Combined Gravitational Action (V): *Extension*

Mohamed E. Hassani[1]
Institute for Fundamental Research
BP.197, CTR, Ghardaïa 47800, Algeria

Abstract: In series of papers relative to the Combined Gravitational Action (CGA) as an alternative gravity theory, we have developed the CGA-formalism, which is exclusively based on a new form of velocity-dependent gravitational potential energy. The present paper is actually an additional exploration, exploitation and extension of the CGA with the main aim of showing (1) the Galilean invariance of CGA-equations; (2) that the CGA is in a certain manner a metric gravity theory; (3) the existence of an important similarity between the CGA and General Relativity Theory; (4) a new CGA-formula for perigee and perihelion precession is derived and proved to be in excellent agreement with observations; (5) the CGA-effects on orbital motion inside and outside the Solar System are investigated; (6) the conversion rate of orbital energy into gravitational power radiation is calculated and its effects are studied.

Keywords: CGA-effects, orbital motion, perigee, perihelion, periastron, Moon, major asteroids, inner planets, binary pulsars, gravitational power radiation

1. Introduction

In previous papers [1,2,3,4] we have, already, investigated the CGA-effects arising from the CGA-formalism as an alternative gravity theory capable of predicting and explaining some old and new gravitational effects and allowed us to resolve –in its context– some unexpected and defiant problems occurred inside and outside the Solar System (SS) like, *e.g.*, the anomalous Pioneer 10's deceleration, the observed secular increase of the Astronomical Unit and the apsidal motion anomaly of the eclipsing binary star systems AS Camelopardalis and DI Herculis. For instance, in the paper [3], we have investigated the CGA-effects on the orbital motion of the planets and in the noncompact and compact stellar objects. In the CGA-fourth part [4], we have shown the existence of the CGA-spin-orbit coupling precession and applied CGA to large-scale structures and the problem of galactic rotation curves has been resolved. Also the Milgrom's theory of Modified Newtonian Dynamics (MOND) [5,6,7,8] as an alternative model to the dark matter (DM) 'hypothesis' became by means of CGA [4] an additional support for DM!

The present work is completely based on CGA-third part [3] with the primary purpose of showing, among other things, the invariance of CGA-equations under Galilean transformations; that the CGA is in certain manner a metric gravity and the existence of an important similarity between the CGA-equations of motion and those of General Relativity Theory (GRT). It is their additional terms that are responsible for the major secular effects as we will see. In paper [3], we have already calculated and listed the numerical values of CGA-effects on the SS' planets (see, Tables 1 and 2 in Ref. [3]). For example, Table 2 illustrated us the excellent agreement between planets' observed perihelion precession and the CGA-predictions. The same appreciation for CGA-apsidal motion in eclipsing binary star systems [3].

[1] E-mail: hassani641@gmail.com

From all that, we can say this is all the more impressive particularly when we take into account the fact that the Brans-Dicke and other alternative gravity theories, except GRT, containing some adjustable parameters that is why are called 'adjustable gravity', however, the CGA has no freedom to adjust its predictions because simply it is by its proper formalism an inadjustable gravity theory.

As we can remark it from the earlier papers [1,2,3,4], the asteroids had been completely neglected that is why an important part of the present work is devoted to them because more recently the author of CGA realized that since some major asteroids are potentially hazardous, for instance, Apollo and Icarus recognized to cross Earth's orbit, for this reason, the asteroids are quite interesting celestial bodies because they can serve as a celestial laboratory enabling us to understand physical process that take place on the asteroids as well as on other similar SS-bodies and also to test more rigorously the alternative gravity theories. Hence, the investigation of CGA-effects on the orbital motion of asteroids is very important. The reader who is interested in the CGA-formalism can see Ref. [3].

The real novelty of the present paper is related to the derivations of two *new* CGA-formulae, one for the perigee and perihelion precession and the other is related to the conversion rate of orbital energy into gravitational power radiation. To begin, it is best for the convenience of the reader, to recall briefly the basic foundation of the CGA-formalism. The CGA as an alternative gravity theory is fundamentally based on the concept of the combined gravitational potential energy (CGPE) which is typically a new form of velocity-dependent-GPE defined by the expression

$$U \equiv U(r,v) = -\frac{k}{r}\left(1+\frac{v^2}{w^2}\right), \tag{1}$$

where $k = GMm$; G being the Newton's gravitational constant; M and m are the masses of the gravitational source A and the moving test-body B; $r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$ is the relative distance between A and B; $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ is the velocity of the test-body B relative to the inertial reference frame of source A; and w is a specific kinematical parameter having the dimensions of a constant velocity defined by

$$w = \begin{cases} c_0, \text{ if } B \text{ is in relative motion inside the vicinity of } A \\ v_{\text{esc}} = \sqrt{2GM/R}, \text{ if } B \text{ is in relative motion outside the vicinity of } A \end{cases}, \tag{2}$$

where c_0 is the light speed in local vacuum and v_{esc} is the escape velocity at the surface of the gravitational source A.

The velocity-dependent-GPE (1) is simply called 'CGPE' because it is, in fact, a combination of the static-GPE $V(r) = -k\,r^{-1}$ and the dynamic-GPE $W(r,v) = -k\,r^{-1}(v/w)^2$. The main difference between the CGPE (1) and the previously well-known velocity-dependent-PGEs is clearly situated in the originality and simplicity of expression (1). For instance, the originality of CGPE is reflected by the fact that CGPE is explicitly depending on r and v but also is implicitly depending on w since the latter is, by definition, a constant specific kinematical parameter. The implicit dependence of CGPE on w is expressed in terms of '*inside the vicinity of A*' and '*outside the vicinity of A*' in (2).

Moreover, the CGPE (1) constitutes a fundamental solution to a system of three second order PDEs, called *'potential equations'* because U is a common solution to these three equations. Indeed, it is easy to show under some appropriate boundary conditions that the combined potential field U is really a fundamental solution to the following PDEs

$$\frac{\partial^2 U}{\partial r^2}+\frac{2}{r}\frac{\partial U}{\partial r}=0, \tag{3}$$

$$\frac{\partial^2 U}{\partial v^2}-\frac{1}{v}\frac{\partial U}{\partial v}=0, \tag{4}$$

$$\frac{\partial^2 U}{\partial r\,\partial v}+\frac{1}{r}\frac{\partial U}{\partial v}=0. \tag{5}$$

Remark, since Eqs.(3-5) are homogeneous and admit the same potential function U as a fundamental solution hence this implies, among other things, that the test-body B is in state of motion at the relative velocity, v, sufficiently far from the main gravitational source A. Furthermore, the same fundamental solution is the *origin* of the CGA-equations of motion and the CGA-field equations because, as we have previously seen in [2,3], the potential function U is the leading term of the CGA-Lagrangian.

