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Proof of zero Johnson noise at zero temperature

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Abstract. The Callen-Welton formula (fluctuation-dissipation theorem) of voltage and current noise of a resistance are the sum of Nyquist's classical Johnson noise equations and a (quantum) zero-point term with power density spectrum proportional to frequency and independent of temperature. At zero temperature, the classical Nyquist term vanishes however the zero-point term produces non-zero noise voltage and current. We show that the claim of zero-point noise directly contradicts to the Fermi-Dirac distribution, which defines the thermodynamics of electrons according to quantum-statistical physics. As a consequence, the Johnson noise must be zero at zero temperature, which is in accordance with Nyquist's original formula. Further investigation shows that Callen-Welton disregarded the Pauli principle during calculating the transition probabilities and, in this way, they produced the zero-point noise artifact.

1. Introduction: The Johnson noise and the second law

In this paper, we prove that the zero-point term in the Johnson noise of resistors is non-existent.

The Johnson (-Nyquist) noise [1,2] of resistors and impedances is a spontaneous voltage and current fluctuation due to the stochastic motion of charge carriers (electrons) in the conductor material at thermal equilibrium. The second law of thermodynamics requires that, in thermal equilibrium, the time average of the instantaneous power flow between two parallel resistors is zero:

$$\left\langle P_{a\Leftrightarrow b}(t,T)\right\rangle_t \quad = \quad 0 \quad , \qquad (1)$$

where $P_{a\Leftrightarrow b}(t,T)$ is the instantaneous power flow between resistors R_a and R_b, see Figure 1, and Equation 1 holds in any frequency band.

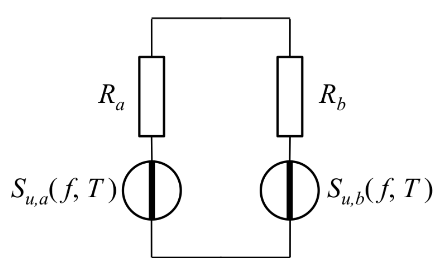

Figure 1. The second law of thermodynamics requires that the net power flow between resistors R_a and R_b is zero. This is provided by the functional form of the power density spectrum $S_u(f,T)$ of Johnson noise (represented by voltage generators in the figure), which is, at any frequency and temperature, proportional to the resistance.

For a passive impedance $Z(f)$ in thermal equilibrium, this condition requires that the functional form of the noise voltage spectrum is:

$$S_u(f) = \mathrm{Re}\left[Z(f)\right]Q(f,T) = R(f)Q(f,T) \quad , \tag{2}$$

where $R(f)$ is the real part of the impedance and $Q(f,T)$ is a universal function of frequency and temperature, which is independent from material properties, geometry or the way of electrical conductance [1-3].

Therefore, if we can prove that, in a given system at zero temperature the function $Q(f,0)$ *is zero, then this finding is a proof that* $Q(f,0) = 0$ *in any system, that is, the Johnson noise is zero at zero temperature.*

2. The Callen-Welton result for the Johnson noise

In the rest of the paper, for the sake of simplicity of notation, we work with frequency-independent resistances, however this does not limit the generality of our treatment because all the equations and conclusions remain valid if $\mathrm{Re}\left[Z(f)\right]$ is substituted into R in these equations and considerations.

The Callen-Welton derivation (fluctuation-dissipation theorem) [4] of Johnson noise results in the sum of the classical Johnson-Nyquist term and a zero-point term:

$$S_{u,q}(f,T) = 4Rhf\left[N(f,T) + 0.5\right] \underset{f<<kT/h}{\rightarrow} 4kTR + 2hfR \tag{3}$$

where $S_{u,q}(f,T)$ is the *one-sided* power density spectrum of the voltage noise on the resistor and h is the Planck constant. The Planck number $N(f,T)$, which already exists in Nyquist's result, is the mean number of hf energy quanta in a linear harmonic oscillator with resonance frequency f, at temperature T:

