A Proof of the ABC Conjecture

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Introduction: The ABC conjecture was proposed by Joseph Oesterle in 1988 and David Masser in 1985. The conjecture states usually that, for any infinitesimal $\varepsilon > 0$, there exists a constant $C_\varepsilon > 0$ such that for any three relatively prime integers a, b and c satisfying

$$a + b = c,$$

the inequality

$$\max(|a|, |b|, |c|) \le C_\varepsilon \prod_{p|abc} p^{1+\epsilon}$$

holds, where p/abc indicates that the product is over primes p which divide the product abc.

This is an unsolved problem hitherto although somebody published papers on the internet claiming proved it.

Abstract

First, we get rid of three kinds from A+B=C according to their respective odevity and gcf (A, B, C) =1. After that, expound relations between C and raf (A, B, C) by the symmetric law of odd numbers. Finally, we have proven $C \le C_\varepsilon$ [raf (A, B, C)] $^{1+\varepsilon}$ in which case A+B=C and gcf (A, B, C) =1.

AMS subject classification: 11A99, 11D99, 00A05.

Keywords: ABC conjecture, A+B=C, gcf (A, B, C) =1, Symmetric law of odd numbers, Sequence of natural numbers, $C \leq C_{\varepsilon}$ [raf (A, B, C)] $^{1+\varepsilon}$.

Values of A, B and C in set A+B=C

For positive integers A, B and C, let raf (A, B, C) denotes the product of all distinct prime factors of A, B and C, e.g. if $A=11^2\times13$, B=13 and C=2×13×61, then raf (A, B, C) =2×11×13×61 =17446. In addition, let gcf (A, B, C) denotes greatest common factor of A, B and C.

Therefrom the ABC conjecture is also able to state that given any infinitesimal non-negative real number $\varepsilon \geq 0$, there exists a constant $C_{\varepsilon} > 0$ such that for every triple of positive integers A, B and C satisfying A+B=C, and gcf (A, B, C) =1, then we have $C \leq C_{\varepsilon}$ [raf (A, B, C)] $^{1+\varepsilon}$.

First let us get rid of three kinds from A+B=C according to their respective odevity and gcf (A, B, C) =1, as listed below.

1. If A, B and C all are positive odd numbers, then A+B is an even number, yet C is an odd number, evidently there is only A+B≠C according to an odd number ≠ an even number.

2. If any two in A, B and C are positive even numbers, and another is a positive odd number, then when A+B is an even number, C is an odd number, yet when A+B is an odd number, C is an even number, so there is only A+B≠C according to an odd number ≠ an even number.

3. If A, B and C, all of them are positive even numbers, then they have at least a common prime factor 2, manifestly this and the given prerequisite of gcf (A, B, C) = 1 are inconsistent, so A, B and C can not be three positive even numbers together.

Therefore we can only continue to have a kind of A+B=C, namely A, B and C are two positive odd numbers and one positive even number. So let following two equalities add together to replace A+B=C in which case A, B and C are two positive odd numbers and one positive even number.

1. $A+B=2^X S$, where A, B and S are three relatively prime positive odd numbers, and X is a positive integer.

2. $A+2^Y V=C$, where A, V and C are three relatively prime positive odd numbers, and Y is a positive integer.

Consequently the proof for ABC conjecture, by now, it is exactly to prove the existence of following two inequalities.

(1). $2^X S \le C_\varepsilon\,[\mathrm{raf}\,(A, B, 2^X S)]^{1+\varepsilon}$ in which case $A+B=2^X S$, where A, B and S are three relatively prime positive odd numbers, and X is a positive integer.

(2). $C \le C_\varepsilon\,[\mathrm{raf}\,(A, 2^Y V, C)]^{1+\varepsilon}$ in which case $A+2^Y V=C$, where A, V and C are three relatively prime positive odd numbers, and Y is a positive integer.

Circumstances Relating to the Proof

Let us divide all positive odd numbers into two kinds of A and B, namely the form of A is 1+4n, and the form of B is 3+4n, where n≥0. From small to large odd numbers of A and of B are arranged as follows respectively.

A: 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69…1+4n …

B: 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63, 67…3+4n …

We list also from small to great natural numbers, well then you would discover that Permutations of seriate natural numbers show up a certain law.

