New Three laws of Planetary Motion

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Abstract: According to improved Newton's gravitational formula, variable dimension fractal method and steepest descent method, this paper presents the new three laws of planetary motion: (1) Accurate gravitational formula between Sun and planet, and accurate gravitational formula between Sun and photon for problem of light bending around Sun; (2) Two kinds of improved Titius-Bode laws; (3) Relation between planetary average moving speed and its average distance to Sun with variable dimension fractal form. By using the second and third laws to predict the un-found planet outside the Neptune (formerly known as the tenth planet), its average distance to Sun equals 59.54 AU, and its average moving speed equals 3.85km/s.
Key words: Planetary motion, new three laws, accurate gravitational formula, improved Titius-Bode law, distance-velocity relation

Introduction

From 1609 to 1618, Kepler discovered the so-called Kepler's three laws of planetary motion: (1) The orbit of every planet is an ellipse with the Sun at one of the two foci; (2) A line joining a planet and the Sun sweeps out equal areas during equal intervals of time; (3) The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

About 400 years later, now we try to present the new three laws of planetary motion: (1) Accurate gravitational formula between Sun and planet, and accurate gravitational formula between Sun and photon for problem of light bending around Sun; (2) Two kinds of improved Titius-Bode laws; (3) Relation between planetary average moving speed and its average distance to Sun with variable dimension fractal form.

1 Accurate gravitational formula between Sun and planet

In References [1] and [2], with the help of the equation derived by Prof. Hu Ning according to general relativity, and Binet's formula, we present the following improved Newton's formula of universal gravitation

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2mp}{c^2r^4} \qquad (1)$$

where: G is gravitational constant, M and m are the masses of the two objects, r is the distance between the two objects, c is the speed of light, p is the half normal chord for the object m moving around the object M along with a curve, and the value of p is given by: $p = a(1-e^2)$ (for ellipse), $p = a(e^2-1)$ (for hyperbola), $p = y^2/2x$ (for parabola).

The improved Newton's universal gravitation formula (Eq.(1)) can give the same results as given by general relativity for the problem of planetary advance of perihelion and the problem of gravitational defection of a photon orbit around the Sun.

It should be noted that, this improved Newton's formula of universal gravitation can also be written as the form of variable dimension fractal (in which the fractal dimension is a variable, instead of a constant).

Suppose

$$F = -\frac{GMm}{r^D} = -\frac{GMm}{r^2} - \frac{3G^2M^2mp}{c^2r^4} \qquad (2)$$

It gives

$$D = -\ln\left(\frac{1}{r^2} + \frac{3GMp}{c^2r^4}\right)/\ln r \qquad (3)$$

Substituting the relevant parameters of the ellipse into Eq.(1), it gives the accurate gravitational formula between Sun and planet

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2ma(1-e^2)}{c^2r^4} \qquad (4)$$

In addition, for the problem of gravitational defection of a photon orbit around the Sun, the accurate gravitational formula between Sun and photon reads

$$F = -\frac{GMm}{r^2} - \frac{1.5GMmr_0^2}{r^4} \qquad (5)$$

where: r_0 is the shortest distance between the light and the Sun, if the light and the Sun is tangent, it is equal to the radius of the Sun.

The funny thing is that, for this problem, the maximum gravitational force given by Eq.(5) (as $r = r_0$) is 2.5 times of that given by the original Newton's law of gravity.

2 Two kinds of improved Titius-Bode laws

The original Titius-Bode law is a hypothesis for the average distance between the Sun and the planet.

It can be expressed as follows: considering the distances to Sun from the near to the far, for the corresponding n-th planet (for Mercury, n is equal to -∞, instead of 1. For Venus, Earth, Mars, Asteroid Belt, Jupiter, Saturn, Uranus, and Neptune; n is equal to 2, 3, 4, 5, 6, 7, 8, and 9 respectively), its average distance to Sun (astronomical unit AU) reads

$$a = 0.4 + 0.3 \times 2^{n-2} \qquad (6)$$

For the original Titius-Bode law, there is an unreasonable point, that is, for Mercury, n is equal to -∞, instead of 1. As we improve the Titius-Bode law, for Mercury, n is equal to 1.