$$N(f,T) = \left[\exp(hf/kT) - 1\right]^{-1} \quad . \tag{4}$$

For the classical physical range $f << kT/h$, the Plank number becomes $N(f,T) \cong kT/(hf)$, which results in the well-known $4kTR$ noise spectrum at low frequencies. In conclusion, the first term of the sum in Equation 3 is the classical physical (Nyquist) result [2] and the second term is its quantum correction (zero-point noise):

$$S_{u,ZP}(f,0) = 2hfR \quad , \tag{5}$$

where the notation reflects that, at zero temperature, the classical term disappears because $N(f,0) = 0$ while the quantum term, the zero-point noise spectrum, is claimed to exist even at zero Kelvin due to the zero-point energy.

The Johnson current noise of the resistance follows from the theory of linear operations on the noise and Ohm's law:

$$S_{i,q}(f,T) = 4Ghf\left[N(f,T)+0.5\right] \underset{f<<kT/h}{\longrightarrow} 4kTG + 2hfG \ , \tag{6}$$

where $S_{i,q}(f,T)$ is the *one-sided* power density spectrum of the current noise of the short-circuited the resistor, where the conductance is given as $G = 1/R$ (or, in the case of impedance, it is the real part of the admittance). Similarly to the voltage noise, the first term of the sum in Equation 6 is the classical physical (Nyquist) result [2] and the second term is its quantum correction (zero-point noise):

$$S_{i,ZP}(f,0) = 2hfG \ , \tag{7}$$

which is again claimed to exist independently from the temperature.

3. Proof that the quantum zero-point term does not exist in the Johnson noise

It follows from the considerations in Section 1, that to prove that the zero-point term does not exist under general conditions, it is enough to create a proof for a special case because that proof will imply that the zero-point noise is zero in any other system, otherwise the second law is violated.

Consider the simplest electron conductor system [5], a metal that does not become superconductor at zero temperature (in superconductors the Johnson noise is singular). Suppose our resistor is made of that. We short-circuit the resistor and measure the current in the loop. The current is the sum of the elementary currents $I(\vec{k})$ of occupied single electron states in the *k*-space:

$$I = \sum_{\{\vec{k}\}} I(\vec{k}) \ , \tag{8}$$

where $\left\{\vec{k}\right\}$ denotes the set of wave vectors of the occupied states. At temperature *T*, the probability of state occupation versus the energy *E* of the state, is given by the Fermi-Dirac distribution [5]:

$$P(E,T) = \frac{1}{1+\exp\left(\dfrac{E-E_{\mathrm{F}}}{kT}\right)} \ , \tag{9}$$

where E_F energy-parameter is the Fermi level. At absolute zero temperature, and zero electrical field, the occupation probability *P* satisfies:

$$P(E,T=0)=\begin{cases}1 & \text{for } E\le E_{\mathrm{F}}\\ 0 & \text{for } E>E_{\mathrm{F}}\end{cases}\quad, \tag{10}$$

that is, all the states are filled below E_F energy and no states are filled beyond E_F. In the k-space, this fact is visualized by the Fermi-surface, which is the surface given by the set of k vectors corresponding to E_F. Due to the symmetry properties of the surface and according to Equation 8, the net current in the material is zero because for each positive occupied k value in any direction, there is an occupied negative k with the same absolute value, too [5]. At zero temperature and a non-zero electrical field in the negative direction along the x axis, the occupied states shift in the positive direction on the k_x axis, representing an electrical current in the x direction [5].

It is obvious from this picture that nonzero electrical current, for example Johnson noise, requires breaking of the symmetry of the set of occupied states, such as the dashed circle shows in Figure 2 [5]. This cannot happen at zero temperature and zero electrical field because that would violate the zero-temperature limit of Fermi-Dirac distribution, see Equation 10.

In conclusion, the material is totally "silent" at zero temperature and no Johnson noise can occur then. Consequently the zero-point term of Johnson noise claimed by the Callen-Welton formula (in Equations 3,5,6,7) does not exist.

