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Why one can maintain that there is a probability loophole in the CHSH

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Abstract: In the paper it is demonstrated that the particular compact form of CHSH, $S = E\{A(1)[B(1) - B(2)] - A(2)[B(1) + B(2)]\}$ with, S maximally 2 and minimally-2, for A and B functions $\in \{-1, 1\}$, is not generally valid for local models. The nonzero probability that local hidden extra parameters violate the CHSH, is not eliminated with basic principles derived from the CHSH.

Keywords: Clauser, Horne, Shimony and Holt criterion, quantum mechanical entanglement.

Contents

1 Introduction and test of the CHSH

The CHSH inequality is an important element in the discussion about the existence or nonexistence of additional local hidden parameters [1]. The CHSH inequality [2] is derived from Bells formula for the correlation [3], $E(a, b)$, between distant spin measurements with setting setting parameters a and b. Generally,

$$E(a, b) = \int d\lambda \rho_\lambda A_\lambda(a) B_\lambda(b) \tag{1.1}$$

In (1.1) we can identify the probability density $\rho_\lambda \geq 0$, with $\int d\lambda \rho_\lambda = 1$. The λ are introduced to explain the correlation and need to have a local effect. This can e.g. be accomplished [5] if a λ_1 is assigned to the A measurement instrument and λ_2 to the B instrument. Furthermore, the measurement functions $A_\lambda(a)$ and $B_\lambda(b)$ both project in $\{-1, 1\}$ to represent binairy spin variables (e.g. up=1, down=-1). The CHSH inequality is based on the following expression,

$$S = E(1, 1) - E(1, 2) - E(2, 1) - E(2, 2) \tag{1.2}$$

The quartet of setting pairs $\mathcal{Q} = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$ occurs random in a series of N spin measurements of entangled particle pairs. Alice and Bob are two assitents in the experiment who, per trial or particle pair measurement, randomly select the setting of their measurement instrument. The argument in favor of the CHSH inequality [4] and against a possible probability loophole [5] is as follows. From (1.1) and (1.2) we may write, suppressing the hidden variables index λ, notation for the moment,

$$S = E\{A(1)[B(1) - B(2)] - A(2)[B(1) + B(2)]\}. \tag{1.3}$$

According to [4], because, A and B are both $\in \{-1, 1\}$, we see that when $B(1) = B(2)$, then $S = \pm 2$, while, when $B(1) = -B(2)$, it again flollows, $S = \pm 2$. Hence, $|S|$ based on (1.1) cannot be larger than 2 and therefore the nonzero probability of $|S| > 2$ with a local hidden variables model of [5] must be based on a mistake. It will be demonstrated in the next section that this claim is untrue. In the paper we show that this argument does not hold in general. The loophole paper [5] has the intention to derive a test of the strength of conclusions that can be derived from the CHSH inequality. Tests of strength are not uncommon in statistcs. In [5] this is done via a reformulation of Bells formula. Let us define sets based on the difference $E(a,b) - E(x,y)$, (a,b) and (x,y) are different settings. We have, $\Omega_+(a,b;x,y) = \{\lambda \,|\, A_\lambda(a)B_\lambda(b) = A_\lambda(x)B_\lambda(y) = +1\}$, together with $\Omega_-(a,b;x,y) = \{\lambda \,|\, A_\lambda(a)B_\lambda(b) = A_\lambda(x)B_\lambda(y) = -1\}$ and $\Omega_0(a,b;x,y) = \{\lambda \,|\, A_\lambda(a)B_\lambda(b) = -A_\lambda(x)B_\lambda(y) = \pm 1\}$. The three sets are disjoint and if Λ denotes the universe set of the λ variables we also have $\Lambda = \Omega_+(a,b;x,y) \cup \Omega_-(a,b;x,y) \cup \Omega_0(a,b;x,y)$. Note that in $E(a,b) - E(x,y)$ only the $\lambda \in \Omega_0(a,b;x,y)$ contribute. Therefore

$$E(a,b) - E(x,y) = -2\int_{\lambda \in \Omega_0(a,b;x,y)} A_\lambda(x)B_\lambda(y)d\lambda \tag{1.4}$$