Now we present the two forms' improve Titius-Bode laws.

The first form is similar to the original Titius-Bode law (while the constants will be newly selected), the planetary average distance to Sun (AU) reads

$$a_n = x_1 + x_2 \times x_3^{\,n-x_4} \qquad (7)$$

where: for Mercury, Venus, Earth, Mars, Asteroid Belt, Jupiter, Saturn, Uranus, Neptune and so on; n is equal to 1, 2, 3, 4, 5, 6, 7, 8, 9 and so on respectively.

Now we utilize the least-squares method to determine the undetermined constants in Eq.(7).

Assume that corresponding to the real average distance a_n', the calculated value is a_n', we consider that the quadratic sum of the relative errors of the calculated values and the real values reaches the minimum, that is, applying the following variational principle to determine the undetermined constants.

$$\Pi = \sum_{1}^{9} \left(\frac{a_n' - a_n}{a_n'}\right)^2 = \min \qquad (8)$$

Substituting the calculated values and the real values into Eq.(8), after applying the steepest descent method, we get the following optimal solution.

x_1=0.1804362， x_2=0.5869832， x_3=1.84711， x_4=2.486026

It gives the first kind of improved Titius-Bode law

$$a_n = 0.1804362 + 0.5869832 \times 1.84711^{\,n-2.486026} \qquad (9)$$

The quadratic sum of the relative errors calculated by Eq.(9) is equal to $\Pi = 4.771078 \times 10^{-2}$, while the quadratic sum of the relative errors calculated by the original Titius-Bode law is equal to $\Pi_0 = 9.272933 \times 10^{-2}$. Obviously, the improved result is better than the original result.

Table 1 shows the comparison between the first kind of improved Titius-Bode law and the original Titius-Bode law.

Table 1. Comparison between the first kind of improved Titius-Bode law and the original Titius-Bode law

Celestial Body	Real average distance to Sun	Result of original formula	Result of Eq.(9)
Mercury	0.39	0.4	0.42
Venus	0.72	0.7	0.62
Earth	1	1	0.99
Mars	1.52	1.6	1.67
Asteroid Belt	2.9	2.8	2.93
Jupiter	5.20	5.2	5.25
Saturn	9.54	10	9.55
Uranus	19.18	19.6	17.48

Neptune	**30.06**	**38.8**	**32.14**

From Table 1 we can see that, for Neptune, the relative error calculated by the original Titius-Bode law is equal to -29.1%, while the relative error calculated by the improved Titius-Bode law Eq.(9) is only equal to -6.9%.

By using Eq.(9), the un-found planet outside the Neptune (formerly known as the tenth planet) can be predicted. Substituting $n=10$ into Eq.(9), we can predict that its average distance to Sun is as follows

$$a_{10}=59.21\ (\text{AU}) \tag{10}$$

It should be noted that, on November 11, 1940, Liu Zihua published his French monograph entitled "Cosmos of the Eight Diagrams — Prediction of A New Planet", in which he predicted that the tenth planet's average distance to Sun is equal to 7.4 billion km (49.3 AU), this result is comparatively close to the predicted result given by Eq.(9) (Liu Zihua's result is about 16.7% less than that of Eq.(9)).

Now we discuss the second improved Titius-Bode laws with the form of variable dimension fractal, its expression only contains 3 undetermined constants as follows

$$a_n=\frac{C}{n^{d_0+d_1 n}} \tag{11}$$

Substituting the calculated values and the real values into Eq.(8), after applying the steepest descent method, we get the following optimal solution.