1, 2^1, 3, 2^2, 5, 2^1×3, 7, 2^3, 9, 2^1×5, 11, 2^2×3, 13, 2^1×7, 15, 2^4, 17, 2^1×9, 19, 2^2×5, 21, 2^1×11, 23, 2^3×3, 25, 2^1×13, 27, 2^2×7, 29, 2^1×15, 31, 2^5, 33, 2^1×17, 35, 2^2×9, 37, 2^1×19, 39, 2^3×5, 41, 2^1×21, 43, 2^2×11, 45, 2^1×23, 47, 2^4×3, 49, 2^1×25, 51, 2^2×13, 53, 2^1×27, 55, 2^3×7, 57, 2^1×29, 59, 2^2×15, 61, 2^1×31, 63, 2^6, 65, 2^1×33, 67, 2^2×17, 69, 2^1×35, 71, 2^3×9, 73, 2^1×37, 75, 2^2×19, 77, 2^1×39, 79, 2^4×5, 81, 2^1×41, 83, 2^2×21, 85, 2^1×43, 87, 2^3×11, 89, 2^1×45, 91, 2^2×23, 93, 2^1×47, 95, 2^5×3, 97, 2^1×49, 99, 2^2×25, 101, 2^1×51, 103 …→

Of course, even numbers contain prime factor 2, yet others are odd numbers in the sequence of natural numbers above-listed.

After each of odd numbers in the sequence of natural numbers is replaced by the belongingness of itself, the sequence of natural numbers is changed into the following form.

A, 2^1, B, 2^2, A, 2^1×3, B, 2^3, A, 2^1×5, B, 2^2×3, A, 2^1×7, B, 2^4, A, 2^1×9, B, 2^2×5 A, 2^1×11, B, 2^3×3, A, 2^1×13, B, 2^2×7, A, 2^1×15, B, 2^5, A, 2^1×17, B, 2^2×9, A 2^1×19, B, 2^3×5, A, 2^1×21, B, 2^2×11, A, 2^1×23, B, 2^4×3, A, 2^1×25, B, 2^2×13, A 2^1×27, B, 2^3×7, A, 2^1×29, B, 2^2×15, A, 2^1×31, B, 2^6, A, 2^1×33, B, 2^2×17, A 2^1×35, B, 2^3×9, A, 2^1×37, B, 2^2×19, A, 2^1×39, B, 2^4×5, A, 2^1×41, B, 2^2×21, A

$2^1\times43$, B, $2^3\times11$, A, $2^1\times45$, B, $2^2\times23$, A, $2^1\times47$, B, $2^5\times3$, A, $2^1\times49$, B, $2^2\times25$, A, $2^1\times51$, B …→

Thus it can be seen, leave from any given even number >2, there are finitely many cycles of B with A leftwards until B=3 with A=1, and there are infinitely many cycles of A with B rightwards.

If we regard an even number on the sequence of natural numbers as a symmetric center of odd numbers, then two odd numbers of every bilateral symmetry are A and B always, and a sum of bilateral symmetric A and B is surely the double of the even number. For example, odd numbers 23(B) and 25(A), 21(A) and 27(B), 19(B) and 29(A) etc are bilateral symmetries respectively whereby even number $2^3\times3$ to act as the center of the symmetry, and there are $23+25=2^4\times3$, $21+27=2^4\times3$, $19+29=2^4\times3$ etc.

For another example, odd numbers 49(A) and 51(B), 47(B) and 53(A), 45(A) and 55(B) etc are bilateral symmetries respectively whereby even number 2×25 to act as the center of the symmetry, and there are $49+51=2^2\times25$, $47+53=2^2\times25$, $45+55=2^2\times25$ etc.

Again give an example, 63(B) and 65(A), 61(A) and 67(B), 59(B) and 69(A) etc are bilateral symmetries respectively whereby even number 2^6 to act as the center of the symmetry, and there are $63+65=2^7$, $61+67=2^7$, $59+69=2^7$ etc.

Overall, if A and B are two bilateral symmetric odd numbers whereby $2^X S$ to act as the center of the symmetry, then there is $A+B=2^{X+1}S$, where $X\geq1$, and

S is an odd number ≥ 1.

The total number of A plus B on the left of $2^X S$ is exactly the total number of pairs of bilateral symmetric A and B for symmetric center $2^X S$. If we regard any finite-great even number $2^X S$ as a symmetric center, then there are merely finitely more pairs of bilateral symmetric A and B.

Namely the total number of pairs of A and B wherewith express $2^{X+1}S$ as the both sum is finite, where $2^{X+1}S$ is a finite-great even number.

Or rather, the total number of pairs of bilateral symmetric A and B for symmetric center $2^X S$ is exactly $2^{X-1}S$.