If subsequently, $E(a,b) = 0$ and we write $\Omega'_0(x,y) = \Omega_0(a,b;x,y)$ and (a,b) such that $E(a,b) = 0$, then

$$E(x,y) = 2\int_{\lambda \in \Omega'_0(x,y)} \rho_\lambda A_\lambda(x)B_\lambda(y)d\lambda \tag{1.5}$$

With $E(x,y) = E_T(x,y)$. Subsequently from $E(a,b) = 0$ it follows [5] that,

$$E_C(x,y) = 2\int_{\lambda \in \Omega'_+(x,y)} \rho_\lambda d\lambda - 2\int_{\lambda \in \Omega'_-(x,y)} \rho_\lambda d\lambda \tag{1.6}$$

and, of course, $E_C(x,y) = E(x,y)$ via $E(a,b) = 0$.

The settings that we employ are, for Alice, $1_A = (1,0,0)^T$ and $2_A = (0,0,1)^T$. The superscript T means transposed of a vector. For Bob we take, $1_B = \frac{1}{\sqrt{2}}(1,1,0)^T$ and $2_B = \frac{1}{\sqrt{2}}(-1,0,-1)^T$. If the A and B indices in 1_A etc, are not necessary they will be omitted. With this selection of setting vectors and taking the quantum correlation innerproduct $\langle a,b\rangle = \sum_{i=1}^{3} a_i b_i$, the S from (1.2) will produce $|S| = \frac{3}{\sqrt{2}} > 2$. Like in [5] we take the probability density, $\rho_\lambda = \rho_{\lambda_1,\lambda_2}$ and $\rho_{\lambda_1,\lambda_2} = \rho_{\lambda_1}\rho_{\lambda_2}$. The separate λ_1, is assigned to A and λ_2, is assigned to B. For, $j = 1,2$,

$$\rho_{\lambda_j} = \{ \begin{matrix} \frac{1}{\sqrt{2}}, & \lambda_j \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] = \Lambda_j \\ 0, & \lambda_j \notin \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \end{matrix} \tag{1.7}$$

with the universal set, $\Lambda = \Lambda_1 \times \Lambda_2$. Furthermore, $\Omega'_\pm(x,y)$, is the Cartesian product of a λ_1 and a λ_2 interval, i.e. $\Omega'_\pm(x,y) = \Omega'_{A\pm}(x) \times \Omega'_{B\pm}(y)$. Similar as in [5] let us take

$$\begin{matrix} \Omega'_{A\pm}(1) \in \left\{\emptyset, \{\lambda_1 \,|\, -1 + \frac{1}{\sqrt{2}} \leq \lambda_1 \leq \frac{1}{\sqrt{2}}\}\right\}, & \Omega'_{B\pm}(1) \in \left\{\emptyset, \{\lambda_2 \,|\, -\frac{1}{\sqrt{2}} \leq \lambda_2 \leq 0\}\right\} \\ \Omega'_{A\pm}(2) \in \left\{\emptyset, \{\lambda_1 \,|\, -\frac{1}{\sqrt{2}} \leq \lambda_1 \leq 1 - \frac{1}{\sqrt{2}}\}\right\}, & \Omega'_{B\pm}(2) \in \left\{\emptyset, \{\lambda_2 \,|\, 0 < \lambda_2 \leq \frac{1}{\sqrt{2}}\}\right\} \end{matrix} \tag{1.8}$$