2. Galilean Invariance of CGA-Equations

It is best to recall that the CGA as an alternative gravity theory is wholly developed in the framework of Galilean relativity principle and Euclidean geometry thus in view of the fact that in the Nature the major physical phenomena manifest at subrelativistic velocities, *i.e.*, velocities that are sufficiently small compared to the light speed in local vacuum. From now, if we confine ourselves to the case where the velocities are subrelativistic we can prove the invariance of CGA-equations under Galilean transformations (GTs). With this aim, let us make use of the following shortcut: since the CGA-formalism is basically founded on the CGPE (1), therefore, the Galilean invariance of CGA-equations implies the invariance of CGPE under GTs. So, supposing two inertial reference frames (IRFs) S and S', which are in uniform relative motion at subrelativistic velocity $\mathbf{u}$ of magnitude u. To simplify the algebra, let the vector velocity $\mathbf{u}$ of IRFs be along their common $x \mid x'$-axis with corresponding parallel planes. Also, the two origins O and O' coincide at the moment $t=t'=0$. The GTs that ensuring the passage from an IRF to another are in vector form:

$$S \to S': \begin{cases} \mathbf{r}'=\mathbf{r}-\mathbf{u}t \\ t'=t \end{cases}, \tag{6}$$

where $\mathbf{r}$ and $\mathbf{r}'$ are position vectors as observed from S and S' at any later time $t'=t$. Since the quantities k and w in [1] are constant, hence the CGPE may be written in the form

$$U \equiv U(\mathbf{r},\mathbf{v})=-\frac{k}{\|\mathbf{r}\|}\left(1+\frac{\|\mathbf{v}\|^2}{w^2}\right). \tag{7}$$

Now, supposing there are two material points P_1 and P_2 whose position vectors are, respectively, $(\mathbf{r}_1,\mathbf{r}_2)$ relative to S and $(\mathbf{r}_1',\mathbf{r}_2')$ relative to S', and their masses $m=m_1$ and $M=m_2$ as observed from the two

IRFs S and S' which is in uniform relative motion at subrelativistic velocity $\mathbf{u}$ with respect to S. Therefore, since under GTs (6), we have

$$\mathbf{r}'_{21} = \mathbf{r}'_2 - \mathbf{r}'_1 = \mathbf{r}_2 - \mathbf{r}_1 = \mathbf{r}_{21} \quad \text{and} \quad \mathbf{v}'_{21} = \mathbf{v}'_2 - \mathbf{v}'_1 = \mathbf{v}_2 - \mathbf{v}_1 = \mathbf{v}_{21}.$$

Hence, we get for the CGPE (7):

$$U(\mathbf{r}'_{21}, \mathbf{v}'_{21}) = U(\mathbf{r}_{21}, \mathbf{v}_{21}). \tag{8}$$

This is clearly illustrating us the invariance of the CGPE under GTs.

3. Is CGA a metric gravity theory?

After having proved the Galilean invariance of CGPE, which led to the invariance of CGA-equations under GTs. Now, we shall try to answer affirmatively the above question in the CGA-context without concerning ourselves with the conceptual details because, for our theory, the metricity has only an heuristic importance . To this end, let us rewrite the CGPE (1) as follows

$$r^2 = \left(\frac{k}{U}\right)^2 \left(1 + \frac{v^2}{w^2}\right)^2, \quad r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}\,, \tag{9}$$

or equivalently

$$dx^2 + dy^2 + dz^2 - \left(\frac{k}{U}\right)^2 = \left(\frac{k}{U}\right)^2 \left(1 + \frac{v^2}{w^2}\right)^2, \tag{10}$$

where

$$dx = x - x_0,\ dy = y - y_0,\ dz = z - z_0.$$

Since the quantity $(k/U)^2$ has the geometrical dimensions of squared length thus, just for convenience, let us define it as $(k/U)^2 = w^2 dt^2$ and $(k/U)^2\left[2(v/w)^2 + (v/w)^4\right] = ds^2$, so after substitution into (10), we get

$$ds^2 = dx^2 + dy^2 + dz^2 - w^2 dt^2. \tag{11}$$

The expression of the quadratic form or equally the space-time interval (11) allows us to say that, in certain sense, the test-body may be gravitationally evolved in flat space-time, called here, CGA-space-time. Let us prove the invariance of (11) under some spatio-temporal transformations more general than GTs. With this aim, considering again two IRFs S and S', which are in uniform relative motion at velocity $\mathbf{u} = u\mathbf{e}_x$ such that $u < w$. Also, the two origins O and O' coincide at the moment $t = t' = 0$. The two IRFs S and S' are linked by the following spatio-temporal transformations given in differential form as follows:

$$S \to S' : \begin{cases} dx' = \eta\left(dx - udt\right) \\ dy' = dy \\ dz' = dz \\ dt' = \eta\left(dt - \dfrac{udx}{w^2}\right) \end{cases} , \quad \eta = \left(1 - u^2/w^2\right)^{-1/2}, \ \ u < w. \tag{12}$$

Remark, by using (12), we obtain the following invariance

$$ds'^2 = dx'^2 + dy'^2 + dz'^2 - w^2 dt'^2 = dx^2 + dy^2 + dz^2 - w^2 dt^2 = ds^2. \tag{13}$$

This means, among other things, that the CGA is –in a certain manner– a metric gravity theory. In passing, and without going into detail about the process of derivation of (12), it is easy to prove that these transformations form an orthogonal-orthochronous group. Also, the same transformations may be reduced to Lorentz transformations for the case when $w = c_0$. Finally, the reader can observe that the classical (Galilean) notion of absolute time –second equation in (6)– agreeing for all the IRFs is not always valid because the fourth-equation in (12) may be reduced to $t' = t$ only if the ratio (u/w) is sufficiently small than unity.

4. Some similarity between CGA and GRT

In the CGA-second part [2], we have derived Eq.(30), which is the general equation of motion in the combined gravitational field and from it, we have derived Eq.(32):

$$\frac{d^2u}{d\varphi^2} + u - \left[\frac{3GM}{c_0^2}\right]u^2 = \frac{GM}{\kappa^2}, \qquad \kappa = r^2\dot{\varphi}, \tag{14}$$

which has the exact form derived under GRT. Further, Eq.(14) allows us ,of course, to study the perihelion advance of Mercury and other planets, and the angular deflection of light in the combined gravitational field [2]. In the CGA-third part [3], we have derived from the CGA-Lagrangian, the CGA-equations of motion in compact form, *i.e.*, Eqs.(10) in Ref.[3]:

$$\frac{d\mathbf{v}}{dt} + \left(1 + \frac{v^2}{w^2}\right)\left(1 + \frac{2GM}{w^2 r}\right)^{-1} \nabla\phi = 0, \tag{15}$$

where ϕ is the Newtonian gravitational potential defined by

$$\phi \equiv \phi(r) = -\frac{GM}{r}, \tag{16}$$

Now, considering the case when the test-body B evolving in the vicinity of the gravitational source A, hence according to (2), Eqs.(15) become

$$\frac{d\mathbf{v}}{dt} + \left(1 + \frac{v^2}{c_0^2}\right)\left(1 + \frac{2GM}{c_0^2 r}\right)^{-1} \nabla\phi = 0\,, \tag{17}$$