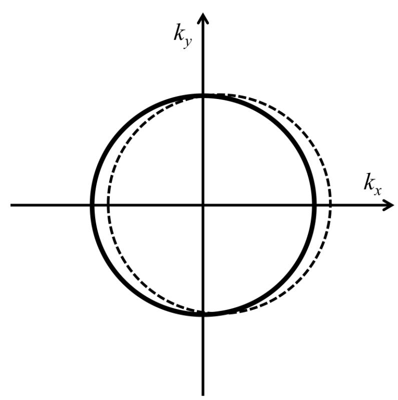

Figure 2. Illustration of the Fermi-surface at 2-dimensions (solid circle). At zero temperature and zero electrical field, electrons occupy all the states within the Fermi-sphere and no states outside of it are occupied. At non-zero current, the symmetry of the occupied states is broken.

For the completeness of the picture, let use see how the classical term of Johnson noise is generated in Equations 4 and 6. The classical (Nyquist) term is also zero at zero temperature in accordance with Equation 4. However, at non-zero temperature the occupation probability is [5]:

$$P(E,T>0)=\begin{cases}1-\varepsilon(E,T) & \text{for } E\le E_{\mathrm{F}}\\ \varepsilon(E,T) & \text{for } E>E_{\mathrm{F}}\end{cases}\quad, \tag{11}$$

where the $\varepsilon(E,T)$ probability ($0<\varepsilon$) characterizing the electron states that are "communicating" and take part in thermal motion (Nyquist noise term) , electronic specific heat, etc [5]. These are the carriers of the classical Johnson noise.

4. Where is the error in the Callen-Welton derivation?

The Callen-Welton derivation neglected the facts that the charge carriers are fermions thus they satisfy the Pauli-principle, which results in the Fermi-Dirac distribution [6]. The transition probabilities used by Callen-Welton do not depend on the occupation level of the state to which the transition occurs, which is the violation of the Pauli-principle [6].

This fundamental mistake produced the zero-point thermal noise artifact.

5. How about the Koch-van_Harlingen-Clark experiment [7]?

It is not our goal to explain how could the Josephson junction based frequency-selective, heterodyne detection experiments [7] lead to the seeming confirmation of the non-existent zero-point Johnson noise. However, it is worthwhile to mention one such attempt by Kleen [8] who, following Heffner's approach [9], gave an estimate about the potential role of the time-energy uncertainty principle in frequency-selective, phase sensitive linear amplifiers. Kleen [8] got the linear frequency dependence and approximately the same values as Equation 5 and the experiments [7].

However, instead of Kleen, we quote here Heffner's results [9], which deduce the noise-temperature T_{ZP} of frequency and phase selective linear amplifiers due to the uncertainty principle at zero thermodynamical temperature:

$$T_{ZP}=\frac{hf/k}{\ln\left(\dfrac{2-1/A}{1-1/A}\right)} \quad . \tag{12}$$

Equation 12 implies that at R source resistance, the equivalent input noise of the amplifier at zero temperature will be

$$S_u=4kT_{ZP}R=4R\frac{hf}{\ln\left(\dfrac{2-1/A}{1-1/A}\right)}=\gamma 4Rhf \quad , \tag{13}$$

where A is the amplification and the $\gamma\approx 0.5$ value would result in the exact zero-point noise artifact of Equation 5.

We evaluated Heffner's formula, see Table 1, and found that amplification A=1.19 yields the artifact value however a wide range of amplification gives similar data.

Table 1. Various amplification and corresponding γ values.

A	1.1	**1.19**	2	10
γ	0.40	**0.50**	0.91	1.34

6. Conclusions

The zero-point Johnson noise term in the fluctuation-dissipation theorem is incorrect and it is the result of neglecting the fermion nature of charge carriers in conductors including the Pauli-principle. The Fermi-Dirac statistics of charge carriers implies that there is no Johnson noise at zero temperature thus the zero-point noise is non-existent in the noise voltage and current of impedances.

Additional work is needed to clarify the exact source of the experimental results, perhaps combined with experiments on impedances, and with direct detection instead of heterodyne detection.

References

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