THE CONCEPTS OF "GORCE" AND "GHEAT": WHAT IS THE PROBLEM WITH THE FIRST LAW OF THERMODYNAMICS? *

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ABSTRACT

An analysis of the movement of a piston under the action of a constant gravitational field g and of the particle collisions of a gas can explain the difficulty of ascribing a physical meaning to the concept of force.

The idea of force very naturally arises out of the anthropomorphic notion of weight. Newton's law $f=\frac{dp}{dt}$ is a tautology as long as we have no definition of f which obviously cannot be defined as being "the product of mass by acceleration" or as "the variation of the quantity of movement in order to time". As pointed out by Feynman, "Gorce is the rate of change of position", meaning that any definition is necessarily right, because for such a law to have physical meaning the concept of Gorce should be introduced independently of the variation of position in order to time (Feynman, Leighton and Sands 1976). In fact, the analysis of the previously mentioned movement of a piston shows that the force quantities are introduced independenty of the $\frac{dp}{dt}$ of the body considered, although they can be related to this quantity. The "force" due to weight results only from the fact that the acceleration of gravity for all bodies is g, $g=\frac{d^2x}{dt^2}$ or obviously $mg=m\frac{d^2x}{dt^2}$. The force due to the particle collisions merely results from the global momentum conservation. This latter force is interpreted as the overlapping of an emitted beam and an absorbed beam. Such an interpretation makes it possible to show that for systems of variable mass f as $\frac{dp}{dt}$ is an entity without physical meaning (Som-

1. THE CONCEPT OF FORCE, THE ENERGY CONSERVATION PRINCIPLE AND THE 1ST PRINCIPLE OF THERMODYNAMICS

1.1. Construction of a model:

Comparison between a piston and a variable mass system

In other previous analyses (Abreu Faro and Abreu, 1987; Abreu 1993 a,b,c) the difficulties associated with the concept of force appeared in connection with a concrete situation, the calculation of the movement of a piston under the action of gravity and the effect of collisions of a gas particles. The idea of force arises quite naturally, out of the anthropomorphic notion of weight. Newton´s law $\mathbf{f}=\frac{d\mathbf{p}}{dt}$ is a tautology as long as f is not defined, and it is obvious that **f** cannot be defined as "the product of mass by acceleration" or "the variation of quantity of movement in order to time".

As pointed out by Feynman, "**Gorce** is the rate of change of position", meaning that a definition is necessarily right because it states absolutely nothing new, since for this law to be of physical interest it would be necessary for the concept of **Gorce** to be introduced independently of the variation of position in order to time (Feynman, Leighton and Sands, 1976).

In fact, when we introduce equation (14), gravitational force is introduced, and the force due to the collisions of the gas particles and these quantities were introduced independently of $\frac{d\mathbf{p}}{dt}$. For example, if the body is at rest, the force due to the particle collisions is f_{co}, which is different from zero, although $\frac{dp}{dt}=0$.

Let us now see how we can construct **f** = m**a** as a quantity derived from the momentum conservation.

We shall, for the purpose, consider the following figures:

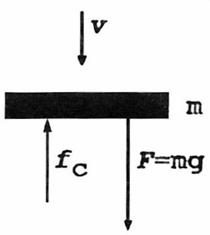

Fig.1 A piston under the action of a dynamic pressure due to the collisions of a particle beam and the force of gravity. The piston moves at velocity **v** and has mass m.

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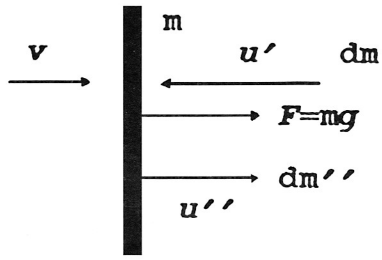

Fig. 2 Mass m under the action of the force due to ejection of mass dm" at velocity u", to absorption of dm' at velocity u', and to the force of gravity **F**. Mass m is schematically represented and moves at velocity **v**.