Note, $\int_{\lambda_j \in \emptyset} \rho_{\lambda_j} d\lambda_j = 0$. The following form will be used in the study of (1.3),

$$E_C(x,y) = \int_{\Omega'_{A+}(x)} d\lambda_1 \int_{\Omega'_{B+}(y)} d\lambda_2 - \int_{\Omega'_{A-}(x)} d\lambda_1 \int_{\Omega'_{B-}(y)} d\lambda_2 \tag{1.9}$$

In order to have a similar approach as in (1.3) we introduce $\theta^{\cdot}_{\cdot,\cdot}(x)$. Those variables are associated to the sets in (1.8). For instance let us define

$$\theta^{\pm}_{A\,\lambda_1}(x) = \{ \begin{matrix} 1, & \lambda_1 \in \Omega'_{A\pm}(x) \neq \emptyset, \\ 0, & \lambda_1 \notin \Omega'_{A\pm}(x). \end{matrix} \tag{1.10}$$

We note that $\theta^{\pm}_{A\,\lambda_1}(x) = 0$, when, $\Omega'_{A\pm}(x) = \emptyset$. Similarly,

$$\theta^{\pm}_{B\,\lambda_2}(y) = \{ \begin{matrix} 1, & \lambda_2 \in \Omega'_{B\pm}(y) \neq \emptyset, \\ 0, & \lambda_2 \notin \Omega'_{B\pm}(y). \end{matrix} \tag{1.11}$$

and $\theta^{\pm}_{B\,\lambda_2}(y) = 0$ when $\Omega'_{B\pm}(y) = \emptyset$. Given the expressions in (1.9) - (1.11), the S in (1.3) can be written as

$$\begin{aligned} S = \int_{\Lambda_1} d\lambda_1 \int_{\Lambda_2} d\lambda_2 [&\theta^{+}_{A\,\lambda_1}(1)\theta^{+}_{B\,\lambda_2}(1) - \theta^{-}_{A\,\lambda_1}(1)\theta^{-}_{B\,\lambda_2}(1) \\ &-\theta^{+}_{A\,\lambda_1}(1)\theta^{+}_{B\,\lambda_2}(2) + \theta^{-}_{A\,\lambda_1}(1)\theta^{-}_{B\,\lambda_2}(2) \\ &-\theta^{+}_{A\,\lambda_1}(2)\theta^{+}_{B\,\lambda_2}(1) + \theta^{-}_{A\,\lambda_1}(2)\theta^{-}_{B\,\lambda_2}(1) \\ &-\theta^{+}_{A\,\lambda_1}(2)\theta^{+}_{B\,\lambda_2}(2) + \theta^{-}_{A\,\lambda_1}(2)\theta^{-}_{B\,\lambda_2}(2)] \end{aligned} \tag{1.12}$$

Suppose we take the following values for the $\theta^{\cdot}_{\cdot,\cdot} \in \{0,1\}$ variables in (1.12).

$$\begin{aligned} &\theta^{+}_{A\,\lambda_1}(1) = \theta^{+}_{B\,\lambda_2}(1) = 1, \theta^{-}_{A\,\lambda_1}(1) = 1, \theta^{-}_{B\,\lambda_2}(1) = 0, \\ &\theta^{-}_{A\,\lambda_1}(2) = \theta^{-}_{B\,\lambda_2}(2) = 1, \theta^{+}_{A\,\lambda_1}(2) = \theta^{+}_{B\,\lambda_2}(2) = 0. \end{aligned} \tag{1.13}$$