Moreover, if the quantity $\left(2GM/c_0^2 r\right)$ is sufficiently small in comparison to unity, therefore, Eqs.(17) may be written in the form

$$\frac{d\mathbf{v}}{dt} = -\frac{GM}{r^3}\mathbf{r} + \frac{GM}{r^3}\left[\frac{2GM}{c_0^2 r}\left(1 + \frac{v^2}{c_0^2}\right)\mathbf{r} - \left(\frac{v}{c_0}\right)^2 \mathbf{r}\right]. \tag{18}$$

We can see that the first term on the right-hand side of Eq.(18) is the well-known Newtonian gravitational acceleration. The remaining terms are the CGA-correction which, in part, gives rise to the CGA-effects like, *e.g.*, the orbital precession (perigee, perihelion and periastron precession) as we will see later. Further, many authors in the field of relativistic gravitational physics have derived –in the context of the post-Newtonian approximation– certain equations of motion similar to Eq.(18). Among them, we can, *e.g.*, choice from Ref.[5] the following equation:

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{\mu}{r^3}\mathbf{r} + \frac{\mu}{r^3}\left[\left(4\frac{\mu}{r} - \frac{v^2}{c^2}\right)\mathbf{r} + 4\frac{(\mathbf{r}\cdot\mathbf{v})\mathbf{v}}{c^2}\right], \quad \mu = Gm_S/c^2\,. \tag{19}$$

Where m_S is the Sun's mass.

Question: why shall CGA arrive at the same results as GRT?

Response: because if we take the concept of the curvature of space-time apart, we find that contrary to the Newton's theory of gravitation, CGA and GRT take, at the same time, in full consideration the relative motion of the test-body and the light speed in local vacuum which in CGA is playing the role of a specific kinematical parameter of normalization and in GRT is supposed to be the speed of gravity propagation. As it was previously mentioned in [3], the principal result of CGA-formalism [1,2,3,4] is the existence of the dynamic gravitational (acceleration) field (DGF), $\mathbf{\Lambda}$, derived from Eq.(28) of Ref. [3] under the form

$$\mathbf{\Lambda} : \begin{cases} \Lambda_x = -\dfrac{v^2}{w^2}\dfrac{GM}{r^2}\dfrac{x - x_0}{r} \\ \Lambda_y = -\dfrac{v^2}{w^2}\dfrac{GM}{r^2}\dfrac{y - y_0}{r} \\ \Lambda_z = -\dfrac{v^2}{w^2}\dfrac{GM}{r^2}\dfrac{z - z_0}{r} \end{cases}, \tag{20}$$

The DGF is actually an induced field, it is more precisely a sort of gravitational induction due to the relative motion of material test-body in the vicinity of the gravitational source. Certainly, the static gravitational field

$$\boldsymbol{\gamma} = -\nabla\phi(r), \tag{21}$$

is generally always stronger than DGF but $\mathbf{\Lambda}$ has its proper role and effects. For example, as an additional field, $\mathbf{\Lambda}$ is responsible for the perihelion advance of Mercury and other planets [1,2,3]. In his 1912 argument [6], Einstein himself noted that the inertia of energy and the equality of inertial and gravitational mass lead us to expect that *"gravitation acts more strongly on a moving body than on the same body in case it is at rest."* It seems Einstein's remark reflects very well the expression of the combined gravitational field [1,2,3]

$$\mathbf{g} = \boldsymbol{\gamma} + \mathbf{\Lambda}. \tag{22}$$

It is clear from (22), that the combined gravitational field, $\mathbf{g}$, may be reduced to the static gravitational field, $\boldsymbol{\gamma}$, only for the case $\mathbf{\Lambda} = 0$, that is, when the material test-body under the action of field is at the relative rest with respect to the main gravitational source. Moreover, as we know from [2,3], the combined gravitational field is derived from the CGPE (1).

Concerning the second result of CGA, namely, the dynamic gravitational force $\mathbf{F}_{\mathrm{D}}$ is defined as the product of mass m of the moving test-body B and $\mathbf{\Lambda}$, that is

$$\mathbf{F}_{\mathrm{D}} = m\mathbf{\Lambda}. \tag{23}$$

Curiously, Lorentz has already arrived at some conclusion very comparable to that of Einstein, but more than one decade before him. In his very influential work [7] entitled 'Considerations on Gravitation' published in 1900, Lorentz wrote *"Every theory of gravitation has to deal with the problem of the influence, exerted on this force by the motion of the heavenly bodies."* To my great surprise, when I read the Lorentz article [7] for the first time, that is, after having written the CGA, I found, among other interesting things, that Lorentz had arrived at an extra-gravitational force (Eqs.(24) in Ref.[7]) whose components are very similar to those of $\mathbf{F}_{\mathrm{D}}$. Again, Lorentz claim and finding reinforcing the fact that the couple $(\mathbf{\Lambda}, \mathbf{F}_{\mathrm{D}})$ is really induced by the relative motion of massive test-body *B* in the gravitational field of central body *A*. Also, Broginsky, Caves and Thorn, in their seminal paper [8] entitled 'Laboratory experiments to test relativistic gravity' published in 1977, they found an extra-gravitational acceleration called by them post-Newtonian gravitational acceleration (Eq.(2.1) in Ref.[8]) whose magnitude is also comparable to that of $\mathbf{\Lambda}$.

5. Effects of $(\mathbf{\Lambda}, \mathbf{F}_{\mathrm{D}})$ on orbital motion

Now, we arrive at the heart of our subject, *viz*., the CGA-effects on orbital motion . As we knew from the papers [1,2,3,4], the CGA-effects consist of average change in orbital period, orbital velocity, radial distance, and secular perigee precession of satellites, secular perihelion precession of planets, angular deflection of light, secular apsidal motion in eclipsing binary star systems, periastron advance of pulsars and CGA-spin-orbit precession. All these CGA-effects are raised from the couple $(\mathbf{\Lambda}, \mathbf{F}_{\mathrm{D}})$.

In order to familiarize the reader with the CGA-effects, it is best to start by showing that CGA-effects are in fact contained in Eq.(18), *i.e*., the terms in the right-hand side of (18). Let us prove the above affirmation, namely, the CGA-corrections to the Newtonian equations of motion are of the form of Eq.(18) by comparing this equation with Newton's one

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{GM}{r^3}\mathbf{r} \ . \tag{24}$$

Hence, simple comparison with Eq.(18) shows us that if we apply Eq.(24) instead of Eq.(18) to, *e.g.*, the Earth's orbital motion, we should have apparently, correct result, which is partly due to the fact that the Earth's orbital velocity is too slow compared to the light speed and the Sun's gravitational field is so weak compared to that of, *e.g.*, neutron star or pulsar. However, in terms of accuracy, the omitted terms on the right-hand side of Eq.(18) would still contribute at least on average by an important amount. Thus, if we take into account these neglected terms, we get the CGA-correction to the Earth's semi-major axis a by an amount of

$$\frac{\delta a}{a} = \left[\frac{2GM_{\mathrm{S}}}{c_0^2 r}\left(1 + \frac{v_{\mathrm{orb}}^2}{c_0^2}\right) - \left(\frac{v_{\mathrm{orb}}}{c_0}\right)^2\right] \cong 10^{-8} \, . \tag{25}$$

In Eq.(25), we have used $M = m_{\mathrm{S}}$ instead of $M = m_{\mathrm{S}} + m_{\mathrm{E}}$ because the mass-ratio, $q = m_{\mathrm{E}}/m_{\mathrm{S}}$, of the system {Sun, Earth} is comparable to zero. The above illustrative example shines a spotlight on the CGA-formalism and pushes us to draw attention of the astronomy community to scrutinize the CGA in order to assess its role and importance as an alternative gravity theory.

