Proof of no Johnson noise at zero temperature

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Abstract. The Callen-Welton formula (fluctuation-dissipation theorem) of voltage and current noise of a resistance are the sum of Nyquist's classical Johnson noise equations and a (quantum) zero-point term with power density spectrum proportional to frequency and independent of temperature. At zero temperature, the classical Nyquist term vanishes however the zero-point term produces non-zero noise voltage and current. We show that the claim of zero-point noise directly contradicts to the Fermi-Dirac distribution, which defines the thermodynamics of electrons according to quantum-statistical physics. As a consequence, the Johnson noise must be zero at zero temperature, which is in accordance with Nyquist's original formula. Further investigation shows that the Callen-Welton derivation has conceptual errors such as neglecting phonon scattering, disregarding the Pauli principle during calculating the transition probabilities and using bosonic (linear oscillator) energies leading to the zero-point noise artifact. Following Kleen's proposal, the possible origin of the heterodyne (Koch - van Harlingen - Clark) experimental results are also discussed in terms of Heffner theory of quantum noise of frequency/phase-sensitive linear amplifiers. Experiments that failed to see the zero-point noise term are also mentioned.

1. Introduction: The Johnson noise and the second law

In this paper, we prove that the zero-point term in the Johnson noise of resistors is non-existent.

The Johnson (-Nyquist) noise [1,2] of resistors and impedances is a spontaneous voltage and current fluctuation due to the stochastic motion of charge carriers (electrons) in the conductor material at thermal equilibrium. The second law of thermodynamics requires that, in thermal equilibrium, the time average of the instantaneous power flow between two parallel resistors is zero:

$$\left\langle P_{a\Leftrightarrow b}(t,T)\right\rangle_t = 0 \quad , \tag{1}$$

where t is time, $P_{a\Leftrightarrow b}(t,T)$ is the instantaneous power flow between resistors R_a and R_b, see Figure 1, and Equation 1 holds in any frequency band.

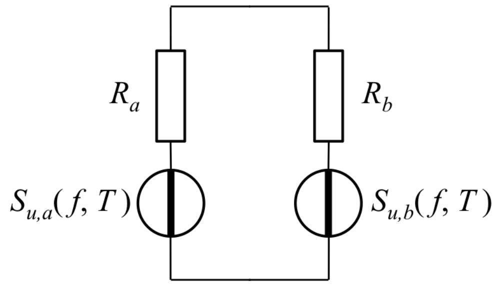

Figure 1. The second law of thermodynamics requires that the net power flow between resistors R_a and R_b is zero. This is provided by the functional form of the power density spectrum $S_u(f,T)$ of Johnson noise (represented by voltage generators in the figure), which is, at any frequency and temperature, proportional to the resistance.

For a passive impedance $Z(f)$ in thermal equilibrium, this condition requires that the functional form of the noise voltage and current spectra are:

$$S_u(f) = \mathrm{Re}\left[Z(f)\right]Q(f,T) = R(f)Q(f,T) \ , \tag{2}$$

$$S_i(f) = \mathrm{Re}\left[Y(f)\right]Q(f,T) = G(f)Q(f,T) \tag{3}$$

where $Y(f) = 1/Z(f)$ if the admittance; $R(f)$ and $G(f)$ are the real part of the impedance and admittance, respectively, $S_u(f)$ and $S_i(f)$ are the Johnson noise voltage and current generators, respectively, where $S_i(f)$ is the Norton-equivalent of $S_u(f)$, $S_i(f) = S_u(f)/\left|Z(f)\right|^2$ (the power density spectrum of the current noise of the short-circuited impedance).

$Q(f,T)$ is a universal function of frequency and temperature, which is independent from material properties, geometry or the way of electrical conductance [1-3].