The schematic representation of Fig.1 and 2 makes it easy to associate the model previously treated of a piston in motion subject to its own weight and the force due to the collisions of a unidimensional beam, f_c, with the model of a mass m emitting mass dm", at velocity u" and absorbing a mass dm' at velocity u'.

In fact it is possible to interpret the beam reflected on the piston surface as being composed of a part which is absorbed, and of another part which is emitted, both with the same mass, and, when the piston is moving, with different velocities on the laboratory frame, u' and u".

We might see the piston mass as composed of a part we can call m', increased by dm', and of another part, m", decreased by dm", the mass, however, remaining constant.

$$m = m' + m'' \qquad dm = 0 = dm' + dm'' \tag{1}$$

If we apply the conservation of the quantity of movement to a mass m ejecting a small mass dm, we obtain the following equation:

$$\frac{d(m\mathbf{v})}{dt} = \mathbf{u}\frac{dm}{dt}, \tag{2}$$

where m**v** is the velocity of mass m and **u** the velocity of mass dm.
If, simultaneously, there exists a force **F** acting upon m, the equation will be (Sommerfeld 1966)

$$\frac{d(m\mathbf{v})}{dt} = \mathbf{u}\frac{dm}{dt} + \mathbf{F}. \tag{3}$$

In this way, and under the conditions of the piston, we can write

$$\frac{d(m\mathbf{v})}{dt} = \mathbf{F} + \mathbf{u}'\frac{dm'}{dt} + \mathbf{u}''\frac{dm''}{dt}. \tag{4}$$

Because the mass is constant,

$$\frac{dm}{dt} = \frac{dm'}{dt} + \frac{dm''}{dt} = 0 \ . \tag{5}$$

We therefore have

$$\left(\frac{dm'}{dt}\right)\mathbf{v} + \left(\frac{dm''}{dt}\right)\mathbf{v} + \frac{md\mathbf{v}}{dt} = \mathbf{F} + \left(\frac{dm'}{dt}\right)\mathbf{u}' + \left(\frac{dm''}{dt}\right)\mathbf{u}'' \tag{6}$$

or

$$\frac{md\mathbf{v}}{dt} = \mathbf{F} + (\mathbf{u}' - \mathbf{v})\frac{dm'}{dt} + (\mathbf{u}'' - \mathbf{v})\frac{dm''}{dt} \ . \tag{7}$$

merfeld 1966), which allows to identify a controversy: *the concept of force and the First Principle of Thermodynamics-the notion of* **gheat**.

RESUMO

A análise do movimento de um êmdolo sob a acção de um campo gravitacional g constante e das colisões das partículas de um gás permite explicitar a dificuldade de atribuir significado físico ao conceito de força.
A ideia de força surge muito naturalmente da noção antropomórfica de peso. A lei de Newton $f = \frac{dp}{dt}$ é uma tautologia enquanto não se definir **f** que evidentemente não pode ser definida como sendo "o produto da massa pela aceleração" ou como "a variação da quantidade de movimento em ordem ao tempo". Como salientou Feynman, "a Gorce is the rate of change of position", querendo com isto afirmar que qualquer definição está necessariamente certa, dado que para que tal lei tivesse significado físico, o conceito de **Gorce** deveria ser introduzido de forma independente da variação de posição em ordem ao tempo (Feynman, Leighton and Sands, 1976). De facto, na análise do movimento do êmbolo anteriormente referido, as grandezas força são introduzidas independentemente de $\frac{dp}{dt}$ do corpo considerado, embora possam ser relacionadas com esta quantidade. A "força" devido ao peso resulta tão somente do facto de a aceleração da gravidade para todos os corpos ser g, $g = \frac{d^2x}{dt^2}$, ou evidentemente $mg = m\frac{d^2x}{dt^2}$. A força devido às colisões das partículas resulta apenas da conservação de momento, em termos globais. Esta ultima força é interpretada como a sobreposição de um feixe emitido e de um feixe absorvido. Tal interpretação permite mostrar que para sistemas de massa variável, f como $\frac{dp}{dt}$ é uma entidade que não tem sentido físico (Sommerfeld 1966), o que permite identificar uma controvérsia: *o conceito de força e o Primeiro Princípio da Termodinâmica*-a noção de **galor**.

