Entanglement And The Special Theory Of Relativity

Ilija Barukčić [1, 2]

[1] Horandstrasse, DE-26441 Jever, Germany.
[2] Corresponding author: Barukcic@t-online.de

Manuscript submitted to viXra.org on Sunday, June 28, 2015.

Abstract: The exact nature of non-locality and entanglement is still a matter of an ongoing controversy. Especially, the concept of non-locality as postulated by the orthodox Copenhagen quantum mechanics is claiming to reflect any non-locality in the quantum realm. Attention should be called to the obvious but very disconcerting fact that the concept of non-locality cannot contradict the theory of special relativity, as long as the same is not refuted theoretically or by experiments. Another way of expressing the peculiar situation is, under conditions of the special theory of relativity it remains rather discomforting to alter the properties of a distant system instantaneously (i. e. no light signal can travel) by acting on a local system. The purpose of this publication is to solve the problem of non-locality and entanglement from the standpoint of the special theory of relativity.

Key words: Quantum theory, relativity theory, unified field theory, causality.

1. Introduction

Einstein's historical critique of the orthodox Copenhagen interpretation of quantum mechanics culminated in May 15, 1935 by an article entitled as “Can Quantum Mechanical Description of Physical Reality Be Considered Complete?” [1] and generally referred to as “EPR paper”. Albert Einstein, Boris Podolsky and Nathan Rosen argued in this paper that quantum theory is incomplete. Schrödinger discussed and extended in 1935 the argument by Einstein, Podolsky, and Rosen and coined the term ‘entanglement’ to describe this very special and peculiar connection between quantum systems. Schrödinger is claiming:
“When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives [the quantum states] have become entangled.” [2]
John Bell's 1964 reconsideration [3] of the EPR argument and the CHSH Inequality [4] generated an ongoing debate on the foundations of quantum mechanics. Meanwhile, both inequalities are refuted [5],[6]. For further discussion of the various kinds and the nature of non-locality postulated by different interpretations of quantum mechanics I must refer the reader to the secondary in literature. In this publication, the compatibility of entanglement with the special theory of relativity will be discussed.

Manuscript submitted to ViXra.org (Sunday, June 28, 2015)

2. Definitions

2.1. **Definition. The Schrödinger equation In General**

The famous Schrödinger equation [6], a partial differential equation which describes how a quantum state of a system changes with time. The Schrödinger equation **for any system**, **no matter whether relativistic or not**, no matter how complicated, has the form

$$_{R}\hat{H} \times {}_{R}\Psi(t) = i\hbar \frac{\partial}{\partial t} {}_{R}\Psi(t), \tag{1}$$

where i is the imaginary unit, $\hbar = \frac{h}{2 \times \pi}$ is Planck's constant h divided by 2xπ, the symbol $\frac{\partial}{\partial t}$ indicates a partial derivative with respect to time t, $_{R}\Psi$ is the wave function of the quantum system, and $_{R}\hat{H}$ is the Hamiltonian operator.

2.2. **Definition. The Schrödinger equation of Alice**

The Schrödinger equation is valid for any system, no matter whether relativistic or not, no matter how complicated, no matter whether described, measured et cetera by Alice or by Bob. The quantum mechanical System as described, measured et cetera by Alice is completely described by the Schrödinger equation

$$_{R}\hat{H}_{A} \times {}_{R}\Psi(t_{A}) = i_{A}\hbar \frac{\partial}{\partial t} {}_{R}\Psi(t_{A}), \tag{2}$$

where i_A is the imaginary unit due to Alice, $\hbar = \frac{h}{2 \times \pi}$ is Planck's constant h divided by 2xπ, the symbol $\frac{\partial}{\partial t}$ indicates a partial derivative with respect to time t, $_{R}\Psi_{A}$ is the wave function of the quantum system of Alice, and $_{R}\hat{H}_{A}$ is the Hamiltonian operator of the quantum system of Alice, t_A is the time as determined by Alice.

2.3. **Definition. The Quantum Mechanical Mathematical Identity $_{R}S_{A}$ of Alice**

Let

$$_{R}S_{A} \equiv {}_{R}\hat{H}_{A} + {}_{R}\Psi(t_{A}), \tag{3}$$

where $_{R}\Psi(t_{A})$ is the wave function of the quantum system of Alice, and $_{R}\hat{H}_{A}$ is the Hamiltonian operator of the quantum system of Alice.

2.4. **Definition. The Schrödinger equation of Bob**

A quantum mechanical System as described, measured et cetera by Bob is completely described by the Schrödinger equation

$$ {}_R\hat{H}_B \times {}_R\Psi(t_B) = i_B \hbar \frac{\partial}{\partial t} {}_R\Psi(t_B), \tag{4}$$

where i_B is the imaginary unit due to Bob, $\hbar = \frac{h}{2\times\pi}$ is Planck's constant h divided by 2xπ, the symbol $\frac{\partial}{\partial t}$ indicates a partial derivative with respect to time t_B, ${}_R\Psi_B$ is the wave function of the quantum system of Bob, and ${}_R\hat{H}_B$ is the Hamiltonian operator of the quantum system of Bob, t_B is the time as determined by Bob.