On the supposition that A and B are a pair of bilateral symmetric odd numbers whereby $2^X S$ to act as the center of the symmetry, then $A+B=2^{X+1}S$.

By now, let A plus $2^{X+1}S$ makes $A+2^{X+1}S$, then B and $A+2^{X+1}S$ are still bilateral symmetry whereby $2^{X+1}S$ to act as the center of the symmetry, and $B+(A+2^{X+1}S) =(A+B)+2^{X+1}S =2^{X+1}S+2^{X+1}S =2^{X+2}S$.

If substitute B for A, let B plus $2^{X+1}S$ makes $B+2^{X+1}S$, then A and $B+2^{X+1}S$ are too bilateral symmetry whereby $2^{X+1}S$ to act as the center of the symmetry, and $A+ (B+2^{X+1}S) =2^{X+2}S$.

Provided both let A plus $2^{X+1}S$ makes $A+2^{X+1}S$, and let B plus $2^{X+1}S$ makes $B+2^{X+1}S$, then $A+2^{X+1}S$ and $B+2^{X+1}S$ are likewise bilateral symmetry whereby $3\times 2^X S$ to act as the center of the symmetry, and $(A+2^{X+1}S)+ (B+2^{X+1}S) =3\times 2^{X+1}S$.

Since there are merely A and B at two odd places of each and every bilateral symmetry on two sides of an even number as the center of the symmetry, then whether $B+(A+2^{X+1}S)=2^{X+2}S$ or $A+(B+2^{X+1}S)=2^{X+2}S$ is $A+B=2^{X+2}S$ entirely. Like that, write $(A+2^{X+1}S) + (B+2^{X+1}S) =3\times2^{X+1}S$ down $A+B= 3\times2^{X+1}S= 2^{X+1}S_t$, where S_t is an odd number ≥3.

Do it like this, not only equalities like as $A+B=2^{X+1}S$ are proven to continue the existence, one by one, but also they are getting more and more along with which X and/or S are getting greater and greater, up to exist infinitely more equalities like as $A+B=2^{X+1}S$ where X≥0, and S is an odd number X≥1.

In other words, added to a positive even number on two sides of $A+B=2^XS$, then we get still such an equality like as $A+B=2^XS$.

Whereas no matter how great a concrete even number 2^XS, there are merely finitely more pairs of bilateral symmetric A and B for symmetric center 2^XS.

If 2^XS is defined as a concrete positive even number, then there is only a part of $A+B=2^XS$ to satisfy gcf (A, B, 2^XS) =1. For example, if $2^XS=18$, then there are 1+17=18, 5+13=18 and 7+11=18 to satisfy gcf (A, B, 2^XS) =1, yet 3+15=18 and 9+9=18 suit not because each has common factor 3.

If add or subtract a positive odd number on two sides of $A+B=2^XS$, then we get another equality like as $A+2^YV=C$, where Y≥1, and V is an odd number ≥1. That is to say, equalities like as $A+2^YV=C$ can come from $A+B=2^{X+1}S$ so as add or subtract a positive odd number on two sides of $A+B=2^{X+1}S$.

Therefore, on the one hand, equalities like as $A+2^YV=C$ are getting more and more along with which equalities like as $A+B=2^{X+1}S$ are getting more and more, up to exist infinite more equalities like as $A+2^YV=C$ along with which infinite more equalities like as $A+B=2^{X+1}S$ appear.

Certainly, we can likewise transform $A+2^YV=C$ into $A+B=2^XS$ so as add or subtract a positive odd number on the two sides of $A+2^YV=C$.

On the other hand, if C is only defined as a concrete positive odd number, then there is merely finitely more pairs of A and 2^YV wherewith express C as the both sum. But also, there is probably a part of $A+2^YV=C$ to satisfy gcf $(A, 2^YV, C) =1$. For example, when C=25, there are merely 1+24=25, 3+22=25, 7+18=25, 9+16=25, 11+14=25and 13+12=25 to satisfy gcf $(A, 2^YV, C) =1$, yet 5+20=25 and 15+10=25 suit not because each has common factor 5.

After factorizations of A, B, S, V and C in $A+B=2^{X+1}S$ plus $A+2^YV=C$, if part prime factors have greater exponents, then there are both $2^{X+1}S \geq$ raf $(A, B, 2^{X+1}S)$ in which case $A+B=2^{X+1}S$ satisfying gcf $(A, B, 2^{X+1}S) =1$, and $C \geq$ raf $(A, 2^YV, C)$ in which case $A+2^YV=C$ satisfying gcf $(A, 2^YV, C) =1$. For example, $2^7 >$ raf $(3, 5^3, 2^7)$ for $3+5^3=2^7$; and for another example, $3^{10} >$ raf $(5^6, 2^5\times23\times59, 3^{10})$ for $5^6+2^5\times23\times59=3^{10}$.