Therefore, if we can prove that, in a given system at zero temperature the function $Q(f,0)$ *is zero, then this finding is a proof that* $Q(f,0) = 0$ *in any system, that is, the Johnson noise is zero at zero temperature.*

2. The Callen-Welton result for the Johnson noise

In the rest of the paper, for the sake of simplicity of notation, we work with frequency-independent resistances, however this does not limit the generality of our treatment because all the equations and conclusions remain valid if $\mathrm{Re}\left[Z(f)\right]$ is substituted into R in these equations and considerations.

The Callen-Welton derivation (fluctuation-dissipation theorem) [4] of Johnson noise results in the sum of the classical Johnson-Nyquist term and a zero-point term:

$$S_{u,q}(f,T) = 4Rhf\left[N(f,T)+0.5\right] \underset{f<<kT/h}{\rightarrow} 4kTR + 2hfR \qquad (4)$$

where $S_{u,q}(f,T)$ is the *one-sided* power density spectrum of the voltage noise on the resistor and h is the Planck constant. The Planck number $N(f,T)$, which already exists in Nyquist's result, is the mean number of hf energy quanta in a linear harmonic oscillator with resonance frequency f, at temperature T:

$$N(f,T) = \left[\exp(hf/kT)-1\right]^{-1} \ . \qquad (5)$$

For the classical physical range $f << kT/h$, the Plank number becomes $N(f,T) \cong kT/(hf)$, which results in the well-known $4kTR$ noise spectrum at low frequencies. In conclusion, the first term of the sum in Equation 4 is the classical physical (Nyquist) result [2] and the second term is its quantum correction (zero-point noise):

$$S_{u,ZP}(f,0) = 2hfR \ , \qquad (6)$$

where the notation reflects that, at zero temperature, the classical term disappears because $N(f,0)=0$ while the quantum term, the zero-point noise spectrum, is claimed to exist even at zero Kelvin due to the zero-point energy.

The Johnson current noise of the resistance follows from the theory of linear operations on the noise and Ohm's law:

$$S_{i,q}(f,T) = 4Ghf\left[N(f,T)+0.5\right] \underset{f<<kT/h}{\rightarrow} 4kTG + 2hfG \ . \qquad (7)$$

Similarly to the voltage noise, the first term of the sum in Equation 7 is the classical physical (Nyquist) result [2] and the second term is its quantum correction (zero-point noise):

$$S_{i,ZP}(f,0) = 2hfG \ , \qquad (8)$$

which is again claimed to exist independently from the temperature.

3. Proof that the quantum zero-point term does not exist in the Johnson noise

It follows from the considerations in Section 1, that to prove that the zero-point term does not exist under general conditions, it is enough to create a proof for a special case because that proof will imply that the zero-point noise is zero in any other system, otherwise the second law is violated.

Consider the simplest electron conductor system [5], a metal with non-zero residual resistance at zero temperature, due to randomly located defects. We study the Johnson noise current in a short-circuited resistor positioned along the x axis by measuring the current in the loop. (Alternatively, without changing the essence of argumentation, a ring-shaped sample could also be used). The current is the sum of the elementary currents $I(\vec{k})$ of occupied single electron states in the *k*-space:

$$I = \sum_{\{\vec{k}\}} I(\vec{k}) \ , \tag{9}$$

where $\{\vec{k}\}$ denotes the set of wave vectors of the occupied states. At temperature *T*, the probability of state occupation versus the energy *E* of the state, is given by the Fermi-Dirac distribution [5]:

$$P(E,T) = \frac{1}{1 + \exp\left(\frac{E - E_{\mathbf{F}}}{kT}\right)} \quad , \tag{10}$$

where E_F energy-parameter is the Fermi level. At absolute zero temperature, and zero electrical field, the occupation probability *P* satisfies:

$$P(E,T = 0) = \begin{cases} 1 & \text{for } E \le E_F \\ 0 & \text{for } E > E_F \end{cases} \ , \tag{11}$$

that is, all the states are filled below E_F energy and no states are filled beyond E_F. In the *k*-space, this fact is visualized by the Fermi-surface, which is the surface given by the set of *k* vectors corresponding to E_F. Due to the symmetry properties of the Fermi-surface and according to Equation 9, the net current in the material is zero because for each positive occupied *k* value in any direction, there is an occupied negative *k* with the same absolute value, too [5], see Figure 2. (Note, at zero temperature and an external non-zero DC current generator drive, the occupied states shift in the positive direction on the k_x axis (see Figure 2) [5].