2.5. **Definition. The Quantum Mechanical Mathematical Identity $_R S_B$ of Bob**

Let

$$ {}_R S_B \equiv {}_R\hat{H}_B + {}_R\Psi(t_B), \tag{5}$$

where ${}_R\Psi(t_B)$ is the wave function of the quantum system of Bob, and ${}_R\hat{H}_B$ is the Hamiltonian operator of the quantum system of Bob.

2.6. **Axioms. Lex identitatis (The identity law).**

Axiom I.
The following theory is based on the following Axiom:

$$ +1 = +1. \tag{Axiom I}$$

3. Theorems

3.1. Theorem. The Normalization Of The Relationship Between The Hamiltonian Of Alice And The Wavefunction Of Alice.

Claim.
The relationship between the Hamiltonian operator of Alice and the wavefunction of Alice can be normalized as

$$1 \equiv \frac{{}_R\hat{H}_A}{{}_RS_A} + \frac{{}_R\Psi(t_A)}{{}_RS_A}. \tag{6}$$

Proof.
Our starting point is the claim that

$$+1 = +1. \tag{7}$$

Multiplying this equation by the wavefunction ${}_R\Psi(t_A)$ of Alice we obtain

$${}_R\Psi(t_A) = {}_R\Psi(t_A). \tag{8}$$

Adding the ${}_R\hat{H}_A$, the Hamiltonian operator of Alice to this equation, it is

$${}_R\hat{H}_A + {}_R\Psi(t_A) = {}_R\hat{H}_A + {}_R\Psi(t_A). \tag{9}$$

Due to our definition above, we obtain

$${}_RS_A \equiv {}_R\hat{H}_A + {}_R\Psi(t_A). \tag{10}$$

We divide this equation by ${}_RS_A$. The normalization of the relationship between the Hamiltonian of Alice and the wavefunction of Alice follows as

$$1 \equiv \frac{{}_R\hat{H}_A}{{}_RS_A} + \frac{{}_R\Psi(t_A)}{{}_RS_A}. \tag{11}$$

Quod erat demonstrandum.

3.2. **Theorem. The Normalization Of The Relationship Between The Hamiltonian Of Bob And The Wavefunction Of Bob.**

Claim.
The relationship between the Hamiltonian operator of Bob and the wavefunction of Bob can be normalized as

$$1 \equiv \frac{{}_R\hat{H}_B}{{}_RS_B} + \frac{{}_R\Psi(t_B)}{{}_RS_B}. \tag{12}$$

Proof.
Our starting point (Axiom I) is the claim that

$$+1 = +1. \tag{13}$$

Multiplying this equation by the wavefunction ${}_R\Psi(t_B)$ of Bob we obtain

$${}_R\Psi(t_B) = {}_R\Psi(t_B). \tag{14}$$

Adding the ${}_R\hat{H}_B$, the Hamiltonian operator of Bob to this equation, it is

$${}_R\hat{H}_B + {}_R\Psi(t_B) = {}_R\hat{H}_B + {}_R\Psi(t_B). \tag{15}$$

Due to our definition above, we obtain

$${}_RS_B \equiv {}_R\hat{H}_B + {}_R\Psi(t_B). \tag{16}$$

We divide this equation by ${}_RS_B$. The normalization of the relationship between the Hamiltonian of Bob and the wavefunction of Bob follows as

$$1 \equiv \frac{{}_R\hat{H}_B}{{}_RS_B} + \frac{{}_R\Psi(t_B)}{{}_RS_B}. \tag{17}$$

Quod erat demonstrandum.

3.3. **Theorem. The Entanglement Between Alice And Bob.**

Claim.
The entanglement of two quantum mechanical systems is determined as

$$\left({}_R\hat{H}_A \times {}_R S_B \right) + \left({}_R\Psi(t_A) \times {}_R S_B \right) \equiv \left({}_R\hat{H}_B \times {}_R S_A \right) + \left({}_R\Psi(t_B) \times {}_R S_A \right). \tag{18}$$

Proof.
Starting with Axiom I it is

$$+1 = +1. \tag{19}$$

Due to our theorem above, it is $1 \equiv \frac{{}_R\hat{H}_B}{{}_R S_B} + \frac{{}_R\Psi(t_B)}{{}_R S_B}$. Thus far, we rearrange the equation and do obtain

$$1 \equiv \frac{{}_R\hat{H}_B}{{}_R S_B} + \frac{{}_R\Psi(t_B)}{{}_R S_B}. \tag{20}$$