Returning again to our main subject and demonstrating the effects of couple $(\mathbf{\Lambda}, \mathbf{F}_{\mathrm{D}})$ on the orbital motion of a material test-body. Firstly, we begin by showing that the orbital eccentricity depends, in part, on the average magnitude of the dynamic gravitational force, $\mathbf{F}_{\mathrm{D}}$, and secondly we should prove that the perigee (if the test-body is a satellite), the perihelion (if the test-body is a planet) and periastron (if the test-body is a star) of any moving material test-body, in combined gravitational field (22), should exclusively depend on the average magnitude of the dynamic gravitational field $\mathbf{\Lambda}$.

5.1. Effect of $\mathbf{F}_{\mathrm{D}}$ on orbital eccentricity

Let us demonstrate that the orbital eccentricity, *e*, is partly depending on the magnitude of the dynamic gravitational force $\mathbf{F}_{\mathrm{D}}$. To this end, suppose the material test-body *B* of mass m orbiting the gravitational source *A* of mass M at the mean radial distance r_{mean} with mean orbital velocity $v_{\mathrm{orb}} = 2\pi\, r_{\mathrm{mean}} P^{-1}$, where P is the mean orbital period. Since the test-body *B* moving inside the vicinity of the gravitational source A, this allows us to use the definition (2) and Eqs.(20). Thus, the magnitude of the dynamic gravitational field $\mathbf{\Lambda}$ –when it plays the role of an extra-gravitational acceleration– should be of the form

$$\Lambda = \frac{GM}{r^2}\left(\frac{v_{\mathrm{orb}}}{c_0}\right)^2 = \frac{4\pi^2 GM}{c_0^2 P^2}\left(\frac{r_{\mathrm{mean}}}{r}\right)^2 . \tag{26}$$

Further, we have the following well-known relation

$$e = \frac{r_{\mathrm{max}} - r_{\mathrm{min}}}{r_{\mathrm{max}} + r_{\mathrm{min}}} , \tag{27}$$

and

$$r_{\text{mean}} = \frac{(r_{\max} + r_{\min})}{2}. \tag{28}$$

Hence, from (26), (27) and (28), we get

$$\Lambda = \frac{\pi^2 GM}{c_0^2 P^2}\left(\frac{\delta r}{e\,r}\right)^2, \tag{29}$$

with $\delta r = r_{\max} - r_{\min}$. Therefore, by multiplying the two sides of (29) by the mass, m, of the moving test-body *B*, we find

$$F_{\mathrm{D}} = m\Lambda = \frac{\pi^2 k}{c_0^2 P^2}\left(\frac{\delta r}{e\,r}\right)^2, \quad k = GMm \tag{30}$$

Finally, from (30), we get the very expected equation that expresses the dependence of e on F_{D}

$$e = \left(\frac{\pi}{c_0 P}\cdot\frac{\delta r}{r}\right)\sqrt{\frac{k}{F_{\mathrm{D}}}}. \tag{31}$$

Eq.(31) shows us more clearly the following possible cases:

i) The orbital eccentricity, *e*, should always be different from zero, that is, $e \neq 0 \Rightarrow \delta r \neq 0$, therefore, the orbit is not circular and as a direct result we have:

$$e \neq 0 \Rightarrow \begin{cases} e < 1, \text{ the orbit is an ellipse} \\ e = 1, \text{ the orbit is a parabola} \\ e > 1, \text{ the orbit is a hyperbola} \end{cases}.$$

ii) Eq.(31) is exclusively valid for the case $e < 1$.

iii) The orbital eccentricity is extremely high if, in terms of average magnitude, $\mathbf{F}_{\mathrm{D}}$ is extremely weak.

iv) The orbital eccentricity is extremely low if, in terms of average magnitude, $\mathbf{F}_{\mathrm{D}}$ is extremely strong.

5.2. Effect of $\mathbf{\Lambda}$ on orbital precession

As previously reported, we have already derived from Eq.(32) of Ref.[2], the formula (32.10):

$$\Delta\varphi = \frac{6\pi GM}{a c_0^2 (1 - e^2)} \quad \text{(rad/rev)}, \tag{32}$$

for orbital precession, which is identical to that derived from GRT. Presently, our purpose is to show the exclusive dependence of orbital precession on the average magnitude of dynamic gravitational field $\mathbf{\Lambda}$. With this aim, we must use (32) only as a *target* that should be explicitly deduced from (29) combined with the well-known equation of ellipse in polar coordinates

$$r = \frac{a\left(1-e^2\right)}{1-e\cos\varphi}\ , \tag{33}$$

All that should be done *via* some simple algebraic calculations as we will see immediately. So let us rewrite (29) as follows

$$r\Lambda = \frac{GM}{r}\cdot\frac{6\pi}{c_0^2}\cdot\frac{\omega^2\delta r^2}{24\pi e^2}\ , \quad \omega = 2\pi/P\,. \tag{34}$$

Or by substituting (33) in the right-hand side of (34), we get

$$r\Lambda = \frac{(1-e\cos\varphi)GM}{a\left(1-e^2\right)}\cdot\frac{6\pi}{c_0^2}\cdot\frac{\omega^2\delta r^2}{24\pi e^2}\,. \tag{35}$$

And from (35) we deduce immediately the formula (32):

$$\frac{r\Lambda}{(1-e\cos\varphi)}\cdot\frac{24\pi e^2}{\omega^2\delta r^2} = \frac{6\pi GM}{ac_0^2\left(1-e^2\right)}\,. \tag{36}$$

Or equivalently

$$\Delta\varphi = \frac{6e^2 r}{(1-e\cos\varphi)}\cdot\frac{\Lambda P^2}{\pi\delta r^2}\,. \tag{37}$$

Further, we have from (33)

$$1\text{-}\,e\cos\varphi = \frac{a\left(1-e^2\right)}{r}\,. \tag{38}$$

Thus, substituting (38) into (37) yields

$$\Delta\varphi = \left(\frac{6e^2P^2}{\pi a\left(1-e^2\right)}\cdot\frac{r^2}{\delta r^2}\right)\Lambda\,. \tag{39}$$

Remark: since $\delta r = r_{\max} - r_{\min} = 2e\,r_{\text{mean}}$, therefore, (39) becomes for the case $r = r_{\text{mean}}$

$$\Delta\varphi = \left[\frac{3P^2}{2\pi a\left(1-e^2\right)}\right]\Lambda\,. \tag{40}$$

Expression (40) is exactly the very expected formula that shows us the dependence of $\Delta\varphi$ on Λ. Moreover, in view of the fact that the perihelion advance by $\Delta\varphi$ *per* revolution, thus in this case the resultant equation for the elliptical orbit should be

$$r = \frac{a\left(1-e^2\right)}{1-e\cos(\varphi+\Delta\varphi)}\ . \tag{41}$$

Eq.(41) may be regarded as a generalization of (33) when the test-body evolving in the combined gravitational field (22). Concerning Λ in (40), is naturally defined by the expression

$$\Lambda = \frac{1}{a}\left[\frac{GM}{c_0 a}\right]^2, \tag{42}$$

since the mean orbital velocity v_{orb} of test-body may be also expressed by $v_{\text{orb}} = \sqrt{GM/a}$ for $r = r_{\text{mean}} = a$. Furthermore, formula (40) means $\Delta\varphi \propto \Lambda$ and the term in bracket is the coefficient of proportionality, which has the physical dimensions of inverse of the average magnitude of acceleration.