The possibility of selection of $\theta^{\cdot}_{\cdot,\cdot} \in \{0,1\}$ such as in (1.13) cannot be rejected. With this selection of $\theta^{\cdot}_{\cdot,\cdot} \in \{0,1\}$ variables, possible confusion of "multiple random models" is avoided. The averaging over models $\mathcal{L}$ such as was done in [4] does not apply to the present case. Its use in [4] was already questionable. In [5] there is only one single fixed model with random input. In the present paper the line of reasoning presented in (1.3) which was also used in [4] to reject the conclusions from [5] leads us to

$$S = \int_{-1+\frac{1}{\sqrt{2}}}^{+\frac{1}{\sqrt{2}}} d\lambda_1 \int_{-\frac{1}{\sqrt{2}}}^{0} d\lambda_2 + \int_{-1+\frac{1}{\sqrt{2}}}^{+\frac{1}{\sqrt{2}}} d\lambda_1 \int_{0}^{\frac{1}{\sqrt{2}}} \lambda_2 + \int_{-\frac{1}{\sqrt{2}}}^{1-\frac{1}{\sqrt{2}}} d\lambda_1 \int_{0}^{\frac{1}{\sqrt{2}}} d\lambda_2 \tag{1.14}$$

Hence, $S = \frac{3}{\sqrt{2}}$ and therefore $|S| > 2$ with a single fixed local hidden variables model where the method of deriving S is similar to the way it is used in [4].

2 Numerical consistency

The previous analysis was done in terms of Ω_+ and Ω_- sets. Here we show that the method is consistent. First we define the measurement functions A and B. We have, $\alpha = \pm 1$ and $\mathrm{sgn}(\xi) = \pm 1$, for ξ real.

$$A_{\lambda_1}(x) = \{ \begin{matrix} \alpha(x), & \lambda_1 \in I(x) \\ \mathrm{sgn}\{\zeta(x) - \lambda_1\}, & \lambda_1 \in \Lambda_1 \backslash I(x) \end{matrix} \tag{2.1}$$

With $I(x) \subset \Lambda_1$ and $\zeta(x) \in \Lambda_1 \backslash I(x)$. For $B_{\lambda_2}(y)$ a similar form is given, $\beta(y) = \pm 1$,

$$B_{\lambda_2}(y) = \{ \begin{matrix} \beta(y), & \lambda_2 \in J(y) \\ \mathrm{sgn}\{\eta(y) - \lambda_2\}, & \lambda_2 \in \Lambda_2 \backslash J(y) \end{matrix} \tag{2.2}$$

With $J(y) \subset \Lambda_2$ and $\eta(y) \in \Lambda_2 \backslash J(y)$. Let us look at the setting pair, $(1,1)$. If $\theta^{+}_{A\,\lambda_1}(1) = 1$ then, $\lambda_1 \in \{\lambda_1 \,|\, -\frac{1}{\sqrt{2}} \leq \lambda_1 < -1+\frac{1}{\sqrt{2}}\} = \Lambda_1 \backslash I(1)$, is available to build Ω_0. For $\theta^{+}_{B\,\lambda_2}(1) = 1$, we have $\lambda_2 \in \{\lambda_2 \,|\, 0 < \lambda_2 \leq \frac{1}{\sqrt{2}}\} = \Lambda_2 \backslash J(1)$. Hence,

$$\begin{aligned} \Omega_0'(1,1) = &\{\lambda_1 \,|\, -1 + \tfrac{1}{\sqrt{2}} \leq \lambda_1 < \tfrac{1}{\sqrt{2}}\} \times \{\lambda_2 \,|\, 0 < \lambda_2 \leq \tfrac{1}{\sqrt{2}}\} \\ &\cup \{\lambda_1 \,|\, -\tfrac{1}{\sqrt{2}} \leq \lambda_1 < -1 + \tfrac{1}{\sqrt{2}}\} \times \{\lambda_2 \,|\, -\tfrac{1}{\sqrt{2}} \leq \lambda_2 \leq 0\} \\ &\cup \{\lambda_1 \,|\, -\tfrac{1}{\sqrt{2}} \leq \lambda_1 < -1 + \tfrac{1}{\sqrt{2}}\} \times \{\lambda_2 \,|\, 0 < \lambda_2 \leq \tfrac{1}{\sqrt{2}}\} \end{aligned} \tag{2.3}$$