On the contrary, there are both $2^{X+1}S \leq$ raf $(A, B, 2^{X+1}S)$ in which case $A+B=2^{X+1}S$ satisfying gcf $(A, B, 2^{X+1}S) =1$, and $C \leq$ raf $(A, 2^YV, C)$ in which case $A+2^YV=C$ satisfying gcf $(A, 2^YV, C) =1$. For example, $2^2\times7 <$ raf $(13, 3\times5, 2^2\times7)$ for $13+3\times5=2^2\times7$; and for another example, $3^4 <$ raf $(11\times7, 2^2, 3^4)$ for $11\times7+2^2 =3^4$.

Since either A or B in $A+B=2^{X+1}S$ plus an even number is still an odd number, and $2^{X+1}S$ plus the even number is still an even number, thereby we can use $A+B=2^{X+1}S$ to express every equality which on two sides of $A+B=2^{X+1}S$ plus an even number makes.

Consequently, there are infinitely more $2^{X+1}S \geq raf (A, B, 2^{X+1}S)$ plus $2^{X+1}S \leq raf (A, B, 2^{X+1}S)$ in which case $A+B=2^{X+1}S$.

Likewise, either 2^YV plus an even number is still an even number, or A plus an even number is still an odd number, and C plus the even number is still an odd number, so we can use equality $A+2^YV=C$ to express every equality which on two sides of $A+2^YV=C$ plus an even number makes.

Consequently, there are infinitely more $C \geq raf (A, 2^YV, C)$ plus $C \leq raf (A, 2^YV, C)$ in which case $A+2^YV = C$.

But, if let $2^{X+1}S \geq raf (A, B, 2^{X+1}S)$ and $2^{X+1}S \leq raf (A, B, 2^{X+1}S)$ in which case $A+B=2^{X+1}S$ separate, and let $C \geq raf (A, 2^YV, C)$ and $C \leq raf (A, 2^YV, C)$ in which case $A+2^YV =C$ separate, then for inequalities like as each of the four kinds, we are unable to deduce their total number whether be actually infinitely more or finitely more.

However, what deserve to be affirmed is that there are $2^{X+1}S \geq raf (A, B, 2^{X+1}S)$ and $2^{X+1}S \leq raf (A, B, 2^{X+1}S)$ in which case $A+B=2^{X+1}S$ satisfying gcf $(A, B, 2^{X+1}S) =1$, and there are $C \geq raf (A, 2^YV, C)$ and $C \leq raf (A, 2^YV, C)$ in which case $A+2^YV = C$ satisfying gcf $(A, 2^YV, C) =1$, according to the preceding illustration with examples.

Proving $C \leq C_{\varepsilon} [raf (A, B, C)]^{1+\varepsilon}$

Hereinbefore, we have known that both there are $2^{X+1}S \leq raf (A, B, 2^{X+1}S)$ and $2^{X+1}S \geq raf (A, B, 2^{X+1}S)$ in which case $A+B=2^{X}S$ satisfying gcf $(A, B, 2^{X+1}S) =1$, and there are $C \leq raf (A, 2^{Y}V, C)$ and $C \geq raf (A, 2^{Y}V, C)$ in which case $A+2^{Y}V=C$ satisfying gcf $(A, 2^{Y}V, C) =1$, whether inequalities of each kind are infinitely more, or are finitely more.

First let us expound a set of identical substitution as the follows. If an even number on the right side of each of above-mentioned four inequalities added to an infinitesimal non-negative real number such as $R \geq 0$, then the result is both equivalent to multiply the even number by another infinitesimal real number, and equivalent to increase an even more infinitesimal real number such as $\varepsilon \geq 0$ to the exponent of the even number, i.e. from this form a new exponent $1+\varepsilon$, but when $R= 0$, the multiplied real number is 1, yet $\varepsilon = 0$.

Actually, aforementioned three ways of doing, all are in order to increase an identical even number into a value and the same, but also we consider such an identical substitution as a rule.

Now that exists the rule of the identical substitution between each other, then we set about proving aforesaid four inequalities, one by one, thereinafter.