Clarification: It is worthwhile to note that when we have derived the formulae (40) and (41), the mass of orbiting test-body B ($m = m_B$) was implicitly supposed to be small than that of gravitational source A ($M = m_A$), *i.e.*, the mass-ratio, $q = m_B / m_A$, of system $\{A, B\}$ is comparable to zero and also the relative distance between A and B is assumed to be sufficiently large. However, if q is not sufficiently comparable to zero and the separation is not enough large, in such a case, we should take $M = m_A + m_B$ as a total mass of system $\{A, B\}$. All that may be summarized below as follows:

$$M = \begin{cases} m_A, \text{ if } q \cong 0 \\ m_A(1+q), \text{ if } q > 0 \end{cases}, \quad q = m_B / m_A . \tag{43}$$

Before, investigating the CGA-effects on orbital motion of major asteroids, it is judged best to familiarize the reader with the process of computation when we would apply the CGA-formalism. To this pedagogical purpose, we have selected the Moon and asteroid Ceres to study the CGA-effects on the orbital motion of these two important celestial bodies.

CGA-effects on orbital motion of Moon: In the system {Earth, Moon}, the Earth playing the role of principal gravitational source A and the Moon has the role of test-body *B*. We have, according to [3], the following orbital and physical parameters of Moon:
$e = 0.0549$, $a = 3.844 \times 10^8$ m, $P =$ 27.32 d $= 2.360580 \times 10^6$ s, $m_B = 7.3477 \times 10^{22}$ kg. For the values of the Earth's mass and of the physical constants, we take $m_A = 5.9722 \times 10^{24}$ kg,
$G = 6.67384 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\text{s}^{-2}$, $c_0 = 299\,792\,458\ \text{ms}^{-1}$, the mass-ratio: $q = m_B / m_A = 0.0123$. As we can remark, q is not sufficiently compared to zero, thus according to (43), we must take the total mass $M = m_A + m_B$ for the average magnitude of dynamic gravitational field, we find, after substituting all the above parameters into (42):

$$\Lambda = 3.188927 \times 10^{-14}\,\text{ms}^{-2}. \tag{44}$$

Hence, in terms of field, the Earth –as the main gravitational source in the system {Earth, Moon}– exerts on the Moon as a test-body un extra-gravitational field $\mathbf{\Lambda}$ of average magnitude (44). Or according to (23), the Earth exerts on the Moon an extra-gravitational force $\mathbf{F}_{\text{D}}$ of average magnitude

$$F_{\text{D}} = 2.343127 \times 10^9\ \text{N}. \tag{45}$$

CGA-secular perigee precession of Moon: In paper [3], we have already investigated the CGA-secular perigee precession of Moon by using the formula (47) in [3] and we obtained a value in good agreement with 0.060 arcsec/cy found by De Sitter [9,10,11] who used GRT and some inaccurate data available at that time (1916). However, if we employ GTR-formula which is identical to (32) and the modern accurate data, we find $\Delta\varphi_{\mathrm{GRT}} = 0.060889$ arcsec/cy. The correct computation of the secular perigee precession of Moon should be seen as a fact of an extreme significance particularly for alternative gravity theories. In what follows we perform the calculation with the help of CGA-formula (40). Hence, by inserting the orbital and physical parameters into (40), we obtain

$$\begin{aligned}\Delta\varphi_{\mathrm{CGA}} &= 2.213616\times10^{-10}\ \text{rad/rev} \\ &= 2.213616\times10^{-10}\times\frac{180}{\pi}\times3600\times\left(\frac{36525}{27.32}\right) = 0.061043\ \text{arcsec/cy}\end{aligned} \quad . \tag{46}$$

This is in good agreement with the above GRT-prediction.

CGA-effects on orbital motion of asteroid Ceres: Ceres is the largest and first discovered asteroid, by Italian astronomer Piazzi on January 1, 1801. The following orbital and physical parameters are from NASA (nssde.gsfc.nasa.gov/planetary/factsheet/asteriodfact.html):
$e = 0.0549$, $a = 2.767\,\mathrm{AU}$, $1\mathrm{AU} = 149.597870\times10^{9}$ m, $P = 4.60\,\mathrm{yr}$, $1\mathrm{yr} = 365.25$ d, $m_{\mathrm{C}} = 8.7\times10^{17}$ kg. For the Sun's mass, we have $m_{\mathrm{S}} = 1.9891\times10^{30}$ kg and for the mass-ratio, we have $q = m_{\mathrm{C}}/m_{\mathrm{S}} = 4.373837\times10^{-13}$. Since $q \cong 0$, thus according to (43), we find, after inserting the orbital and physical parameters into (42):

$$\Lambda = 2.764520\times10^{-12}\,\mathrm{m\,s^{-2}}. \tag{47}$$

This means that the Sun as a principal gravitational source exerts on Ceres, during its orbital motion, an extra-gravitational field $\mathbf{\Lambda}$ of average magnitude (47). Or according to (23), the Sun exerts on Ceres an extra-gravitational force $\mathbf{F}_{\mathrm{D}}$ of average magnitude

$$F_{\mathrm{D}} = 2.405132\times10^{6}\,\mathrm{N}. \tag{48}$$

CGA-secular perihelion precession of Ceres: In addition to the importance of secular perigee precession of Moon, the investigation of Ceres' secular perihelion precession is by itself a significant test-case for alternative theories of gravity. For example, the GRT-secular perihelion precession is $\Delta\varphi_{\mathrm{GRT}} = 0.0303484$ arcsec/cy. This GRT-prediction should be, of course, compared with CGA-prediction in order to be sure of CGA-formalism. So, by substituting the orbital and physical parameters into (40), we obtain

$$\Delta\varphi_{\mathrm{CGA}} = 0.303200\ \text{arcsec/cy}\ . \tag{49}$$

This is in excellent accord with GRT-prediction.

5.3. CGA-effects on orbital motion of four major asteroids

In order to assess more rigorously the theoretical predictions of CGA-formalism, we have selected from JPL small body database (http://ssd.jpl.nasa.gov/sbdb.cgi) and from (nssdc.gsfc.nasa.gov/planetary/ factsheet/asteroidfact.html) four major asteroids. Their masses are extremely small with respect to that of the Sun. this characteristic allows us to follow exactly the same process of computation applied to Ceres.

The orbital and physical parameters are listed in Table 1 and the observed perihelion precession of each asteroid and CGA-effects are listed in Table 2.