Now $(\mathbf{u}' - \mathbf{v}) = \mathbf{v}'$ is the velocity of mass dm' on the piston frame and $(\mathbf{u}'' - \mathbf{v}) = -\mathbf{v}'$ the velocity of dm".
Since

$$\frac{dm'}{dt} = -\frac{dm''}{dt}\ , \tag{8}$$

we have

$$\frac{m d\mathbf{v}}{dt} = \mathbf{F} + 2\mathbf{v}'\frac{dm'}{dt}\ , \tag{9}$$

or

$$\frac{m d\mathbf{v}}{dt} = \mathbf{F} + 2(\mathbf{u}' - \mathbf{v})\frac{dm'}{dt}\ . \tag{10}$$

The term $2(\mathbf{u}' - \mathbf{v})\frac{dm'}{dt}$ can be easily interpreted. It correspons to force $\mathbf{f}_c$. (Abreu Faro and Abreu 1987).
In fact

$$2(\mathbf{u}' - \mathbf{v})\frac{dm'}{dt} = 2\mathbf{u}'\left(\frac{dm'}{dt}\right)\left(1 - \frac{\mathbf{v}}{\mathbf{u}'}\right), \tag{11}$$

corresponds to

$$f_c = f_{c0}\left(1 - \frac{\dot{x}}{u}\right)$$

where

$$2\mathbf{u}'\left(\frac{dm'}{dt}\right) \qquad f_{c0}$$

and

$$\left(1 - \frac{\dot{x}}{u}\right) \qquad \left(1 - \frac{\mathbf{v}}{\mathbf{u}'}\right).$$

We can therefore interpret the force exerted on the piston due to the collisions of gas particles as a superposition of the forces due to an absorbed particle beam and to an emitted particle beam.
On the other hand we see that in a variable mass system the force is given by $(\mathbf{u}' - \mathbf{v})\frac{dm'}{dt}$ for an absorbed beam or, equally, for an emitted beam, as can be checked from (7) to (10).
It is obvious that if the physical meaning is not considered, we can introduce the following quantities which we shall call **gorces**, symbolically represented by **g**:

$$\mathbf{g} = \frac{d\mathbf{p}'}{dt}\ ,$$

where, in the situation previously analysed,

$$\mathbf{g} = \boldsymbol{F} + \boldsymbol{u}\frac{dm'}{dt}.$$

We can, therefore identify several "forces":

1. $\mathbf{f}_1 = \mathbf{F} + \mathbf{f}'_1$

where

$$\boldsymbol{f}'_1 = \boldsymbol{u}\frac{dm'}{dt}.$$

2. $\mathbf{f}_2 = \mathbf{F} + \mathbf{f'}_2$

where

$$\boldsymbol{f'}_2 = (\boldsymbol{u} - \boldsymbol{v}) \frac{dm'}{dt}.$$

Quantities f_1 and f'_1 are entities without physical meaning, while $\mathbf{f}_2$, $\mathbf{f'}_2$ and $\mathbf{F}$ are entities whose physical meaning is ultimately derived from the physical meaning of the momentum conservation and, as we shall see next, from energy conservation.

This "arbitrariness" in the definition of "force" originated a controversy which can be easily solved if the previous analysis is taken into consideration.

1.2 Identification of a controversy:

the concept of force and the First Principle of Thermodynamics – the notion of **gheat**.