(1). For inequality $2^{X+1}S \leq raf (A, B, 2^{X+1}S)$ in which case $A+B=2^{X}S$ satisfying gcf $(A, B, 2^{X+1}S) =1$, $2^{X+1}S$ divided by $raf (A, B, 2^{X+1}S)$ is equal to $2^{X}S_1^{t-1}\sim S_n^{m-1}/A_{raf}B_{raf}$ as a true fraction, where $S_1\sim S_n$ express all distinct prime factors of S; t-1~m-1 are respectively exponents of prime factors $S_1 \sim S_n$ orderly and t-1≥0, … m-1≥0; A_{raf} expresses the product of all distinct prime factors of A; and B_{raf} expresses the product of all distinct prime factors of B.

Undoubtedly $2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}$ as the true fraction is smaller than 1.

After that, even number raf (A, B, $2^{X+1}S$) added to an infinitesimal non-negative real number such as $R \geq 0$ to turn the even number itself into $[\text{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$ according to the above mentioned the rule of the identical substitution. Evidently there is $2^{X+1}S \leq [\text{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$ successively.

By now, multiply $[\text{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$ by $2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}$, then there is still $2^{X+1}S \leq 2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}[\text{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$.

Also let $C_\varepsilon = 2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}$, we get $2^{X+1}S \leq C_\varepsilon[\text{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$.

Manifestly when R=0, it has ε =0, and there is $2^{X+1}S = C_\varepsilon[\text{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$, yet when R>0, it has ε >0, and there is $2^{X+1}S < C_\varepsilon[\text{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$.

(2). For inequality $C \leq \text{raf}(A, 2^Y V, C)$ in which case $A+2^Y V = C$ satisfying gcf (A, $2^Y V$, C) =1, C divided by raf (A, $2^Y V$, C) is equal to $C_1^{j-1} \sim C_e^{f-1}/2A_{raf}V_{raf}$ as a true fraction, where $C_1 \sim C_e$ express all distinct prime factors of C; j-1~f-1 are respectively exponents of prime factors $C_1 \sim C_e$ orderly and j-1≥0, … f-1≥0; A_{raf} expresses the product of all distinct prime factors of A; and V_{raf} expresses the product of all distinct prime factors of V.

Undoubtedly $C_1^{j-1} \sim C_e^{f-1}/2A_{raf}V_{raf}$ as the true fraction is smaller than 1 too.

After that, even number raf (A, $2^Y V$, C) added to an infinitesimal non-negative real number such as $R \geq 0$ to turn the even number itself into $[\text{raf}(A, 2^Y V, C)]^{1+\varepsilon}$ according to the above mentioned the rule of the identical substitution.

Evidently there is $C \leq [\text{raf}(A, 2^Y V, C)]^{1+\varepsilon}$ successively.

By now, multiply $[\text{raf}(A, 2^Y V, C)]^{1+\varepsilon}$ by $C_1^{j-1} \sim C_e^{f-1}/2A_{raf}V_{raf}$, then there is still $C \leq C_1^{j-1} \sim C_e^{f-1}/2A_{raf}V_{raf}\,[\text{raf}(A, 2^Y V, C)]^{1+\varepsilon}$.

Also let $C_\varepsilon = C_1^{j-1} \sim C_e^{f-1}/2A_{raf}V_{raf}$, we get $C \leq C_\varepsilon\,[\text{raf}(A, 2^Y V, C)]^{1+\varepsilon}$.

Manifestly when $R = 0$, it has $\varepsilon = 0$, and there is $C = C_\varepsilon\,[\text{raf}(A, 2^Y V, C)]^{1+\varepsilon}$, yet when $R>0$, it has $\varepsilon > 0$, and there is $C < C_\varepsilon\,[\text{raf}(A, 2^Y V, C)]^{1+\varepsilon}$.

(3). For inequality $2^{X+1}S \geq \text{raf}(A, B, 2^{X+1}S)$ in which case $A+B=2^X S$ satisfying $\text{gcf}(A, B, 2^{X+1}S) = 1$, $2^{X+1}S$ divided by $\text{raf}(A, B, 2^{X+1}S)$ is equal to $2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}$ as a false fraction, where $S_1 \sim S_n$ express all distinct prime factors of S; t-1~m-1 are respectively exponents of prime factors $S_1 \sim S_n$ orderly and $t-1 \geq 0, \ldots\ m-1 \geq 0$; A_{raf} expresses the product of all distinct prime factors of A; and B_{raf} expresses the product of all distinct prime factors of B.