ASTEROID	a (AU)	e	P (yr)	m (kg)
1862 Apollo	1.470110	0.55993	1.785	2.00×10^{12}
2101 Adonis	1.874470	0.76381	2.570	1.80×10^{12}
433 Eros	1.457970	0.22263	1.760	6.69×10^{15}
1566 Icarus	1.077903	0.82683	1.119	1.00×10^{12}

Table 1: The orbital and physical parameters of four major asteroids.

Using the data listed in Table 1 and the CGA-formalism, we get the CGA-predictions which are listed in Table 2 with the observed secular perihelion precession of each asteroid.

		Predicted CGA-effects		
Asteroid	$\Delta\varphi_{\text{obs}}$ (arcsec/cy)	Λ ($\mathrm{m\,s^{-2}}$)	F_{D} (N)	$\Delta\varphi_{\text{CGA}}$ (arcsec/cy)
1862 Apollo	2.1239	1.842055×10^{-11}	36.8411	2.144459
2101 Adonis	1.9079	8.892244×10^{-12}	16.0000	1.918676
433 Eros	1.6000	1.889735×10^{-11}	1.26423×10^{5}	1.573589
1566 Icarus	10.007	4.676354×10^{-11}	46.76354	10.05435

Table 2: The observed secular perihelion precession of each asteroid and the predicted CGA-effects.

Table 2 illustrates us more conclusively that in addition to the average magnitude of couple $\left(\mathbf{\Lambda},\mathbf{F}_{\text{D}}\right)$, the predicted CGA-secular perihelion precession $\Delta\varphi_{\text{CGA}}$ of each asteroid is in excellent agreement with the observed value $\Delta\varphi_{\text{obs}}$.

The reader, who is already familiarized with CGA-formalism, has the natural right to ask about the status of the CGA vis-à-vis the scientific community. Certainly, the CGA as an alternative gravity theory could be regarded as a generalization of the Newton's theory of gravitation as we have previously seen in [3]. The CGA is in fact a newborn gravitational model (formulated in 2009) compared to the old ones. However, if the CGA is capable of predicting and explaining some old and new gravitational effects [1,2,3,4] this is due to the originality, simplicity and coherence of its formalism, which has no adjustable parameters. The well-informed reader can judge CGA in its own context. Historically, GRT succeeded in providing a numerical value for perihelion precession of Mercury [12] and attributed this additional rotation of the line of apsides to the space-time that curves around the Sun. However, as it was demonstrated in [3], CGA has predicted and explained the same effect in the framework of Euclidean geometry and Galilean relativity principle. We have already shown in the present paper that the CGA-effects are mainly due to the couple $(\mathbf{\Lambda}, \mathbf{F}_D)$. Again, in order to convince more conclusively the reader of the excellent agreement between the CGA-predictions and observations let us investigate the perihelion precession of the inner planets by using the newly derived formula (40). The average magnitude of the couple $(\mathbf{\Lambda}, \mathbf{F}_D)$ has been previously calculated (see, *e.g.*, Table1in Ref.[3]).

5.4. CGA-secular perihelion precession of inner planets

In Table 3, are listed the orbital and physical parameters of each inner planet. Some data are adopted from [3].

PLANET	a (m)	e	P (yr)
Mercury	57.92×10^9	0.2056	0.241
Venus	108.25×10^9	0.0068	0.615
Earth	149.60×10^9	0.0167	1.000
Mars	227.95×10^9	0.0934	1.881

Table 3: The orbital and physical parameters of inner planets.

With the help of the data listed in Table 3 and the newly derived formula (40), we get the CGA-predictions $\Delta\varphi_{\text{CGA}}$ displayed in Table 4 with the observed secular perihelion precession $\Delta\varphi_{\text{obs}}$ of each inner planet.

PLANET	$\Delta\varphi_{\text{obs}}$ (arcsec/cy)	$\Delta\varphi_{\text{CGA}}$ (arcsec/cy)
Mercury	43.1000	43.042580
Venus	8.0000	8.622262
Earth	5.0000	3.844435
Mars	1.3624	1.352923

Table 4: The observed secular perihelion precession and CGA-prediction for each inner planet are listed for comparison.

Like before, from Table 4, the reader can easily observe the good agreement between CGA-theoretical predictions and observations. Since GRT has already arrived at the same results, thus it is necessary to say a few words about CGA and GRT. In the CGA-context, the secular perigee precession of Moon and secular perihelion precession of planets and asteroids as, extra-gravitational effects, are originally caused by the action of couple $(\mathbf{\Lambda}, \mathbf{F}_\mathrm{D})$, however, in the GRT-context, the above mentioned secular effects are causally attributed to the curvature of space-time around the Sun. Concerning CGA, the reader can return to Eqs.(31) and (40) to see the dependence of orbital eccentricity *e* on the average magnitude of $\mathbf{F}_\mathrm{D}$, and the dependence of perigee *and/or* perihelion precession $\Delta\varphi$ on the average magnitude of $\mathbf{\Lambda}$. Remembering the discussion in connection with certain similarity between CGA and GRT, particularly Eqs.(18) and (19) containing CGA-effects and GRT-effects, respectively.

5.5. CGA-effects on orbital motion of binary pulsars

Before the advent of the CGA, it was usually claimed that the study of compact stellar objects like, *e.g.*, neutron stars and pulsars is exclusively belonging to GRT-domain because their strong compactness is enough to bend the local space-time in such a way that some observable GRT-effects should occur. However, this claim has already been refuted in [3]. Once again, we consider the reader as an intellectual witness in order to focus his attention on the fact that the CGA is very capable of investigating its proper effects, even, in compact stellar objects. More precisely, we will see that in addition to the couple $(\mathbf{\Lambda}, \mathbf{F}_\mathrm{D})$, there is another important effect, namely, the CGA-apsidal motion in binary pulsar systems which, in reality, has been studied earlier in [3]. This effect is very similar to perihelion precession and consequently should be defined by the same newly derived formula (40). Indeed, if we take the usual notation for the apsidal motion rate $\dot{\omega}$ instead of $\Delta\varphi$, we obtain

$$\dot{\omega} = \left[\frac{3P^2}{2\pi a\left(1-e^2\right)}\right]\Lambda . \tag{50}$$

The high compactness of pulsars implies that the resulting gravitational fields near the pulsar' surface is large, thus enabling strong-field tests of alternative gravity theories. Further, pulsars and their orbiting companions are generally compact enough that their motion can be treated as that of two point-masses. Therefore, in the CGA-context, we can logically consider each pulsar as the main gravitational source and the companion as the test-body. Consequently, the causal source of CGA-effects in the binary pulsar systems is exactly of the same nature as for ordinary (noncompact) eclipsing binary star systems [2,3,4].

Hence, the combined gravitational field (22) becomes stronger as the pulsar and its companion are so close together that an ordinary star like the Sun could not fit in their orbits. As a direct result, the couple $(\mathbf{\Lambda}, \mathbf{F}_\mathrm{D})$ should have its intensity amplified drastically. That is why, *e.g.*, the value of CGA-apsidal motion rate of binary pulsar systems should more significantly than that of ordinary eclipsing binary star systems. Exactly like previous investigation [3], *i.e.*, the determination of GCA-effects in binary pulsars should show us, among other things, that the usual GRT-interpretation of gravity as a deformation of space-time is not a physical reality but a pure topological property of Riemann geometry which is conceptually non-Euclidian.