As already noted, we can consider various forces with physical meaning. One of these, gravitational force $\mathbf{F}$, may be associated with the potential energy $E_{pot} = V$,

$$\mathbf{F} = -\mathbf{d}V \qquad (14)$$

where $\mathbf{d}V$ is the external differential of V (Misner, Thorne and Wheeler 1972).

The work of $\mathbf{F}$ depends only on the position (Abreu 1987). That is since V depends only on the position coordinates – height, in the case of the piston – the work of $\mathbf{F}$ depends only on the final and inicial heights.

If $\mathbf{F} = -\mathbf{d}V$ is the only force acting upon mass m, and considering that $dW = -dV = dE_{cin} = dT$, we have

$$dT + dV = 0 \qquad (15)$$

If the internal energy U is considered, we can define a *dissipative force* (see 1.1) and obtain

$$dT + dV + dU = 0, \qquad (16)$$

$$dW_{total} = dW + dW_{diss.} = dT, \qquad (17)$$

$$dW = -dV, \qquad (18)$$

and

$$dW_{diss.} = dT + dV_{diss.} = -dU. \qquad (19)$$

Between two equilibrium points we have

$$W = -W_{diss.} = \Delta U. \qquad (20)$$

The mentioned controversy will now be generically characterized.

Let us consider an entity in general, which we shall call "force", $\mathbf{f}$, and its work dW. If we take the energy E which in any transformation considered is related to "force" $\mathbf{f}$, we can always write

$$dE = dW + dQ, \qquad (21)$$

it sufficing, for the purpose, to define entity dQ as what is lacking in work, dW, to obtain the energy variation dE. The situation is similar to the one previously referred to when we were dealing with the concept of **gorce**. For the same reason we are going to call this entity **gheat** ...

Obviously, if the "force" has physical meaning and is conveniently related to the energy variation, entity dQ has physical meaning.

As it will be verified, this is precisely what happens in the analysis we shall now proceed to make for variable mass systems, both in the classical and in the relativistic formulation (Abreu 1983).

If the mass is constant, we find that

$$\mathbf{F} = \frac{d\mathbf{p}}{dT} = m\mathbf{a}\,, \tag{22}$$

and therefore

$$\mathbf{v}.\frac{d}{dt}(m\mathbf{v}) = \mathbf{v.F} \tag{23}$$

or

$$\frac{d}{dt}[(1/2)mv^2] = \mathbf{v.F} \tag{24}$$

By integrating (23) we can write

$$(1/2)mv_2^2 - (1/2)mv_1^2 = W. \tag{25}$$

Copeland rightly calls attention (Copeland 1982) to the fact that (25) is usually rewritten in the form

$$(1/2)m_2v_2^2 - (1/2)m_1v_1^2 = W - Q, \tag{26}$$

and that (26) is not a solution for (24) because the mass not being constant, Force F, with physical meaning, is not related to the momentum by

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}\,. \tag{27}$$

Copeland is right, even if equation (26) is formally defined, if we define *gorce* ("force") by (27) and *gheat* ("heat") by (26).

In fact, in (3), as previously seen,

$$\mathbf{F} + \mathbf{u}\,\frac{dm}{dt}$$

does not possess the meaning of force.
From (3) we obtain

$$\mathbf{v}.\left(\frac{d}{dt}\right)(m\mathbf{v}) = \mathbf{v.F} + \mathbf{v.u}\left(\frac{dm}{dt}\right) \tag{28}$$

or

$$m\mathbf{v}.\frac{d\mathbf{v}}{dt} + v^2\frac{dm}{dt} = \mathbf{v.F} + \mathbf{v.u}\left(\frac{dm}{dt}\right) \tag{29}$$

But

$$\frac{d}{dt}\,[(1/2)m\mathbf{v}.\mathbf{v}] = m\mathbf{v}.\left(\frac{d}{dt}\right)\mathbf{v} + (1/2)\,v^2\left(\frac{dm}{dt}\right) \quad (30)$$

or

$$\frac{d}{dt}(1/2mv^2) = \mathbf{v}.\mathbf{F} + \mathbf{v}.\left[(\mathbf{u} - (1/2)\mathbf{v})\left(\frac{dm}{dt}\right)\right]. \quad (31)$$

