

An elementary proof of the Riemann Hypothesis

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One of the possible ways to deal with RH is to deal with Mertens function, and check it for all possible powers $(\frac{1}{2}+\varepsilon)$ of N when the integer N tends to infinity and the real $(\frac{1}{2} + \varepsilon)$ is between $\frac{1}{2}$ and 1 excluded

The key point is that, even tending to infinite, N always stays finite, and finite stays the amount of primes and composite integers. Thus the factor parity checked by Mertens function may be affected by integers only for rational values of the exponent $(\frac{1}{2}+\varepsilon)$, ruling out each ε not rational. Then we can discard all the not rational values of ε .

Let us now consider separately each subset of possible integers N contributing to each still allowed rational ε : every rational value greater than $\frac{1}{2}$ and lower than 1 may be represented as a fraction among integers. After possible simplifications, its minimal form will exhibit a numerator not lower than 2. Thus, all the integers not ruled out by the denominator and then possibly belonging to each of such subsets will not be squarefree: they can all be discarded in the computation of the Mertens function: then no ε different from 0 does imply integers contributing to the considered exponent, then ε has to be taken equal to 0 thus proving Riemann Hypothesis.