It is obvious from this picture that, at zero temperature in thermal equilibrium, a non-zero Johnson noise current through the short-circuited resistor would violate the zero-temperature Fermi-Dirac distribution, see Equation 11 and the solid curve in Figure 2, because that would require spontaneous violations of the symmetry of the Fermi-Dirac distribution in random directions at different instants of time.

In conclusion, the material is totally "silent" at zero temperature and no Johnson noise can occur then. Consequently the zero-point term of Johnson noise claimed by the Callen-Welton formula (in Equations 4,6,7,8) does not exist.

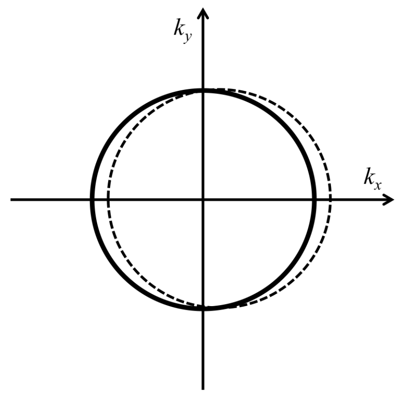

Figure 2. Illustration of the Fermi-surface at 2-dimensions (solid circle). In thermal equilibrium, at zero temperature and zero current, electrons occupy all the states within the Fermi-sphere and no states outside of it are occupied. The dashed line illustrates the situation at zero temperature with external DC current generator driving, which results in a non-equilibrium state and a broken symmetry of the occupied states.

For the completeness of the picture, let use see how the classical term of Johnson noise is generated in Equations 5 and 7. The classical (Nyquist) term is also zero at zero temperature in accordance with Equation 5. However, at non-zero temperature, in thermal equilibrium, the occupation probability is [5]:

$$P(E,T>0)=\begin{cases}1-\varepsilon(E,T) & \text{for } E\le E_{\mathrm{F}}\\ \varepsilon(E,T) & \text{for } E>E_{\mathrm{F}}\end{cases}\quad, \tag{12}$$

where the $\varepsilon(E,T)$ probability ($0<\varepsilon$) characterizing the electron states that are "communicating" and take part in thermal motion (Nyquist noise term) , electronic specific heat, etc [5]. These are the carriers of the classical Johnson noise.

4. Where is the error in the Callen-Welton derivation?

So, where is the mistake in the Callen-Welton derivation [4] that produced the zero-point thermal noise artifact?

First of all, to determine energy dissipation and impedance (see their Equation 2.6 and the related equations [4]) Callen-Welton calculate the transition probabilities of charge carriers to new energy states that are shifted by $\pm hf$ energy compared to the original state, where f is the frequency of the sinusoidal voltage drive $U_0\sin(2\pi ft)$ of the resistor, where U_0 is the voltage amplitude and t is time. To do that, they use the time-dependent perturbation energy $qU_0\sin(2\pi ft)$, where q is the charge of the electron. Unfortunately, this whole concept is inappropriate. In a solid, the energy dissipation is not based on $\pm hf$ energy shifts. Particularly at a low frequency f, the electrons in the conductance band are randomly scattered into a wide range of energies with the order of kT random absolute energy shift. The characteristic lifetime of a state, mean free time [5], is much shorter than the sinusoidal period $1/f$; it is in the order of 10^{-13} seconds at room temperature. Thus time-dependent perturbation theory, which assumes that the state persist for many sinusoidal periods $1/f$, is completely inappropriate. Neither the lifetime of the state is long-enough, nor the scattering takes place with $\pm hf$ energy shifts but a much larger random energy shift, in accordance with standard solid state physics [5].