We have displayed in Table 5 the orbital and physical parameters of each binary pulsar. The Sun's mean radius $(R_S = 695508\ \text{km})$ and mass $(m_S = 1.9891 \times 10^{30}\ \text{kg})$ are used as units for each binary pulsar's semi-major axis and mass.

PULSAR	P (d)	e	a/R_S	m_A/m_S	m_B/m_S	REF.
PSR B 1913+16	0.322997	0.6171	2.803849	1.4414	1.3867	a
PRS B 1534+12	0.420	0.2740	3.280619	1.3400	1.3400	b
PSR J 0737-3039	0.102251	0.0877	1.265262	1.3380	1.2490	c

Table 5: The orbital and physical parameters of each pulsar and its companion
Ref.: a) Weisberg and Taylor [12]; b) Nice *et al.*, [13]; c) Kramer *et al.*,[14]

Since the mass-ratio, $q = m_B / m_A$, of each binary pulsar is of the order of unity, thus according to (43), we can take $M = m_A + m_B$ as a total mass of system for the average magnitude of $\boldsymbol{\Lambda}$ and also for CGA-apsidal motion rate $\dot{\omega}_{\text{CGA}}$. So, with the aid of Table 5, we have calculated and displayed in Table 6 the CGA-effects with the observed apsidal motion rate $\dot{\omega}_{\text{obs}}$ of each binary pulsar.

		Predicted CGA-effects		
PULSAR	$\dot{\omega}_{\text{obs}}$ (deg/yr)	$\dot{\omega}_{\text{CGA}}$ (deg/yr)	Λ (m s^{-2})	F_D (N)
PSR B 1913+16	4.226595	4.223614	2.114670×10^{-4}	5.832862×10^{26}
PRS B 1534+12	1.756	1.760	1.185548×10^{-4}	3.159952×10^{26}
PSR J 0737-3039	16.900	16.83559	1.925609×10^{-3}	4.783955×10^{27}

Table 6: Observed apsidal motion rate and predicted values of CGA-effects.

As the reader can remark it easily, in addition to the other CGA-effects, Table 6 reveals us the excellent agreement between CGA-apsidal motion rate and the observed one for each binary pulsar.

6. Conversion Rate of Orbital Energy into Gravitational Power Radiation

In the GRT-context, the orbital energy loss is supposed to be due to the gravitational radiation, *i.e.*, gravitational waves of bounded binary systems. Firstly, in the CGA-context, we prefer using the term *conversion* as a replacement for the word *loss* because, causally, the CGA attributing the mechanism of the conversion –of orbital energy into gravitational power radiation– to the variation of work done by the dynamic gravitational force $\mathbf{F}_D$ during the orbital motion of binary system $\{A, B\}$ for the reason that $\mathbf{F}_D$ should play a double role: an extra-gravitational force and a perturbation force.

Explanation: Since the (relative) orbit of the main gravitational source A of mass m_A and the test-body B of mass m_B is elliptical, more precisely, the binary system $\{A, B\}$ may contain either a single elliptical orbit or each body would travel in its own separate elliptical orbit as is illustrated in Figure 1 below.

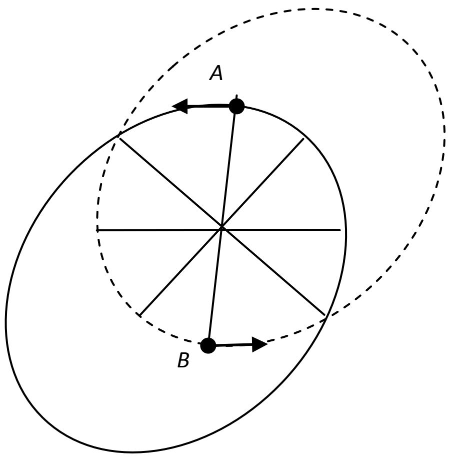

Figure 1: The orbit of the hypothetical binary star system $\{A, B\}$.

Thus the two stars are closer together at some times than at others, so that the couple $(\mathbf{\Lambda}, \mathbf{F}_{\mathrm{D}})$ alternately strengthens at periastron and weakens at apastron. Hence, dynamically, there is a certain amount of work δW performed by $\mathbf{F}_{\mathrm{D}}$ and is defined like this

$$\delta W = \mathbf{F}_{\mathrm{D}} \cdot \delta \mathbf{s}, \tag{51}$$

where $\delta \mathbf{s}$ is a displacement vector.

Or equivalently, we can rewrite (51) as follows

$$\delta W = F_{\mathrm{D}} \cdot \delta s \, \cos\theta, \tag{52}$$

where θ is the angle between the force vector and displacement vector and

$$F_{\mathrm{D}} = \frac{\mu}{a}\left[\frac{GM}{c_0 a}\right]^2, \ \mu = m_A m_B M^{-1}, \ M = m_A + m_B. \tag{53}$$

The presence of the reduced mass μ is quite natural since we are dealing with binary system consists of two bodies rotating about their common center of mass (Fig.1). Now, supposing that the Kepler's third law $\left(P^2 / r^3 = 4\pi^2 / GM\right)$ is enough accurate to be applied here. Thus, according to the above considerations, δW should have its maximum value at periastron $r \to r_{\min}$ and its minimum value at apastron $r \to r_{\max}$. Inversely, at periastron the orbital period should be $P \to P_{\min}$ and at apastron $P \to P_{\max}$. By taking into account the fact that $\mathbf{F}_{\mathrm{D}}$ as a perturbation force will cause a change in the work δW during the time interval $\delta P = P_{\max} - P_{\min}$. Therefore, the rate of change in δW is called, in the CGA-context, *average gravitational power* and is given by

$$\dot{\mathscr{E}} = \frac{\delta W}{\delta P}. \tag{54}$$

Because δs should be infinitesimal compared to the relative separation between A and B, hence for our purpose it is best to take $\delta s = \frac{3}{4} r_{\mathrm{G}}$ where $r_{\mathrm{G}} = GM / c_0^2$ is the gravitational radius of system $\{A, B\}$. Thus for the case $\theta \cong 0°$, Eq.(52) becomes $\delta W = F_{\mathrm{D}} \cdot \delta s$ and consequently Eq.(54) takes the form

$$\dot{\mathscr{E}} = F_{\mathrm{D}} \frac{\delta s}{\delta P} = \frac{3\mu}{4a} \left[\frac{GM}{c_0 a} \right]^2 \frac{r_{\mathrm{G}}}{\delta P}. \tag{55}$$

Let us focus our attention on the orbital period. we have from Kepler's third law:

$$P_{\min}(r \to r_{\min}) = \frac{2\pi r_{\min}^{3/2}}{\sqrt{GM}} \quad \text{and} \quad P_{\max}(r \to r_{\max}) = \frac{2\pi r_{\max}^{3/2}}{\sqrt{GM}}.$$