C **=0.3803499，** d_0 **=-0.3061203，** d_1 **=-0.1890803**

It gives the second kind of improved Titius-Bode law

$$a_n=\frac{0.3803499}{n^{-0.3061203-0.1890803n}} \tag{12}$$

The quadratic sum of the relative errors calculated by Eq.(12) is equal to $\Pi=5.42992\times10^{-2}$, while the quadratic sum of the relative errors calculated by the original Titius-Bode law is equal to $\Pi_0=9.272933\times10^{-2}$. Obviously, this improved result is also better than the original result.

Table 2 shows the comparison between the second kind of improved Titius-Bode law and the original Titius-Bode law.

Table 2. Comparison between the second kind of improved Titius-Bode law and the original Titius-Bode law

Celestial Body	Real average distance to Sun	Result of original formula	Result of Eq.(9)
Mercury	**0.39**	**0.4**	**0.38**
Venus	**0.72**	**0.7**	**0.61**
Earth	**1**	**1**	**0.99**
Mars	**1.52**	**1.6**	**1.66**
Asteroid Belt	**2.9**	**2.8**	**2.85**

Jupiter	5.20	5.2	5.03
Saturn	9.54	10	9.07
Uranus	19.18	19.6	16.70
Neptune	30.06	38.8	31.34

From Table 2 we can see that, for Neptune, the relative error calculated by the original Titius-Bode law is equal to -29.1%, while the relative error calculated by the improved Titius-Bode law Eq.(12) is only equal to -4.3%.

By using Eq.(12), the un-found planet outside the Neptune (formerly known as the tenth planet) can also be predicted. Substituting $n=10$ into Eq.(12), we can predict that its average distance to Sun is as follows

$$a_{10} = 59.86 \text{ (AU)} \quad (13)$$

This result is very close to the result of 59.21 given by Eq.(10), the difference is only equal to 1.1%. Taking the average of the two values, it gives

$$a_{10} = 59.54 \text{ (AU)} \quad (14)$$

3 Relation between planetary average moving speed and its average distance to Sun with variable dimension fractal form

By using the formula that $\frac{GMm}{r^2} = \frac{mv^2}{r}$, many scholars have derived the following approximate formula for the relation between planetary average moving speed and its average distance to Sun.

$$r_{av} \approx \frac{GM}{v_{av}^2} \quad (15)$$

Supposing that the unit of the planetary average distance to Sun is taken as the astronomical unit (AU), the unit of the planetary average moving speed is taken as km/s, and taking the accurate data as follows: the mass of Sun M=1.98892x10^{30}kg, the gravitational constant G=6.67221937N·m^2/kg^2, and 1AU=1.4959787x10^8 km, then Eq.(15) can be reduced as follows

$$r_{av} \approx \frac{887.079}{v_{av}^2} \quad (15')$$

If we want to get an accurate formula, it can be written as the following form of variable dimension fractal

$$r_{av} = \frac{GM}{v_{av}^D} \quad (16)$$

And the accurate value of D can be written as follows

$$D = \frac{\ln GM - \ln r_{av}}{\ln v_{av}} \quad (17)$$

For different planets, the accurate values of D are shown in table 3.

Table 3. The accurate values of D

Celestial Body	Accurate value of D
Mercury	2.0036
Venus	2.0013
Earth	2.0001
Mars	2.0020
Jupiter	2.0001
Saturn	1.9994
Uranus	1.9986
Neptune	1.9983

Taking average, it gives

$$D_{av} = 2.000 \quad (18)$$

According to Eq.(16), the planetary average moving speed reads

$$v_{av} = \exp\left(\frac{\ln GM - \ln r_{av}}{D}\right) \quad (19)$$

Substituting $a_{10} = 59.54$ (AU) into Eq.(19), for the un-found planet outside the Neptune (formerly known as the tenth planet), we can predict that its average moving speed is as follows

v_{av} **=3.85km/s**

Eq.(19) can also be utilized to test that whether or not some data are compatible. For example, besides predicted that the tenth planet's average distance to Sun is equal to 7.4 billion km (49.3 AU), Liu Zihua also predicted that its average moving speed is equal to 2km/s. While according to Eq.(19), its average moving speed should be equal to 4.24km/s.

References

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