By integrating

$$(1/2)m_2v_2^2 - (1/2)m_1v_1^2 = W + \left[\mathbf{v}.\{\mathbf{u} - (1/2)\mathbf{v})\}\left(\frac{dm}{dt}\right)\right] \quad (32)$$

where Copeland writes

$$W = \mathbf{v}.\mathbf{F}$$

where **F** is the force due to the gravitational field.
If we compare (32) with (26), we obtain

$$Q = -\int[\mathbf{v}.(\mathbf{u} - (1/2)\mathbf{v})]\,dm, \quad (33)$$

"but this is not quite true", says Copeland, since

$$\frac{d}{dt}\left[(1/2)mv^2\right] = \mathbf{v}.\mathbf{F} + (1/2)u^2)\,\frac{dm}{dt} - (1/2)(\mathbf{u} - \mathbf{v})^2\,\frac{dm}{dt}, \quad (34)$$

and (32) becomes

$$(1/2)m_2v_2^2 - (1/2)m_1v_1^2 = W + \int(1/2)u^2dm - \int(1/2)(\mathbf{u} - \mathbf{v})^2dm. \quad (35)$$

Copeland introduces **his** notion of heat

$$Q = \int(1/2)(\mathbf{u} - \mathbf{v})^2\,dm, \quad (36)$$

which corresponds the the kinetic energy of the beam on the piston frame and which, because the beam is stopped (absorbed) by the piston, is transformed into "heat" (*kinetic energy lost by the accrued mass as it comes to rest* (Copeland 1983)), writing (35) in the form

$$\Delta T = W + \int(1/2)\,u^2dm - Q. \quad (37)$$

The term $\int(1/2)\,u^2dm$ corresponds to the kinetic energy of the beam on the laboratory frame which "disppears" on this frame since dm is absorbed by the piston.

In the analysis previously developed on 1.1 we verified that the superposition of an emitted beam and of an absorbed beam gives rise to a *dissipative force* which satisfies equation

$$\Delta T = W + W_{diss.} \quad (38)$$

Since $\Delta T + \Delta V + \Delta U = 0$,

and

$$W_{diss.} = -\Delta U, \quad (39)$$

we see that $W_{diss.}$ corresponds to $(1/2)u^2dm$ of the emitted and of the absorbed beam, i.e., to the variation of the gas energy on the laboratory frame.

This term with energetic meaning corresponds to affirming the dissipation of gravitational energy into internal energy (heat) of the gas and not to the variation of internal energy of the piston (Abreu 1985).

In this situation, that of the piston, it becomes obvious that

$$W_{total} = \Delta T$$

and that in Copeland's analysis, only the force due to the field is introduced in W.

IN CONCLUSION

The expression of the "1st Principle" dE = dW + dQ is a tautology, reason why the physical interpretations which are attributed to it **are not equivalent** and some of them are incorrect.

REFERENCES

[1] Feynman, R. Leighton, R. Sands, M. The Feynman Lectures on Physics (Addison-Wesley, Reading) Vol.I, 12-1 (1976).

[2] Sommerfeld, A. Mechanics, (Academic Press, New York), 5 (1966).

[3] Abreu Faro, M. and Abreu, R. A one dimensional model of irreversibility, Relatório Interno, CEL 4, IST (1987).

[4] Abreu, R. Técnica n.º único, p. 15(a), 35(b), 65(c), 69(d) (1993).

[5] Misner, C.W. Thorne and Wheeler, J.A. Gravitation (W.H. Freemnn and Company, New York), 59 (1970).

[6] Abreu, R. Técnica1, 50 (1987).

[7] Copeland, J. Am. J. Phys. 50, 7 (1982).