Due to the theorem of Alice, it is $1 \equiv \frac{{}_R\hat{H}_A}{{}_R S_A} + \frac{{}_R\Psi(t_A)}{{}_R S_A}$ and it follows that

$$\frac{{}_R\hat{H}_A}{{}_R S_A} + \frac{{}_R\Psi(t_A)}{{}_R S_A} \equiv \frac{{}_R\hat{H}_B}{{}_R S_B} + \frac{{}_R\Psi(t_B)}{{}_R S_B}. \tag{21}$$

Rearranging equation yields

$$\left({}_R\hat{H}_A \times {}_R S_B \right) + \left({}_R\Psi(t_A) \times {}_R S_B \right) \equiv \left({}_R\hat{H}_B \times {}_R S_A \right) + \left({}_R\Psi(t_B) \times {}_R S_A \right). \tag{22}$$

Quod erat demonstrandum.

Scholium.
The Hamiltonian of Alice can but must not be equivalent with the Hamiltonian of Bob and vice versa. The wave function as determined by Alice can but must not be identical with the wave function as determined by Bob. The mathematical identity of Alice (${}_R S_A$) can but must not be identical with the mathematical identity of Bob (${}_R S_B$).

Manuscript submitted to ViXra.org (Sunday, June 28, 2015)

4. Discusssion

In our analysis, we followed strictly Schrödinger's proposal of two systems. The state of both systems is known very precisely by their respective representatives (i. e. every system has its own Hamiltonian and its own wave function). Based on the theorem $1 \equiv \frac{{}_R\hat{H}_B}{{}_R S_B} + \frac{{}_R\Psi(t_B)}{{}_R S_B}$ and the theorem $1 \equiv \frac{{}_R\hat{H}_A}{{}_R S_A} + \frac{{}_R\Psi(t_A)}{{}_R S_A}$ we obtain the following picture. The following illustration by a 2x2 table may show the relationships once again.

Fig.		Alice		
		yes	no	
Bob	yes	**a**	b	$\frac{{}_R\hat{H}_B}{{}_R S_B}$
	no	c	d	$\frac{{}_R\Psi(t_B)}{{}_R S_B}$
		$\frac{{}_R\hat{H}_A}{{}_R S_A}$	$\frac{{}_R\Psi(t_A)}{{}_R S_A}$	1

The straightforward question is, are there circumstances where **a**, the joint distribution function between $\frac{{}_R\hat{H}_B}{{}_R S_B}$ and $\frac{{}_R\hat{H}_A}{{}_R S_A}$ is equal to a = 0. In the case of independence, there should be no entanglement and **a** should be determined as $a = \frac{{}_R\hat{H}_B \times {}_R\hat{H}_A}{{}_R S_B \times {}_R S_A}$. Both systems may enter into any kind of physical interaction (i. e. out of itself or by a measurement et cetera). Even if after a time of mutual influence the systems separate again, both systems are described by the equation

$$\left({}_R\hat{H}_A \times {}_R S_B\right) + \left({}_R\Psi(t_A) \times {}_R S_B\right) \equiv \left({}_R\hat{H}_B \times {}_R S_A\right) + \left({}_R\Psi(t_B) \times {}_R S_A\right). \qquad (23)$$

each of them with a representative of its own.

5. Conclusion

Entanglement can be described from the standpoint of the special theory of relativity too without any contradiction.

Acknowledgment

None.

Appendix

None.

References

[1] Albert Einstein, Boris Podolsky and Nathan Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?", *Physical Review*, vol. 47, Issue 10, pp. 777-780, 1935.

[2] Erwin Rudolf Josef Alexander Schrödinger, "Discussion of Probability Relations Between Separated Systems," *Proceedings of the Cambridge Philosophical Society*, vol. 31, issue 04, pp. 555–563, 1935.

[3] J. S. Bell, "On the Einstein-Podolsky-Rosen paradox," *Physics* 1, pp. 195-200, 1964.

[4] Clauser, J. F., Horne, M. A., Shimony, A. and Holt, R. A., "Proposed experiment to test local hidden-variable theories," *Physical Review Letters*, 23, pp. 880-884, 1969.

[5] Ilija Barukčić, "Anti-Bell - Refutation of Bell's theorem," *American Institute of Physics - Conference Proceedings*, Vol. 1508, No. 1, pp. 354-358, 2012.

[6] Ilija Barukčić, "Anti CHSH - Refutation Of The CHSH Inequality," *http://vixra.org/abs/1411.0046*, pp. 1-10, 2015.

[7] Erwin Schrödinger, "An Undulatory Theory of the Mechanics of Atoms and Molecules", *Physical Review*, vol. 28, no. 6, pp. 1049-1070, 1926.