From equation (1.5) and definitions in (2.1) and (2.2) employing (2.3) the following expression obtains

$$\begin{aligned} E_T(1,1) = &\alpha(1) \left(\int_0^{\eta(1)} d\lambda_2 - \int_{\eta(1)}^{1/\sqrt{2}} d\lambda_2 \right) + \tfrac{\beta(1)}{\sqrt{2}} \left(\int_{-1/\sqrt{2}}^{\zeta(1)} d\lambda_1 \,-\, \int_{\zeta(1)}^{-1+1/\sqrt{2}} d\lambda_1 \right) \\ &+ \left(\int_{-1/\sqrt{2}}^{\zeta(1)} d\lambda_1 \,-\, \int_{\zeta(1)}^{-1+1/\sqrt{2}} d\lambda_1 \right) \left(\int_0^{\eta(1)} d\lambda_2 - \int_{\eta(1)}^{1/\sqrt{2}} d\lambda_2 \right) \end{aligned} \tag{2.4}$$

This gives,

$$E_T(1,1) = \alpha(1) \left(2\eta(1) - \frac{1}{\sqrt{2}} \right) + \frac{\beta(1)}{\sqrt{2}} \left(2\zeta(1) + 1 \right) + \left(2\zeta(1) + 1 \right) \left(2\eta(1) - \frac{1}{\sqrt{2}} \right) \tag{2.5}$$

Here we note that, $\eta(1) \in \{\eta \,|\, 0 < \eta \leq \frac{1}{\sqrt{2}}\}$ and $\zeta(1) \in \{\zeta \,|\, -\frac{1}{\sqrt{2}} \leq \zeta < -1 + \frac{1}{\sqrt{2}}\}$. In the numerical study we found, $\alpha(1) = \beta(1) = -1$ and $\zeta(1) \approx -0.63786$, and $\eta(1) \approx 0.15284$. This gives $E(1,1) \approx 0.70707$. This agrees with $1_A = (1,0,0)^T$ and $1_B = \frac{1}{\sqrt{2}}(1,1,0)^T$. For setting pair $(2,1)$ we have in a similar way for Ω_0,

$$\begin{aligned} \Omega_0'(2,1) = &\{\lambda_1 \,|\, -\tfrac{1}{\sqrt{2}} \leq \lambda_1 < 1 - \tfrac{1}{\sqrt{2}}\} \times \{\lambda_2 \,|\, 0 < \lambda_2 \leq \tfrac{1}{\sqrt{2}}\} \\ &\cup \{\lambda_1 \,|\, 1 - \tfrac{1}{\sqrt{2}} < \lambda_1 < \tfrac{1}{\sqrt{2}}\} \times \{\lambda_2 \,|\, -\tfrac{1}{\sqrt{2}} \leq \lambda_2 \leq 0\} \\ &\cup \{\lambda_1 \,|\, 1 - \tfrac{1}{\sqrt{2}} < \lambda_1 < \tfrac{1}{\sqrt{2}}\} \times \{\lambda_2 \,|\, 0 < \lambda_2 \leq \tfrac{1}{\sqrt{2}}\} \end{aligned} \tag{2.6}$$

In a similar manner it follows that

$$E_T(2,1) = \alpha(2) \left(2\eta(1) - \frac{1}{\sqrt{2}} \right) + \frac{\beta(1)}{\sqrt{2}} \left(2\zeta(2) - 1 \right) + \left(2\zeta(2) - 1 \right) \left(2\eta(1) - \frac{1}{\sqrt{2}} \right) \tag{2.7}$$

