Photon's mass

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Abstract
The double deflection of a light ray near an attracting center, which was predicted by Einstein, is explained by the space curvature.

1. Introduction

Einstein confirmed that radiation conveys *inertia* between the emitting and absorbing bodies [1]. But previously, it had been understood that light radiation was attracted to Earth and had *gravitational mass.* Being based on this understanding, in 1801, Soldner [2] used Newton laws with inertial and gravitational masses

$$F = ma, \quad F = kmM / r^2 . \tag{1}$$

He reasoned in this way as Rutherford reasoned after 110 years about the deflection of alpha particles. Soldner found that a mass M deflects a light ray on the angle α:

$$tg\frac{\alpha}{2} = \frac{kM}{c^2 R}, \quad \alpha \approx \frac{2kM}{c^2 R} \tag{2}$$

(here R is the impact parameter). However, in 1909, Eddington observed the double deflection in accordance with Einstein's General Relativity.

Ginzburg [3] noted that this fact is explained by the space curvature, which Soldner could not foreknow. But Okun' [4,5] concluded that the gravitational mass of a relativistic particle depends on the direction of its velocity. In particular, for a horisontally moving photon above the Earth or Sun, its grravitational mass is twice as large as that for a vertically moving photon.

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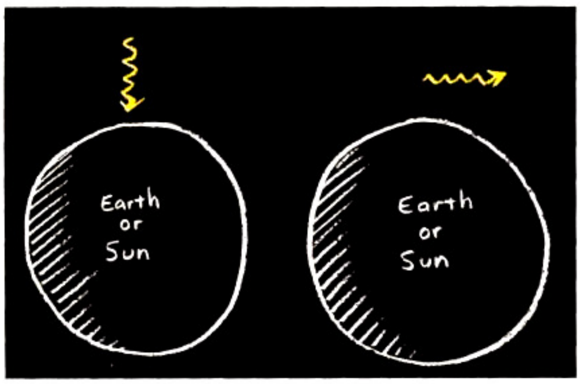

Gravitational force attracting a horizontally moving photon to the Earth or Sun is twice as large as that attracting a vertically moving photon.

Because of this controversy, it is appropriate to demonstrate that the inertial mass determines the gravitational force in any situation. I.e. there is no difference between inertial and gravitation masses. Gravitational force parallel to velocity was considered in [6]; here we consider gravitational force, which is perpendicular to velocity, i.e. the case of a light ray, which is near an attracting mass.

2. Calculation

Consider round orbits in the Schwarzschild space-time with coordinates t, r, θ, φ [7, (100,14)], metric

$$ds^2 = \frac{r-1}{r}dt^2 - \frac{r}{r-1}dr^2 - r^2 d\theta^2 - r^2\sin^2\theta d\varphi^2 \qquad (3)$$

(we put $c = r_g = 2M = 1$, $\sin^2\theta = 1$), and equations for geodesic lines using t as a parameter, $\{t, r(t)\}$:

$$\frac{D}{dt}\frac{dx^i}{dt} \equiv \frac{d^2x^i}{dt^2} + \Gamma^i_{jk}\frac{dx^j}{dt}\frac{dx^k}{dt} = \alpha\frac{dx^i}{dt}, \quad \Gamma^r_{tt} = \frac{r-1}{2r^3}, \ \Gamma^r_{rr} = \frac{-1}{2r(r-1)}, \ \Gamma^r_{\varphi\varphi} = -(r-1). \qquad (4)$$

As opposed to [1, (87, 3)], there is no zero on the right-hand side but there is a quantity, which is proportional to the tangent vector, because t is not a canonical parameter. In this case a geodesicness of the line is provided by the curvature vector, which is directed along the line.

For round orbits, $i = r$, $dr/dt \equiv 0$, and equation (4) gives

$$\Gamma^r_{tt} + \Gamma^r_{\varphi\varphi}\left(\frac{d\varphi}{dt}\right)^2 = 0, \quad \frac{r-1}{2r^3} = (r-1)\left(\frac{d\varphi}{dt}\right)^2, \quad \left(\frac{d\varphi}{dt}\right)^2 = \frac{1}{2r^3}. \qquad (5)$$

It is easy to calculate that if $r = 3/2$, world line (5) is an isotropic world line, i.e. it represents an photon's orbit. Really, taking into account (5) yields

$$ds^2 \equiv \frac{r-1}{r}dt^2 - r^2 d\varphi^2 = \left(\frac{r-1}{r} - \frac{r^2}{2r^3}\right)dt^2 = 0, \quad r = \frac{3}{2}. \qquad (6)$$

World line (5) is a geodesic line of space-time, which represents a moving on a circle in the space. The centripetal acceleration of such a moving is determined by the second derivative with respect to time of the *deviation* of this circle, $r = \text{Const}$, from the *tangential geodesic* line in the space with the metric

$$dl^2 = \frac{r}{r-1}dr^2 + r^2 d\varphi^2 \qquad (7)$$

We write the equation of such geodesic line, $r(\varphi)$, using the general equation (4) with φ as the parameter

$$\frac{d^2r}{d\varphi^2} + \Gamma^r_{rr}\left(\frac{dr}{d\varphi}\right)^2 + \Gamma^r_{\varphi\varphi} = \alpha\frac{dr}{d\varphi}. \qquad (8)$$

Now, taking into account that $dr/d\varphi = 0$ for the tangential line at the point $\varphi = 0$ yields for $\varphi = 0$:

$$\frac{d^2r}{d\varphi^2} = (r-1). \qquad (9)$$

This equation gives the second derivative of the deviation of our circle from the tangential geodesic line, but with respect to φ. The required second derivative with respect to t is obtained with (5):

$$\frac{d^2r}{dt^2} = \frac{(r-1)}{2r^3}. \qquad (7)$$

Now we can find the centripetal acceleration on any round orbit. It is g:

$$a = \frac{\sqrt{g_{rr}}}{g_{tt}}\frac{d^2r}{dt^2} = \frac{1}{2r^2}\sqrt{\frac{r}{r-1}} = g. \qquad (8)$$

The point is the curvature of a motionless body world-line, $r = \text{Const}, \varphi = 0$, equals

$$\frac{D^2r}{dt^2} = \Gamma^r_{tt} = \frac{r-1}{2r^3}. \qquad (9)$$

So, from viewpoint of GR, the motionless body has acceleration

$$g = \frac{\sqrt{g_{rr}}}{g_{tt}}\frac{D^2r}{dt^2} = \frac{1}{2r^2}\sqrt{\frac{r}{r-1}}. \qquad (12)$$

This is the free fall acceleration from our viewpoint, g.

Conclusion

The double deflection of a light ray near an attracting center, which was predicted by Einstein, is explained not by double gravitational force attracting a horizontally moving photon. The gravitational mass of a photon always equals the inertial mass (which is named by E/c^2). The second half of the deflection is from space curvature.

Reference

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