Since we are dealing with elliptical orbit, hence, we have $r_{\max} = a(1+e)$ and $r_{\min} = a(1-e)$, and δP takes the form

$$\delta P = P_{\max} - P_{\min} = \frac{2\pi a^{3/2}}{\sqrt{GM}} \left[(1+e)^3 - (1-e)^3 \right]. \tag{56}$$

By substituting (56) into (55), and performing some algebraic calculation, we get

$$\dot{\mathscr{E}} = \frac{3 f(e)}{8\pi} \frac{\mu}{\sqrt{GMa}} \left[\frac{GM}{c_0 a} \right]^4, \tag{57}$$

with

$$f(e) = \left[(1+e)^3 - (1-e)^3 \right]^{-1}. \tag{58}$$

Because of the principle of conservation of energy, the average gravitational power $\dot{\mathscr{E}}$ should be equal to the change in the total orbital energy $\dot{E}$ of system under consideration, *i.e.*, we should have

$$\dot{\mathscr{E}} = \dot{E}. \tag{59}$$

In view of the fact that the average orbital velocity is $v_{\mathrm{orb}} = \sqrt{GM/a}$,hence, according to (57) the change in orbital energy is of the order $(v_{\mathrm{orb}} / c_0)^4$. As we can remark it, Eq.(57) defines us the average rate of gravitational emission energy from system $\{A, B\}$. This average rate $\dot{\mathscr{E}}$ has the physical dimensions of power thus its units are $\mathrm{J\,s^{-1}}$ or Watt. In the CGA-context, we call $\dot{\mathscr{E}}$ gravitational power. Also, $\dot{\mathscr{E}}$ is strictly positive, this fact is not new because the natural *sign* of power –in electricity and in mechanics– is always positive . Moreover, Peters and Mathews have, in their very interesting work [15], derived a formula for average rate at which the system radiate energy. This rate is also strictly positive. In [15], their formula is numbered (16) and located on page 437, first column. Here, we rewrite it exactly in its original form as follows:

$$\langle p \rangle = \frac{32}{5}\frac{G^4}{c^5}\frac{m_1^2 m_2^2 (m_1 + m_2)}{a^5 (1-e^2)^{7/2}}\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right).$$

Thus, originally, Peters and Mathews derived a formula with strictly positive sign and used $\langle p \rangle$ as an average power. Also, many authors confounded the Peters-Mathews' notation $\langle p \rangle$ with the orbital period of system and that's why they took the first time derivative of period equals to the expression on the right-hand side of the above formula.

6.1. Consequences of Gravitational Power Radiation

Five consequences may be occurred from the gravitational power radiation, namely, (1) a decrease in the orbital period: The progressive conversion of orbital energy of the binary system {*A*, *B*} into gravitational power radiation *causes* a gradual decay of orbital period; (2) a decrease in the orbital separation: Exactly like the case (1), *i.e.*, the conversion of orbital energy leads to a decrease in the orbital separation between *A* and *B*; (3) change in the orbital eccentricity; (4) change in the orbital angular momentum; (5) gravitational coalescence: Because of the conversion of orbital energy, the orbits are gradually shrinking and the coalescence should be imminent at least in the long term.

It is worthwhile to note that in the framework of Newton's gravity theory these orbital parameters are constant of motion. However, in the CGA-context they will be functions of time which will be gradually varying.

6.1.2. Orbital period decay rate

The parameters a, P and e are related, respectively, to the total orbital energy E, Kepler's third law and orbital angular momentum through the following equations:

$$E = -\frac{G m_A m_B}{2a}, \tag{60}$$

$$\frac{P^2}{a^3} = \frac{4\pi^2}{GM}, \tag{61}$$

$$\ell^2 = G m_A m_B \mu a (1 - e^2). \tag{62}$$

By combining (60) and (61) *via* differentiation with respect to time, and by taking into account (59), we get –after substitution and some algebraic calculation– the following expression for the orbital period decay:

$$\dot{P} = \frac{9}{4}\left[\frac{GM}{c_0^2 a}\right]^2 f(e). \tag{63}$$

6.1.3. Semi-major axis decay rate

By following, exactly, the above process, we obtain an expression for the semi-major axis decay rate:

$$\dot{a} = \frac{3}{4\pi c_0^4}\left[\frac{GM}{a}\right]^{5/2} f(e), \tag{64}$$

or equivalently

$$\dot{a} = \frac{3}{2}\left[\frac{GM}{c_0^2 a}\right]^2 \frac{a}{P} f(e). \tag{65}$$

6.1.4. Secular change in the orbital eccentricity

To calculate the rate of secular change in the orbital eccentricity, we must differentiate Eq.(56) with respect to time and with the help of Eq.(64), we find

$$\dot{e} = -\frac{3(GM)^{5/2}}{16\pi c_0^4 a^{7/2}\left(1+e^2\right)}. \tag{66}$$

6.1.5. Secular change in the orbital angular momentum

By differentiating Eq.(62) with respect to time, we get

$$\dot{\ell} = \ell\left[\frac{\dot{a}}{2a} - \frac{e}{\left(1-e^4\right)}\dot{e}\right]. \tag{67}$$

Substituting (64) and (66) into (67), yields the following expected expression

$$\dot{\ell} = \frac{3(GM)^{5/2}\ell}{16\pi c_0^4 a^{7/2}}\left[2f(e) + g(e)\right], \tag{68}$$

where

$$f(e) = \left[(1+e)^3 - (1-e)^3\right]^{-1} \quad \text{and} \quad g(e) = e\left(1-e^4\right)^{-1}$$

6.1.6. Gravitational coalescence mean-time

In the CGA-context, the gravitational coalescence mean-time is the mean-lifetime of binary system's orbit. We can derive an expression for the gravitational coalescence mean-time by integrating Eq.(64), and we obtain –after omitting the integration constant– the following equation:

$$t_{\mathrm{GC}} = \frac{8\pi}{21}\frac{c_0^4 a^{7/2}}{(GM)^{5/2}} f^{-1}(e). \tag{69}$$

Eq.(69) is called "gravitational coalescence mean-time" because it only predicts the mean-time for the radius of the orbit to shrink considerably before reaching zero, *i.e.*, total coalescence. This consideration is based on the fact that during the process of integration of Eq.(69), we have intentionally omitted the constant of integration. Moreover, if we take into account the fact that, in the long term, the orbital velocity will become a significant fraction of the light speed and as a direct result, Eq.(69) may be interpreted as an estimation of gravitational coalescence mean-time .

7. Conclusion

Based solely on the CGA-formalism, we have demonstrated the causal dependence of orbital eccentricity on the average magnitude of dynamic gravitational force, also a new formula for the secular perigee and perihelion precession had been derived. This formula is proved to be exclusively depended on the average magnitude of dynamic gravitational field and is successfully applied inside and outside the Solar System. Furthermore, it is found that the CGA-predictions are, at the same time, in excellent agreement with observations and GRT-predictions. The last fact is mainly due to the existence of certain similarity between the approximate GRT-equations of motion and CGA-ones. Finally, the conversion rate of orbital energy into gravitational power radiation is calculated and its effects are studied.

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