The $\eta(1) \in \{\eta \,|\, 0 < \eta \leq \frac{1}{\sqrt{2}}\}$ and is already established approximately, $\eta(1) \approx 0.15284$. For $\zeta(2)$ we must see $\zeta(2) \in \{\zeta \,|\, 1 - \frac{1}{\sqrt{2}} < \zeta < \frac{1}{\sqrt{2}}\}$. Numerical study gives approximately, $\zeta(2) \approx 0.68110$, together with $\alpha(2) = -1$. The result of the numerical values for $\alpha(2), \beta(1), \zeta(2)$ and $\eta(1)$ is that $E(2,1) \approx -8.1913 \times 10^{-5}$. This agrees with $2_A = (0,0,1)^T$ and $1_B = \frac{1}{\sqrt{2}}(1,1,0)^T$. For pair $(1,2)$ we similarly have

$$\begin{aligned} \Omega_0'(1,2) = &\{\lambda_1 \,|\, -1 + \tfrac{1}{\sqrt{2}} \leq \lambda_1 \leq \tfrac{1}{\sqrt{2}}\} \times \{\lambda_2 \,|\, -\tfrac{1}{\sqrt{2}} \leq \lambda_2 \leq 0\} \\ &\cup \{\lambda_1 \,|\, -\tfrac{1}{\sqrt{2}} \leq \lambda_1 < -1 + \tfrac{1}{\sqrt{2}}\} \times \{\lambda_2 \,|\, 0 < \lambda_2 \leq \tfrac{1}{\sqrt{2}}\} \\ &\cup \{\lambda_1 \,|\, -\tfrac{1}{\sqrt{2}} \leq \lambda_1 < -1 + \tfrac{1}{\sqrt{2}}\} \times \{\lambda_2 \,|\, -\tfrac{1}{\sqrt{2}} \leq \lambda_2 \leq 0\} \end{aligned} \tag{2.8}$$

Hence,

$$E_T(1,2) = \alpha(1)\left(2\eta(2) + \frac{1}{\sqrt{2}}\right) + \frac{\beta(2)}{\sqrt{2}}\left(2\zeta(1)+1\right) + \left(2\zeta(1)+1\right)\left(2\eta(2) + \frac{1}{\sqrt{2}}\right) \quad (2.9)$$

We already had, $\zeta(1) \in \{\zeta \,|\, -\frac{1}{\sqrt{2}} \le \zeta < -1 + \frac{1}{\sqrt{2}}\}$ and numerically, $\zeta(1) \approx -0.63786$, together with $\alpha(1) = -1$. For $\eta(2)$ we found in numerical study, $\eta(2) \approx -3.7137 \times 10^-5$ and it is required that $\eta(2) \in \{\eta \,|\, -\frac{1}{\sqrt{2}} \le \eta \le 0\}$. This gives $E(1,2) \approx -0.70701$, with, $\beta(2) = -1$. This agrees with $1_A = (1,0,0)^T$ and $2_B = \frac{1}{\sqrt{2}}(-1,0,-1)^T$. Finally, we have all α, β, ζ and η numerical values and need to check for $E(2,2)$. The Ω_0 is

$$\begin{aligned} \Omega_0'(2,2) = &\{\lambda_1 \,|\, -\tfrac{1}{\sqrt{2}} \le \lambda_1 < 1 - \tfrac{1}{\sqrt{2}}\} \times \{\lambda_2 \,|\, -\tfrac{1}{\sqrt{2}} \le \lambda_2 \le 0\} \\ &\cup \{\lambda_1 \,|\, 1 - \tfrac{1}{\sqrt{2}} < \lambda_1 < \tfrac{1}{\sqrt{2}}\} \times \{\lambda_2 \,|\, 0 < \lambda_2 \le \tfrac{1}{\sqrt{2}}\} \\ &\cup \{\lambda_1 \,|\, 1 - \tfrac{1}{\sqrt{2}} < \lambda_1 < \tfrac{1}{\sqrt{2}}\} \times \{\lambda_2 \,|\, -\tfrac{1}{\sqrt{2}} \le \lambda_2 \le 0\} \end{aligned} \quad (2.10)$$

