities. Calculation for our solution:**

The quantum inequalities are [13]:

$$\frac{t_0}{\pi} \int_{-\infty}^{+\infty} \frac{\langle T^{ik} u_i u_k \rangle}{t^2 + t_0^2} dt \geq -\frac{3}{32 \pi^2 t_0^4} \tag{14}$$

for $(h, c, G = 1)$ and $u_i$ is the quadrivelocity in a Eulerian or moving reference system. In our case using the International System of Units we get as Pfenning solution [4] :

$$\Delta \leq \frac{1}{\alpha^2} v^2 L_{Planck} \tag{15}$$

where Pfenning in his paper [4] chooses $\alpha = 1/10$ and therefore

$$\Delta \leq 10^2 v^2 L_{Planck} \tag{16}$$

I believe that the concentration of energy in a very small volume is due to the bad choice of the $f = f(r)$ function,or of the parameter $\alpha$ by Pfenning [4], since the quantum inequalities are valid for all values of $t_0$ [13] and this leads to $0 < \alpha \ll 1$ therefore $\alpha$ to an arbitrary value. We can conclude that $\alpha$ can assume an arbitrarily small value, and the results presented in [13] provide indirect evidence of this.

**3 Conclusion:** The calculations seem to suggest that the modified Alcubierre propulsion system allows to reach superluminal speeds without problems in the energy density and components of the energy-impulse tensor, and that the negative energy can be arbitrarily reduced. This energy is compared with that of the Casimir effect in a parallel plane condenser, to investigate the experimental feasibility. Quantum inequalities are then calculated in orded to investigate the spacial distribution of the negative energy in the Pfenning zone. In the Alcubierre solution a event horizon is present at $v > 1$ while in our solution it is present at $v \geq 1$ . In our next paper this problem will be investigated [16].

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