Moreover the Callen-Welton derivation [4] directly violates the Pauli-principle. They neglected the fact that the charge carriers are fermions thus they satisfy the Pauli-principle, which results in the Fermi-Dirac distribution [5,6]. The transition probabilities used by Callen-Welton do not depend on the occupation level of the state to which the transition occurs. The transition probabilities in their Equation 2.6 and in the related equations [4] should have been multiplied by (1-*n*), in accordance with the Pauli-principle, where *n* is the occupation number (0 or 1) of the end-state of the transition, see the detailed treatment in [6]. Without that step, the system of electrons cannot converge to a Fermi-Dirac statistics in thermal equilibrium.

Another questionable step in the Callen-Welton derivation is that, when they calculate the energy of the system at a given frequency, they use the energy of quantum linear harmonic oscillator at the given temperature. However, again, the system in question is the system of electrons. The linear harmonic oscillator describes bosons with Bose-Einstein statistics (phonons, photons, etc.) while electrons are fermions with the completely different Fermi-Dirac statistics [6].

5. On the Koch - van Harlingen - Clark experiment [7]

It is not our main concern to explain how could the Josephson junction based frequency-selective, heterodyne detection experiments [7] lead to the seeming confirmation of the non-existent zero-point Johnson noise. However, it is worthwhile to mention one such attempt by Kleen [8] who, following Heffner's approach [9], gave an estimate about the potential role of the time-energy uncertainty principle in frequency-selective, phase sensitive linear amplifiers. Kleen [8] got the linear frequency dependence and approximately the same values as Equation 6 and the experiments [7].

However, instead of Kleen, we quote here Heffner's results [9], which deduce the noise-temperature T_{ZP} of frequency and frequenecy/phase selective linear amplifiers due to the uncertainty principle at zero thermodynamical temperature:

$$T_{ZP} = \frac{hf/k}{\ln\left(\frac{2-1/A}{1-1/A}\right)} \quad . \tag{13}$$

Equation 13 implies that at R source resistance, the equivalent input noise of the amplifier at zero temperature will be

$$S_u = 4kT_{ZP}R = 4R\frac{hf}{\ln\left(\frac{2-1/A}{1-1/A}\right)} = \gamma 4Rhf \quad , \qquad (14)$$

where A is the amplification and the $\gamma \approx 0.5$ value would result in the exact zero-point noise artifact of Equation 6.

We evaluated Heffner's formula, see Table 1, and found that amplification A=1.19 yields the artifact value however a wide range of amplification gives similar data.

Table 1. Various amplification and corresponding γ values.

A	1.1	**1.19**	2	10
γ	0.40	**0.50**	0.91	1.34

7. Experiments indicating the non-existence of the zero-point term

Voss and Webb [10] evaluated the shot noise of Josephson junctions and found that the results have excellent fit with a thermal activation model while these experimental results and 4 - 10 orders of magnitude below the noise level implied by the zero-point Johnson noise. Unfortunately, they suspected that the reason for deviations was the Langevin model of Josephson junctions, and did not mention/explore the possibility that the zero-point term of Fluctuation-Dissipation Theorem is incorrect.

It is also relevant to note that, back at the beginning of the 1980s, van der Ziel et al, did not see the zero-point term via direct (non-heterodyne) measurements [11] of Hanbury Brown-Twiss type microwave circuitry [12] at 1 Kelvin temperature and up to 95 GHz frequency, even though this frequency limit at this temperature is about 5 times beyond the *kT*/*h* classical/quantum boundary.

7. Conclusions

The zero-point Johnson noise term in the Fluctuation-Dissipation-Theorem is incorrect. It is the result of neglecting the fermion nature of charge carriers in conductors including the Pauli-principle. The Fermi-Dirac statistics of charge carriers implies that there is no Johnson noise at zero temperature thus the zero-point noise term must be non-existent.

Additional work is needed to clarify the exact source of the Koch - van Harlingen - Clarke experimental results, perhaps combined with experiments on impedances, and with direct wideband detection instead of heterodyne/selective one.

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