This then leads us to

$$E_T(2,2) = \alpha(2)\left(2\eta(2) + \frac{1}{\sqrt{2}}\right) + \frac{\beta(2)}{\sqrt{2}}\left(2\zeta(2)-1\right) + \left(2\zeta(2)-1\right)\left(2\eta(2) + \frac{1}{\sqrt{2}}\right) \quad (2.11)$$

We already found $\alpha(2) = \beta(2) = -1$ and $\zeta(2) \approx 0.68110$, with, $\eta(2) \approx -3.7137 \times 10^-5$. This resulted in $E(2,2) \approx -0.70706$ and this agrees with $2_A = (0,0,1)^T$ and $2_B = \frac{1}{\sqrt{2}}(-1,0,-1)^T$.

Plugging the results of this single fixed and definitely *not* random model into the CHSH criterion we find that $S = E(1,1) - E(2,1) - E(1,2) - E(2,2) \approx 2.12122 > 2$. The numerical error fluctuates in the sixth decimal. The local hidden variables model presented in the preceding pages violates the CHSH criterion. In the present section numerical consistency was demonstrated. The numerical consistency supports the Ω_+ and Ω_- analysis of the previous section. This nullifies the argument implicit in [4] that only (random) variating models can violate the criterion. It also rejects the claim that "compact" version $S = E\{A(1)[B(1) - B(2)] - A(2)[B(1) + B(2)]\}$ forbids locality to violate with nonzero probability the CHSH criterion such as given in [5].

3 Conclusion & Discussion

We conclude that the conjecture in [4] that "there must be a mistake in [5]" is unjustified. There is a single fixed local variables model that can violate the ± 2 CHSH criterion. A possible objection that A and B functions do not exist is definitely unfounded. Both E_C as well as E_T are equivalent to the same Bell formula. Conclusions for E_C are valid for E_T and vice versa. We devoted a chapeter to the numerical underpinning of the latter claim.

In this paper it was demonstrated that the step from term-by-term: $S = E(1,1) - E(1,2) - E(2,1) - E(2,2)$, to compact $S = E\{A(1)[B(1) - B(2)] - A(2)[B(1) + B(2)]\}$ and therefore S for A and B both $\in \{-1, 1\}$ exclusively varying between and including -2 and 2, does not hold in all local hidden variable model cases. If the step from term-by-term to compact is allowed in the original expression of Bells formula then it is allowed in the Ω set analysis of Bells formula in [5].

In [5] it was demonstrated that a nonzero probability for $2\sqrt{2}$ violation of the CHSH exists with local hidden variables. In the present paper we showed with a single fixed model that violation amounts to $\frac{3}{\sqrt{2}}$. The latter is, of course, larger than 2 and shows that the "there must be a mistake" argument in [4] based on $|S| \leq 2$, is false. The reader should note that the objection "it is not possible to violate maximally" was already dealt with by showing nonzero probability in [5]. Shifting the goal-post from 2 in the direction of $2\sqrt{2}$ and therewith reject the possibility that CHSH has a probability loophole, can be judged as an extremely weak defense of a criterion that sternly eliminates certain types of models. Furthermore, the model in [5] is fixed and only the data used in the model varies randomly. Hence, the $\mathcal{L}$ argument of [4] certainly does not apply to the present case and is also invalid for the case of [5].

Concludingly, it was demonstrated that a single fixed model of local hidden variables may violate the CHSH, ± 2, bounds. Therefore, local hidden variables are possible. We may think of a form of 'tHoofts predeterminism [6], interpreted as random events that are, to nature, a logical progression of the unfolding of events. We can also mention the possible existence of mirror matter [7], [8], [9], [10], [11] as an unknown in the explanation of the entanglement. However, hypothetical mirror matter and matter exchange information via gravity and the mixing of mirror-photons and photons [8]. In entanglement, the mixing of the two types of photons then should be held responsible.

Finally, perhaps that there is no explanation for entanglement. It is then however reasonable to expect that this conclusion is arrived at with the most stringently possible tested statistics.

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