There are infinitely many quadruples of primes

Diego Liberati
Consiglio Nazionale delle Ricerche

With the exception of (5 7 11 13), every quadruple of primes is centered on odd multiples of 15, like (11 13 17 19), (101 103 107 109) and so on, but not all the odd multiples of 15 are the center of a prime quadruple: this is because some of them are sieved by primes higher than 5.

In fact, every prime p higher than 5 does sieve such a candidate quadruple, in one of its four positions, 4 times every p candidates, thus not sieving p-4 quadruples every p candidates.

Then, in order to compute how many candidate quadruples are not sieved by any of such infinite primes, one need to compute the product of the fraction p-4/p over all the primes p greater than 5.

When the number of primes tends to the infinity, both the numerator and the denominator of such product tend to the infinity with the same strength (even if the fraction tends to zero, quite slowly indeed, because p-4/p tends to increase toward 1 when p increases).

Thus the numerator proves that there are infinitely many quadruples of primes not sieved by any lower prime, then proving the conjecture.

It follows that there are infinitely many twin pairs, being a quadruple formed by a pair of twin pairs. Thus Polignac conjecture is true for n=2.

Moreover, it follows that Polignac conjecture is also true for n=4, being every center of the quadruple an isolated composite, thus centering a consecutive prime gap of 4.