Undoubtedly $2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}$ as the false fraction is greater than 1.

After that, even number $\text{raf}(A, B, 2^{X+1}S)$ added to an infinitesimal non-negative real number such as $R \geq 0$ to turn the even number itself into $[\text{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$ according to the above mentioned the rule of the identical substitution.

By now, multiply $[\text{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$ by $2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}$, then there is $2^{X+1}S \leq 2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}\,[\text{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$.

Let $C_\varepsilon = 2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}$, we get $2^{X+1}S \le C_\varepsilon\,[raf\,(A, B, 2^{X+1}S)]^{1+\varepsilon}$.

Manifestly when R=0, it has ε =0, and there is $2^{X+1}S = C_\varepsilon\,[raf\,(A, B, 2^{X+1}S)]^{1+\varepsilon}$, yet when R>0, it has ε >0, and there is $2^{X+1}S < C_\varepsilon\,[raf\,(A, B, 2^{X+1}S)]^{1+\varepsilon}$.

(4). For inequality $C \ge raf\,(A, 2^Y V, C)$ in which case $A+2^Y V = C$ satisfying gcf $(A, 2^Y V, C) = 1$, C divided by raf $(A, 2^Y V, C)$ is equal to $C_1^{j-1} \sim C_e^{f-1}/2A_{raf}V_{raf}$ as a false fraction, where $C_1 \sim C_e$ express all distinct prime factors of C; j-1~f-1 are respectively exponents of prime factors $C_1 \sim C_e$ orderly and j-1≥0, … f-1≥0; A_{raf} expresses the product of all distinct prime factors of A; and V_{raf} expresses the product of all distinct prime factors of V.

Undoubtedly $C_1^{j-1} \sim C_e^{f-1}/2A_{raf}V_{raf}$ as the false fraction is greater than 1.

After that, even number raf $(A, 2^Y V, C)$ added to an infinitesimal non-negative real number such as R ≥ 0 to turn the even number itself into $[raf\,(A, 2^Y V, C)]^{1+\varepsilon}$ according to the above mentioned the rule of the identical substitution.

By now, multiply $[raf\,(A, 2^Y V, C)]^{1+\varepsilon}$ by $C_1^{j-1} \sim C_e^{f-1}/2A_{raf}V_{raf}$, then there is $C \le C_1^{j-1} \sim C_e^{f-1}/2A_{raf}V_{raf}\,[raf\,(A, 2^Y V, C)]^{1+\varepsilon}$.

Let $C_\varepsilon = C_1^{j-1} \sim C_e^{f-1}/2A_{raf}V_{raf}$, we get $C \le C_\varepsilon\,[raf\,(A, 2^Y V, C)]^{1+\varepsilon}$.

Manifestly when R= 0, it has ε = 0, and there is $C = C_\varepsilon\,[raf\,(A, 2^Y V, C)]^{1+\varepsilon}$, yet when R>0, it has ε >0, and there is $C < C_\varepsilon\,[raf\,(A, 2^Y V, C)]^{1+\varepsilon}$.

We have concluded $C_\varepsilon = 2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}$ and $C_\varepsilon = C_1^{j-1} \sim C_e^{f-1}/2A_{raf}V_{raf}$ in the preceding proof. Thus it can be seen, C_ε in each of aforementioned four inequalities is a constant because it consists of known numbers.

Besides, for an infinitesimal non-negative real number $R \geq 0$, actually, it is merely comparatively speaking, if raf (A, B, $2^{X+1}S$) or raf (A, $2^{Y}V$, C) is an infinite-great positive even number, then regard any concrete positive real number as R, it is still an infinitesimal non-negative real number such as $R=21113999919998999961999299992387641543265499999999999999999$ $72212347888618764972287683516511155643786543765678\,2015.31115\sqrt{2}\pi$.

To sum up, we have proven that there are both $2^{X+1}S \leq C_{\varepsilon}$ [raf (A, B, $2^{X+1}S$)]$^{1+\varepsilon}$ in which case $A+B=2^{X}S$ satisfying gcf (A, B, $2^{X+1}S$) =1, and $C \leq C_{\varepsilon}$ [raf (A, $2^{Y}V$, C)]$^{1+\varepsilon}$ in which case $A+2^{Y}V=C$ satisfying gcf (A, $2^{Y}V$, C) =1, where $X \geq 1$, $Y \geq 1$, S is an odd number ≥ 1, and C is an odd number ≥ 1.

The proof is completed by now. As a consequence, the ABC conjecture is tenable.